Electrical Circuit- I

Course Code: EEE 0713-1101
Course Title: Electrical Circuit-I

Noor Md. Shahriar
Senior Lecturer & Deputy Head, Dept. of EEE
University of Global Village (UGV), Barishal

Electrical Circuit I (EEE 0713-1101)

3 Credit Course

Class:	**17 weeks (2 classes per week) =34 Hours**
Preparation Leave (PL):	02 weeks
Exam:	04 weeks
Results:	02 weeks
Total:	**25 Weeks**

Attendance:

Students with more than or equal to 70% attendance in this course will be eligible to sit for the Semester End Examination (SEE). SEE is mandatory for all students.

SYNOPSIS / RATIONALE

Electrical Circuit 1 forms the foundation of electrical engineering education, introducing students to the principles and analysis techniques of electrical circuits. This course is essential for understanding the behavior and properties of various electrical components and their interactions within circuits. By mastering the fundamentals of circuit analysis, students develop problem-solving skills crucial for tackling more complex electrical engineering topics. Understanding this course is essential for students pursuing careers in electrical engineering, providing them with the necessary knowledge to design, analyze, and troubleshoot electrical circuits in diverse applications.

Course Objective

- Understand basic electrical circuit concepts and laws.
- Analyze simple resistive circuits using Ohm's Law and Kirchhoff's Laws.
- Apply nodal and mesh analysis techniques to analyze complex circuits.
- Learn the use of circuit simulation software for analysis and design.
- Develop skills in troubleshooting and debugging electrical circuits.

Course Learning Outcome (CLO)

Serial No.	Course Learning Outcome (CLO)	**Blooms Taxonomy Level**
CLO-1	Explain the basic operation of different circuit parameters and their characteristics to solve complex engineering problems.	**1,2 Remembering, Understanding**
CLO-2	Compare different laws and circuit analysis.	**3 Applying**
CLO-3	Understand the impact and advantage of electrical devices on societal and environmental aspects.	**4 Analyzing**
CLO-4	Apply the knowledge of designing circuits and to solve real life engineering problems such as blood vessels.	**2,5,6 Understanding, Evaluating, Creating**

ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

Bloom's Category Marks (out of 90)	Tests (45)	Quizzes (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	08	08	Bloom's Affective Domain: (Attitude or will) Attendance: 15 Copy or attempt to copy: −10 Late Assignment: -10
Understand	08	07	
Apply	08		
Analyze	08		
Evaluate	08		
Create	05		

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	10
Understand	10
Apply	10
Analyze	10
Evaluate	10
Create	10

EEE-0713-1101 **Lectures:2 hours/week**

Electrical Circuit I **Credits: 3**

Serial No.	Content of Course	Hours	CLOs
1	Circuit Variables and Elements: Voltage, current, power, energy, independent and dependent sources, resistance.	8	CLO-1
2	Basic Laws: Ohm's law, Kirchhoff's current and voltage laws, simple resistive circuits, series and parallel circuits, voltage and current division, Wye-Delta transformation, linearity property.	8	CLO-2, CLO-3
3	Techniques of Circuit Analysis: Nodal and mesh analysis including super node and super mesh.	9	CLO-3. CLO-4
4	Network Theorems: Source transformation, superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem, reciprocity theorem, and Millman's theorem.	9	CLO-1, CLO-4

Outline of Course

- 1 Current, Voltage, Power & Energy
- 2 Ohm’s Law & Kirchhoff's Law
- 3 Series, Parallel, Y,Δ, CDR, VDR
- 4 Nodal Analysis
- 5 Nodal Analysis with Source
- 6 Mesh Analysis
- 7 Source Transformation
- 8 Thevenin’s Theorem
- 9 Thevenin’s & Norton’s Theorem with Dependent Source
- 10 Maximum Power Transfer Theorem
- 11 Superposition theorem
- 12 Inductors & Capacitors
- 13 Phasor

Course Schedule

Course plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Content of Course	ASG/Quiz/Pr	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Fundamental types of energy, electrical energy discussion		Lecture, Discussion	Written Exam, Class Participation	CLO-0
2	DC & AC Generation, transmission, AC vs DC comparison		Lecture, Visual Aids, Group Discussion	Quiz, Written Exam	CLO-1
3	Electrical quantities, DC voltage, resistance, power, measuring unit, Ohm's law, problems		Lecture, Practical Examples	Problem Solving	CLO-1
4	Kirchhoff's Current Law (KCL), Kirchhoff's Voltage Law (KVL)		Lecture, Group Problem Solving	Quiz, Written Exam	CLO-2
	Quiz-1	Quiz-01			
	Mid Term Exam				

Course Schedule

Course plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Content of Course	ASG/Quiz/Pr	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
5	Series circuits, voltage, current, power, energy problems		Lecture, Case Studies, Problem Practice	Assignment, Problem Solving	CLO-3
6	Parallel circuits, voltage divider rule, problem-solving	ASG	Lecture, Hands-on Examples	Quiz, Problem-Solving Exam	CLO-3
7	Current divider rule, parallel circuits with multiple branches	Assignment	Lecture, Group Activities	Assignment, Oral Presentation	CLO-4
8	Wye-Delta transformation and simplification		Lecture, Board Work, Practical Problems	Problem Solving, Classwork	CLO-3
9	Nodal analysis, theory and problems		Lecture, Visual Presentation, Examples	Written Exam, Assignment	CLO-3
10	Mesh analysis, theory and problems		Lecture, Practice Problems, Case Studies	Written Exam, Problem Solving	CLO-4
	Quiz-2	Quiz-02			

Course Schedule

SL	Content of Course	ASG/Quiz/Pr	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
	Mid Term Exam				
11	Theory and problem solution on Thevenin's Theorem		Lecture, Whiteboard Examples, Problem-Solving	Quiz, Written Exam	CLO-2
12	Theory and problem solution on Norton's Theorem		Lecture, Group Problem Solving	Assignment, Problem Solving	CLO-2
13	Theory and problem solution on Superposition Theorem		Lecture, Case Studies, Group Activities	Quiz, Problem Solving	CLO-3
14	Theory and problem solution on Maximum Power Transfer Theorem	Assignment-2	Lecture, Practical Examples, Problem Solving	Problem-Solving Exam	CLO-3

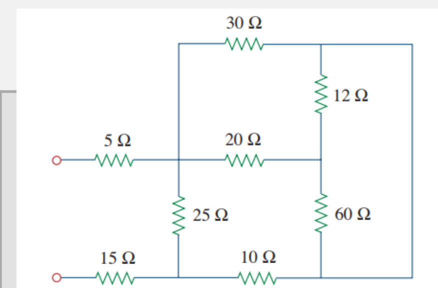
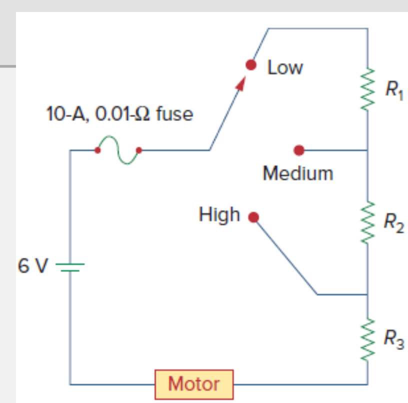
Course Schedule

SL	Content of Course	ASG/Quiz/Pr	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
	Quiz-3	Quiz-03			
15	Inductors and their series-parallel combinations		Lecture, Visual Demonstration	Problem Solving, Assignment	CLO-1
16	Capacitors and their series-parallel combinations		Lecture, Visual Aids, Problem-Solving	Written Exam, Class Participation	CLO-1
17	Phasor Analysis		Lecture, Phasor Diagram Demonstration	Quiz, Written Exam	CLO-2
	Mid Term Exam				

Reference books

1. **“Fundamentals of Electric Circuits"** by Charles K. Alexander, Matthew N.O. Sadiku.
2. **"Electric Circuits"** by James W. Nilsson, Susan A. Riedel.
3. **"Engineering Circuit Analysis"** by William H. Hayt, Jack E. Kemmerly, Steven M. Durbin.
4. **"Introductory Circuit Analysis"** by Robert L. Boylestad.
5. **"Circuit Analysis: Theory and Practice"** by Allan H. Robbins, Wilhelm C. Miller.
6. **"The Analysis and Design of Linear Circuits"** by Roland E. Thomas, Albert J. Rosa, Gregory J. Toussaint.

A Sample Question

Sl. no.	Question	Figure	Marks	CO
1.(a)	While learning to solve an electrical circuit one of the most important laws is Kirchhoff's current law. State that law.		2	C1
(b)	The current entering the positive terminal of a device is i(t) = 6e−2t mA and the voltage across the device is v(t) = 10di⁄dtV. (i) Find the charge delivered to the device between t = 0 and t = 2 s. (ii) Calculate the power absorbed. (iii) Determine the energy absorbed in 3 s.		5	C1, C2
(c)	Let's assume you found a bunch of tangled wires which is illustrated in the figure below. When you measured the resistance of two untangled ends with a multimeter, you found R_{eq}. Calculate the value of $R_{eq.}$		4	C3,C4
(d)	The circuit in the following figure is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 2 Ω. Determine the series dropping resistances R1, R2, and R3.		4	C5, C6

Bloom Taxonomy Cognitive Domain Action Verbs

Remembering (C1)	Choose • Define • Find • How • Label • List • Match • Name • Omit • Recall • Relate • Select • Show • Spell • Tell • What • When • Where • Which • Who • Why
Understanding (C2)	Classify • Compare • Contrast • Demonstrate • Explain • Extend • Illustrate • Infer • Interpret • Outline • Relate • Rephrase • Show • Summarize • Translate
Applying (C3)	Apply • Build • Choose • Construct • Develop • Experiment with • Identify • Interview • Make use of • Model • Organize • Plan • Select • Solve • Utilize
Analyzing (C4)	Analyze • Assume • Categorize • Classify • Compare • Conclusion • Contrast • Discover • Dissect • Distinguish • Divide • Examine • Function • Inference • Inspect • List • Motive • Relationships • Simplify • Survey • Take part in • Test for • Theme
Evaluating (C5)	Agree • Appraise • Assess • Award • Choose • Compare • Conclude • Criteria • Criticize • Decide • Deduct • Defend • Determine • Disprove • Estimate • Evaluate • Explain • Importance • Influence • Interpret • Judge • Justify • Mark • Measure • Opinion • Perceive • Prioritize • Prove • Rate • Recommend • Rule on • Select • Support • Value
Creating (C6)	Adapt • Build • Change • Choose • Combine • Compile • Compose • Construct • Create • Delete • Design • Develop • Discuss • Elaborate • Estimate • Formulate • Happen • Imagine • Improve • Invent • Make up • Maximize • Minimize • Modify • Original • Originate • Plan • Predict • Propose • Solution • Solve • Suppose • Test • Theory

Course Code: EEE 0713-1101

Course Title: Electrical Circuit -I

Week-1

Page (13-31)

What are Current and Voltage?

Overview

In this part, we will cover:

- Definitions of current and voltage
- Hydraulic analogies to current and voltage
- Reference polarities and actual polarities

Current: Formal Definition

- Current is the net flow of charges, per time, past an arbitrary "plane" in some kind of electrical device.
- We will only be concerned with the flow of positive charges. A negative charge moving to the right is conceptually the same as a positive charge moving to the left.
- Mathematically, current is expressed as…

$$i = \frac{dq}{dt}$$

Current, typically in Amperes [A]

Charge, typically in Coulombs [C]

Time, typically in seconds [s]

The Ampere

- The unit of current is the [Ampere], which is a flow of 1 [Coulomb] of charge per [second], or:

$$1[A] = 1[Coul/sec]$$

- Remember that current is defined in terms of the flow of **positive** charges.

One [coulomb] of positive charges per [second] flowing from left to right

- is equivalent to -

one [coulomb] of negative charges per [second] flowing from right to left.

Voltage: Formal Definition

- When we move a charge in the presence of other charges, energy is transferred. Voltage is the change in potential energy, per charge, as we move between two points; it is a ***potential difference***.
- Mathematically, this is expressed as…

$$v = \frac{dw}{dq}$$

Voltage, typically in Volts [V]

Energy, typically in Joules [J]

Charge, typically in Coulombs [C]

What is a [Volt]?

- The unit of voltage is the [Volt]. A [Volt] is defined as a [Joule per Coulomb].
- Remember that voltage is defined in terms of the energy gained or lost by the movement of **positive** charges.

One [Joule] of energy is lost from an electric system when a [Coulomb] of positive charges moves from one potential to another potential that is one [Volt] lower.

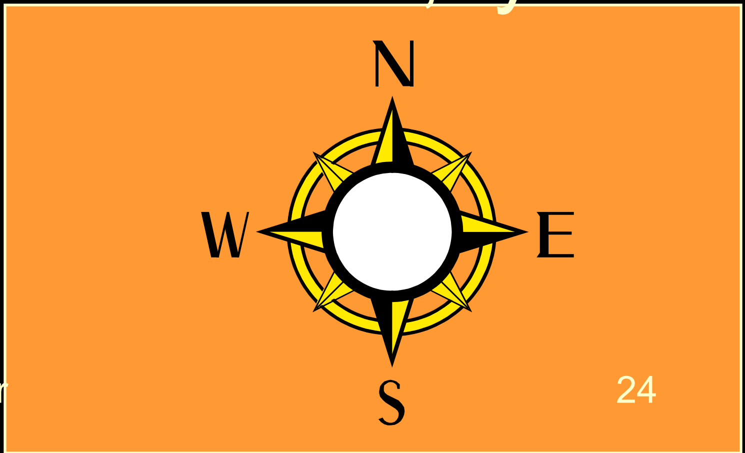
Polarities

It is extremely important that we know the polarity, or the sign, of the voltages and currents we use. Which way is the current flowing? Where is the potential higher? To keep track of these things, two concepts are used:

1. Reference polarities, and
2. Actual polarities.

Reference Polarities

The reference polarity is a direction chosen for the purposes of keeping track. It is like picking North as your reference direction, and keeping track of your direction of travel by saying that you are moving in a direction of 135 degrees. This only tells you where you are going with respect to north, your reference direction.



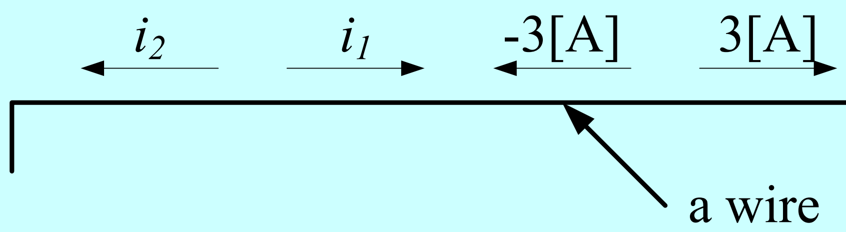
Actual Polarity

The actual polarity is the direction something is actually going. We have only two possible directions for current and voltage.

- If the actual polarity is the same direction as the reference polarity, we use a positive sign for the value of that quantity.
- If the actual polarity is the opposite direction from the reference polarity, we use a negative sign for the value of that quantity.

Polarities for Currents

- For current, the reference polarity is given by an arrow.
- The actual polarity is indicated by a value that is associated with that arrow.
- In the diagram below, the currents i_1 and i_2 are not defined until the arrows are shown.
- Use lowercase variables for current. Uppercase subscripts are preferred.



i_1 = 3[A]

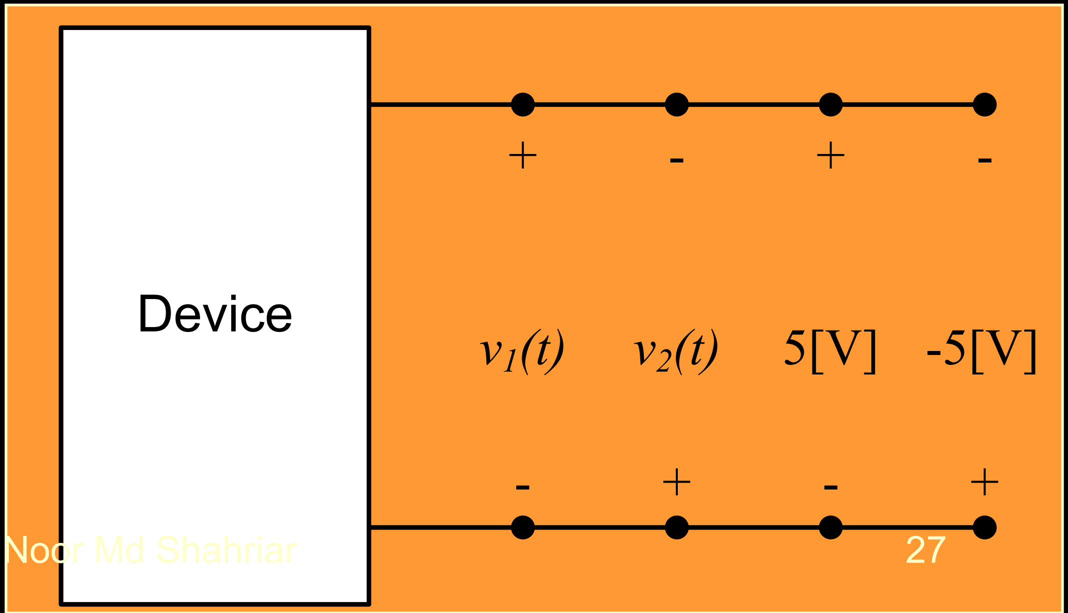
i_2 = -3[A]

These are all different ways to show the same thing, a current of 3 [Coulombs] per [second] of positive charges moving from left to right through this wire.

The arrow shows a reference polarity, and the sign of the number that goes with that arrow shows the actual polarity.

Polarities for Voltages

- For voltage, the reference polarity is given by a variable v with a subscript, and a + sign and a – sign, at or near the two points involved.
- The actual polarity is indicated by the sign of the value of that variable v, or by the sign of the value that is placed between the + and - symbols.
- In the diagram below, the voltages v_1 and v_2 are not defined until the + and – symbols are shown.
- Use lowercase variables for voltage. Uppercase subscripts are preferred.

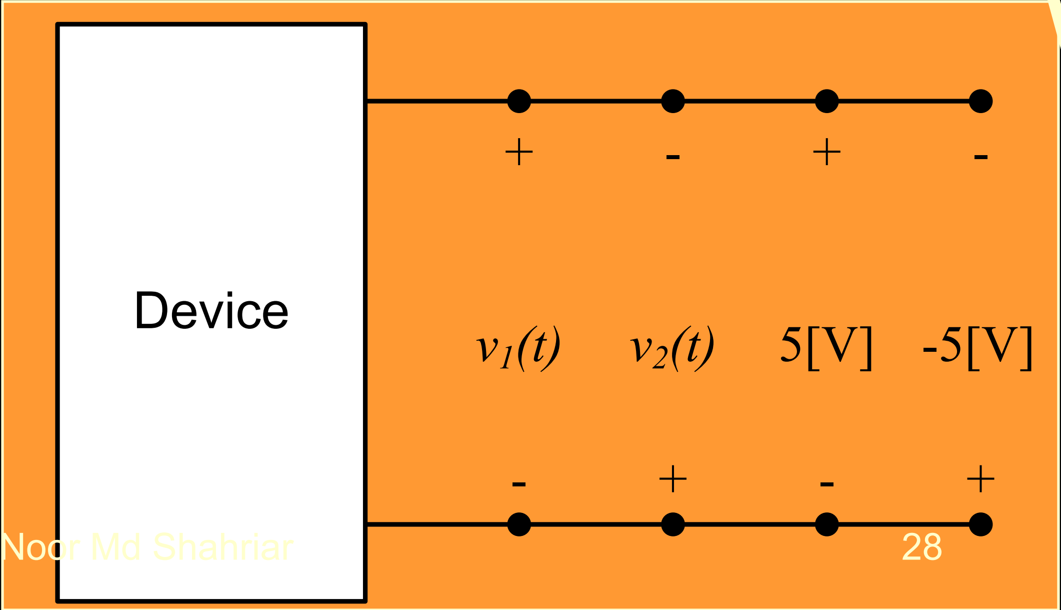


Defining Voltages

- For voltage, the reference polarity is given by a variable v with a subscript, and a + sign and a – sign, at or near the two points involved.

In the diagram below, the voltages v_1 and v_2 are not defined until the + and – symbols are shown.

In this case,
$v_1 = 5[V]$
and
$v_2 = -5[V]$.
These four labels all mean the same thing.



Energy

This is the definition found in most dictionaries, although it is dangerous to use nontechnical dictionaries to define technical terms. For example, some dictionaries list force and power as synonyms for energy, and we will not do that!

- Energy is the ability or the capacity to do work.
- It is a quantity that can take on many forms, among them heat, light, sound, motion of objects with mass.

Joule Definition

- The unit for energy that we use is the [Joule], abbreviated as [J].
- A [Joule] is a [Newton-meter].
- In everything that we do in circuit analysis, energy will be conserved.
- One of the key concerns in circuit analysis is this: Is a device, object, or element absorbing energy or delivering energy?

Go back to Overview slide.

Week -2

Page- (32-54)

Power

- Power is the rate of change of the energy, with time. It is the rate at which the energy is absorbed or delivered.
- Again, a key concern is this: Is power being absorbed or delivered? We will show a way to answer this question.
- Mathematically, power is defined as:

$$p = \frac{dw}{dt}$$

Power, typically in Watts [W]

Energy, typically in Joules [J]

Time, typically in seconds [s]

Watt Definition

- A [Watt] is defined as a [Joule per second].
- We use a capital [W] for this unit.
- Light bulbs are rated in [W]. Thus, a 100[W] light bulb is one that absorbs 100[Joules] every [second] that it is turned on.

Noor Md Shahriar

Power from Voltage and Current

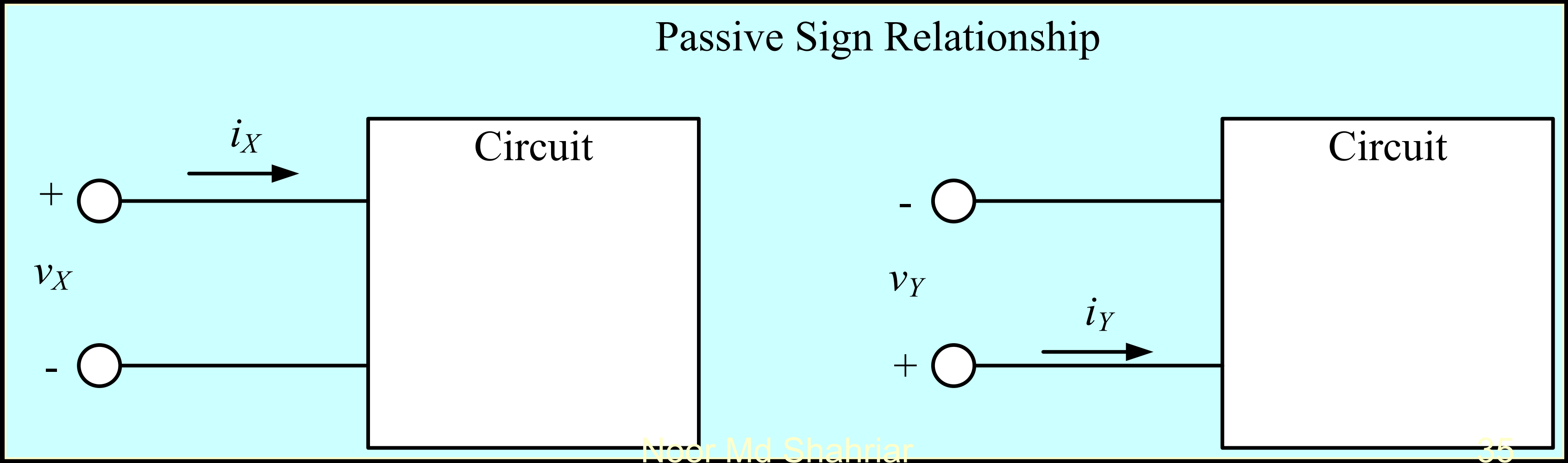
Power can be found from the voltage and current, as shown below. Note that if voltage is given in [V], and current in [A], power will come out in [W].

$$p = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt} = vi$$

Go back to Overview slide.

Passive Sign Relationship – Discussion of the Definition

- The two circuits below have reference polarities which are in the passive sign relationship.
- Notice that although they look different, these two circuits have the same **relationship** between the polarities of the voltage and current.



Using Sign Relationships for Power Direction – The Rules

We will use the **sign relationships** to determine whether power is absorbed, or power is delivered.

- When we use the **passive sign relationship** to assign reference polarities, *vi* gives the power absorbed, and –*vi* gives the power delivered.
- When we use the **active sign relationship** to assign reference polarities, *vi* gives the power delivered, and –*vi* gives the power absorbed.

Using Sign Relationships for Power Direction – The Rules

We will use the **sign relationships** to determine whether power is absorbed, or power is delivered.

- When we use the **passive sign relationship** to assign reference polarities, *vi* gives the power absorbed, and –*vi* gives the power delivered.
- When we use the **active sign relationship** to assign reference polarities, *vi* gives the power delivered, and –*vi* gives the power absorbed.

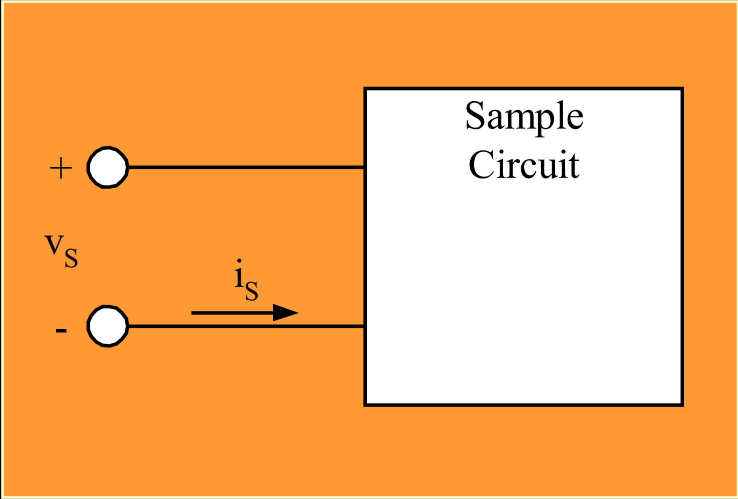
	Passive Relationship	**Active Relationship**
Power absorbed	$p_{ABS} = vi$	$p_{ABS} = -vi$
Power delivered	$p_{DEL} = -vi$	$p_{DEL} = vi$

Example of Using the Power Direction Table – **Step 1**

We want an expression for the power absorbed by this Sample Circuit.

1. **Determine which sign relationship has been used to assign reference polarities for this Sample Circuit.**

	Passive Relationship	Active Relationship
Power absorbed	$p_{ABS} = vi$	$p_{ABS} = -vi$
Power delivered	$p_{DEL} = -vi$	$p_{DEL} = vi$



Example of Using the Power Direction Table – **Step 2**

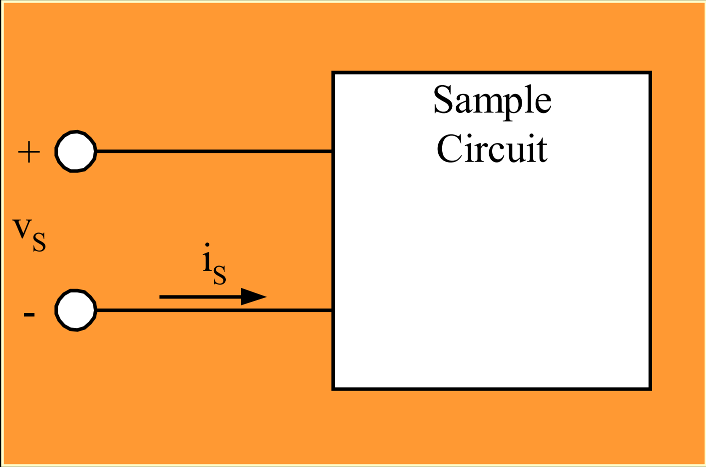
We want an expression for the power absorbed by this Sample Circuit.

1. Determine which sign relationship has been used.

This is the active sign relationship.

2. **Next, we find the cell that is of interest to us here in the table. It is highlighted in red below.**

	Passive Relationship	Active Relationship
Power absorbed	$p_{ABS} = vi$	$p_{ABS} = -vi$
Power delivered	$p_{DEL} = -vi$	$p_{DEL} = vi$



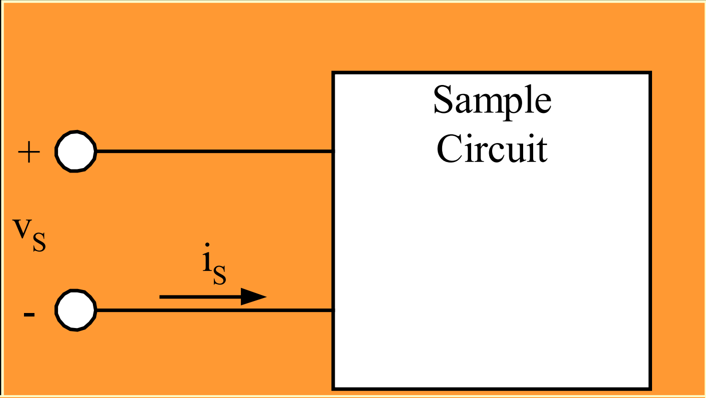
Example of Using the Power Direction Table – **Step 3**

We want an expression for the power absorbed by this Sample Circuit.

1. Determine which sign relationship has been used.
2. Find the cell that is of interest to us here in the table. This cell is highlighted in red.
3. **Thus, we write** $p_{ABS.BY.CIR} = -v_S i_S$.

Go back to Overview slide.

	Passive Relationship	Active Relationship
Power absorbed	$p_{ABS} = vi$	$p_{ABS} = -vi$
Power delivered	$p_{DEL} = -vi$	$p_{DEL} = vi$



This is the active sign **Relationship**.

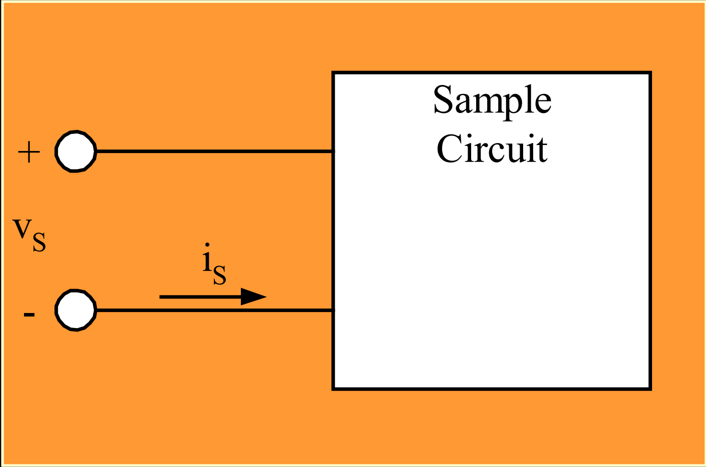
Example of Using the Power Direction Table – **Note on Notation**

We want an expression for the power absorbed by this Sample Circuit.

1. Determine which sign **relationship** has been used.
2. Find the cell that is of interest to us here in the table. This cell is highlighted in red.
3. **Thus, we write** $p_{ABS.BY.CIR} = -v_S i_S$.

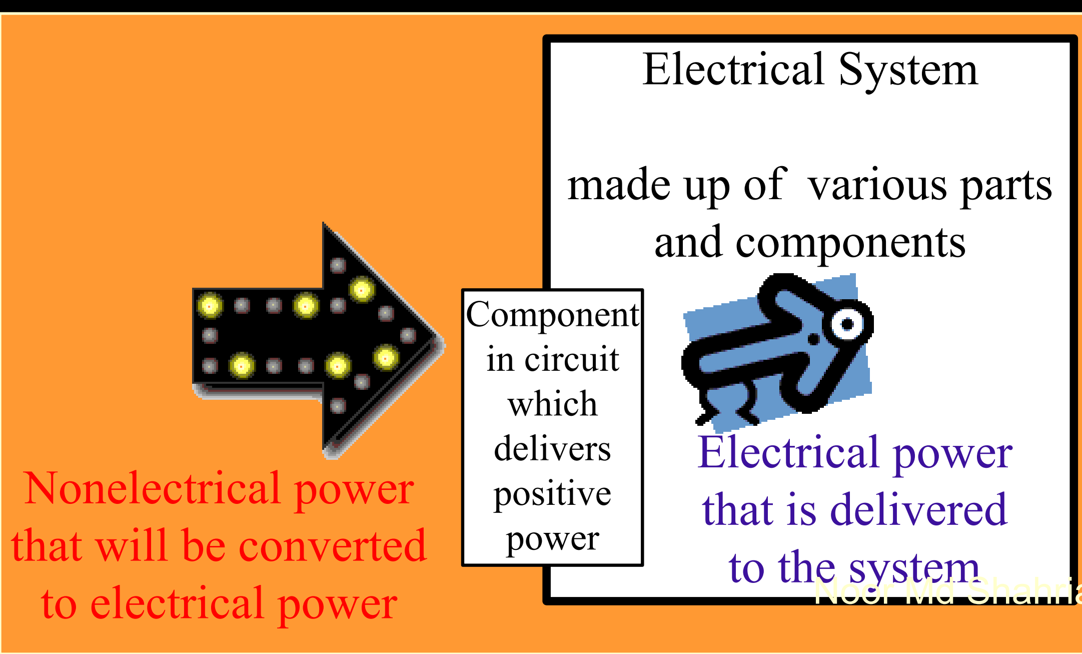
Go back to Overview slide.

In your power expressions, always use lowercase variables for power. Uppercase subscripts are preferred. Always use a two-part subscript for all power and energy variables. Indicate whether abs or del, and by what.



Power Directions Assumption #1

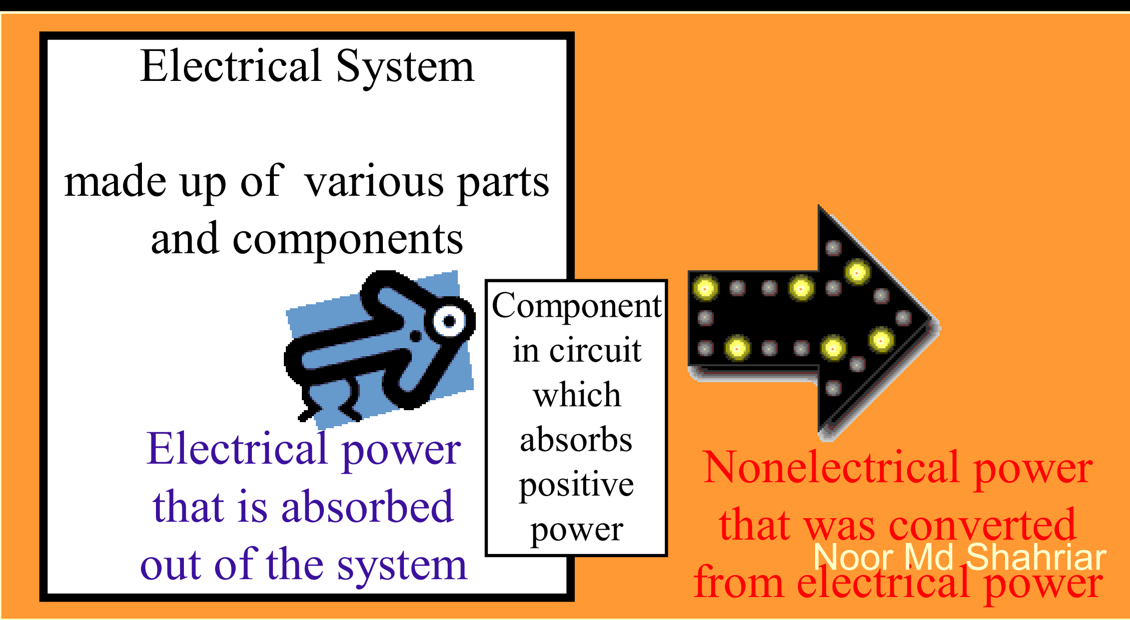
- So, a key assumption is that when we say power delivered, we mean that there is power taken from someplace else, converted and delivered to the electrical system. This is the how this approach gives us direction.
- For example, in a battery, this power comes from chemical power in the battery, and is converted to electrical power.
- Remember that energy is conserved, and therefore power will be conserved as well.



Positive power delivered by something means that power from somewhere else **enters** the electrical system as electrical power, through that something. In this diagram, the red power (nonelectrical) is being changed to the blue power (electrical).

Power Directions Assumption #2

- So, a key assumption is that when we say power absorbed, we mean that there is power from the electrical system that is converted to nonelectrical power. This is the how this approach gives us direction.
- For example, in a lightbulb, the electrical power is converted to light and heat (nonelectrical power).
- Remember that energy is conserved, and therefore power will be conserved as well.

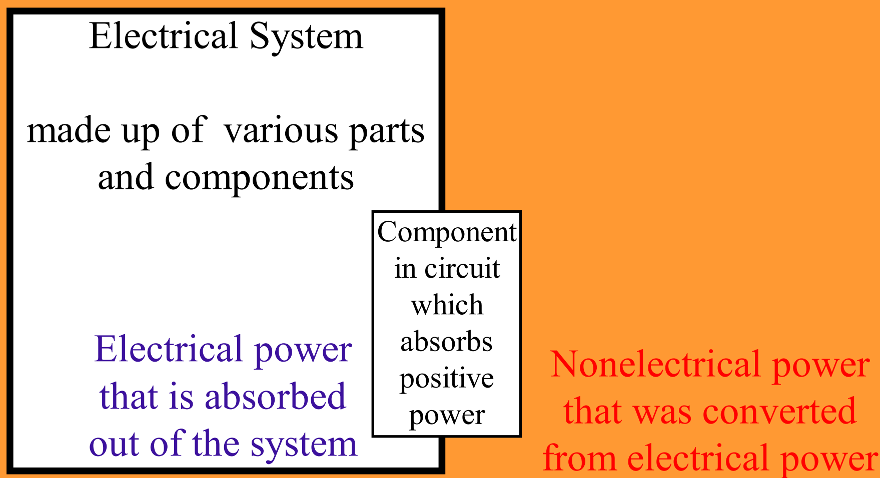


Positive power absorbed by something means that power from the electrical system **leaves** as nonelectrical power, through that something. In this diagram, the blue power (electrical) is being changed to the red power (nonelectrical).

Power Directions Terminology – Synonyms

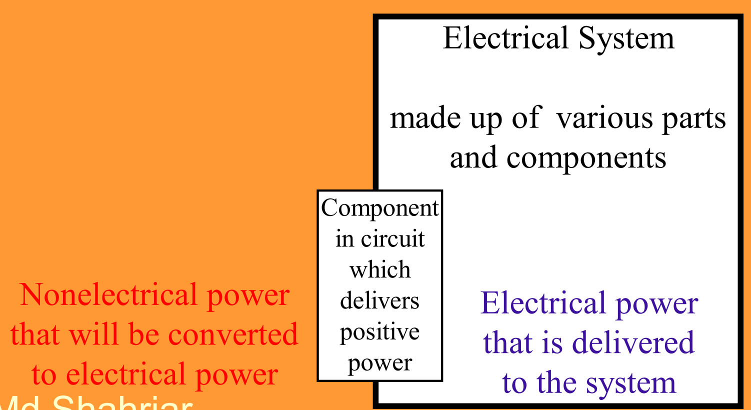
There are a number of terms that are synonyms for **power absorbed**. We may use:

- Power absorbed by
- Power consumed by
- Power delivered to
- Power provided to
- Power supplied to
- Power dissipated by



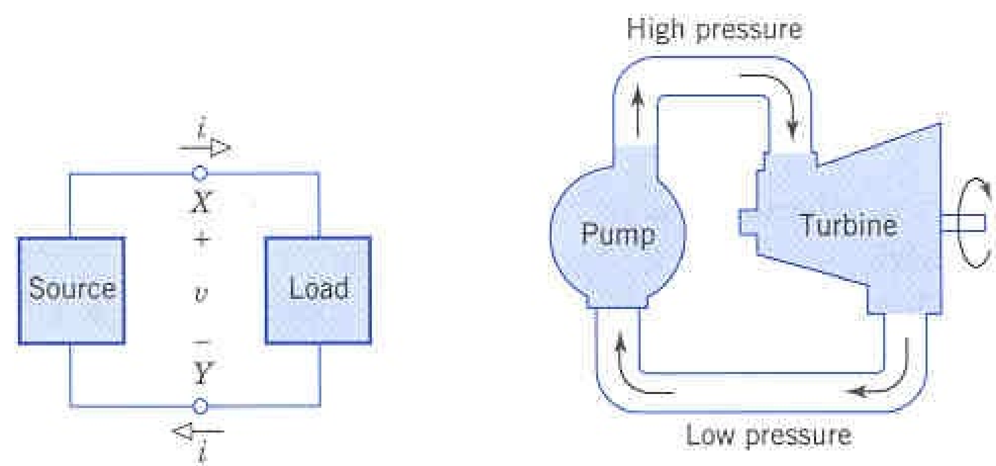
There are a number of terms that are synonyms for **power delivered**. We may use:

- Power delivered by
- Power provided by
- Power supplied by



Another Hydraulic Analogy

- Another useful hydraulic analogy that can be used to help us understand this is presented by A. Bruce Carlson in his textbook, **Circuits**, published by Brooks/Cole. The diagram, Figure 1.9, from page 11 of that textbook, is duplicated here.

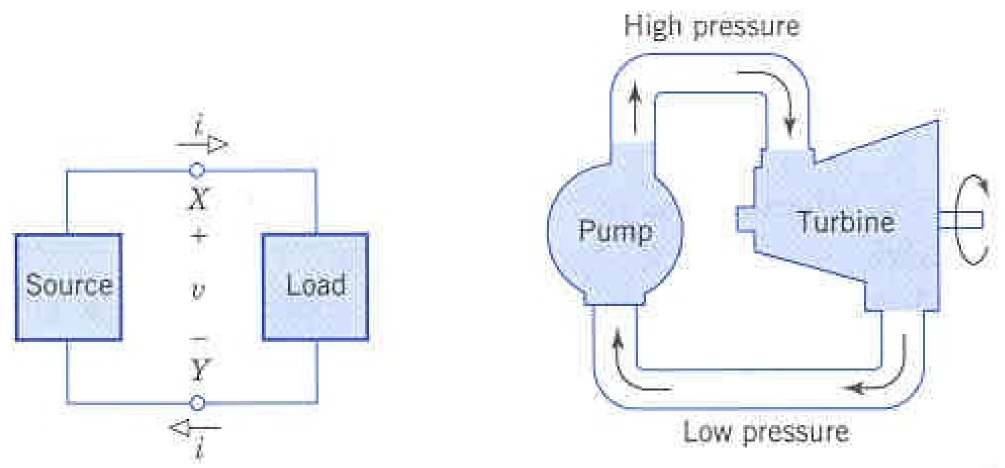


(a) Source-load circuit (b) Analogous hydraulic system

Figure 1.9

Another Hydraulic Analogy – Details

- In this analogy, the electrical circuit is shown at the left, and the hydraulic analog on the right.
- As Carlson puts it, "The pump (*source*) forces water flow (*current*) through pipes (*wires*) to drive the turbine (*load*). The water pressure (*potential*) is higher at the inlet port of the turbine than at the outlet."



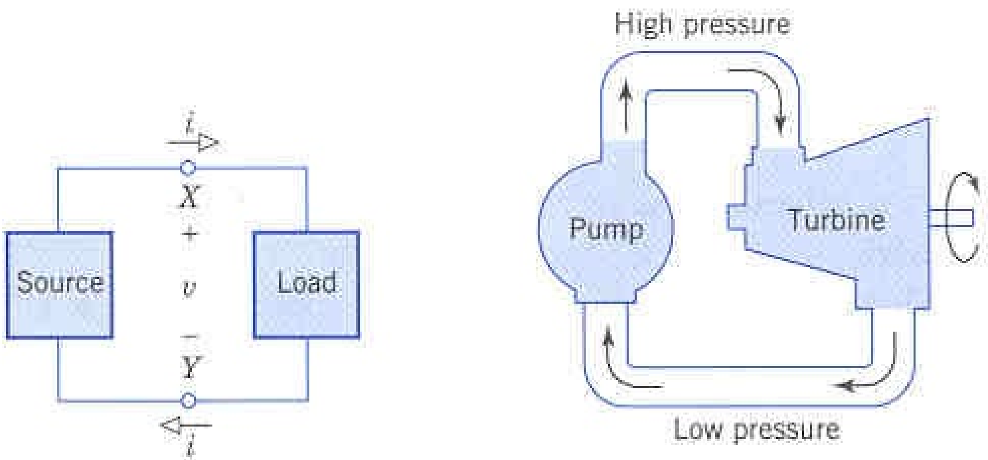
(a) Source-load circuit (b) Analogous hydraulic system

Figure 1.9

Note that the Source is given with reference polarities in the active sign relationship, and the Load with reference polarities in the passive sign relationship. As a result, in this case, since all quantities are positive, the Source delivers power, and the Load absorbs power.

Another Point on Terminology

- We always need to be careful of our context. When we say things like "the Source delivers power", we implicitly mean "the Source delivers positive power".



(a) Source-load circuit

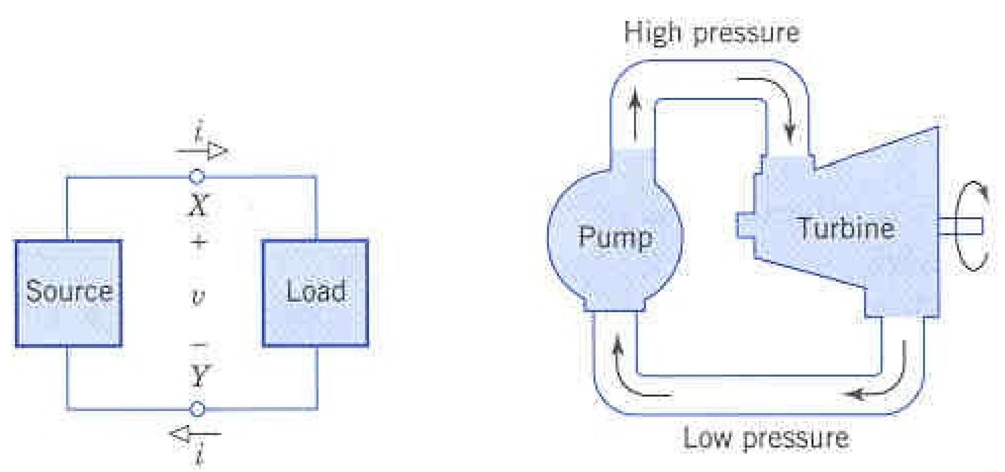
(b) Analogous hydraulic system

Figure 1.9

Note that the Source is given with reference polarities in the active sign relationship, and the Load with reference polarities in the passive sign relationship. As a result, in this case, since all quantities are positive, the Source delivers power, and the Load absorbs power.

Another Point on Terminology

- At the same time, it is also acceptable to write expressions such as $p_{ABS.BY.SOURCE}$ = -5000[W]. This is the same thing as saying that the power delivered is 5000[W].
- However, unless the context is clear, it is ambiguous to just write p = 5000[W]. Your answer must be clear, because the direction is important!



(a) Source-load circuit

(b) Analogous hydraulic system

Figure 1.9

Note that the Source is given with reference polarities in the active sign relationship, and the Load with reference polarities in the passive sign relationship. As a result, in this case, since all quantities are positive, the Source delivers power, and the Load absorbs power.

Why bother with Sign Relationships?

- Students who are new to circuits often question whether sign relationships are intended just to make something easy seem complicated. It is not so; using sign conventions helps.
- The key is that often the direction that power is moving is not known until later. We want to be able to write expressions now that will be valid no matter what the actual polarities turn out to be.
- To do this, we use sign relationships , and the actual directions come out later when we plug values in.

Go back to Overview slide.

Sample Problem

The components of a cell phone are shown in Figure 1. Assume that the charge carriers are electrons.

a) Find the power absorbed by the battery at t = 3[ms].

b) Find the energy delivered by the charger during the third [millisecond], counting [milliseconds] starting at t = 0.

c) Determine whether the electrons flowing through the charger at t = 3[ms] are gaining or losing energy. Explain your answer.

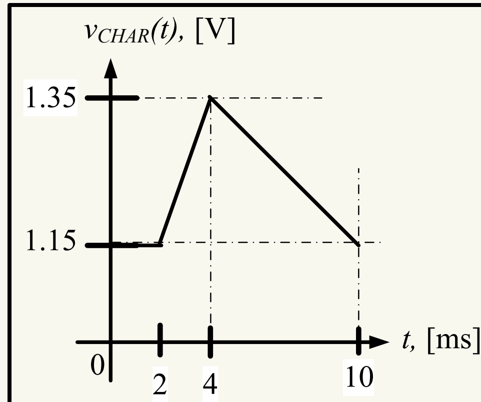


Figure 2

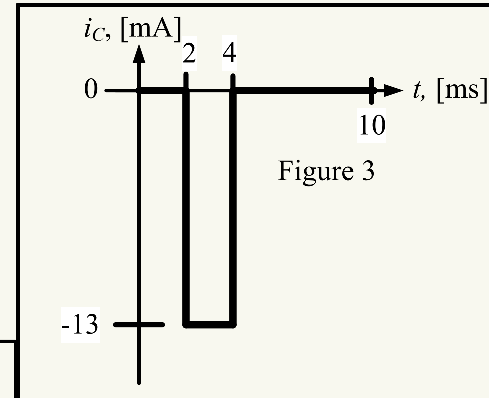


Figure 3

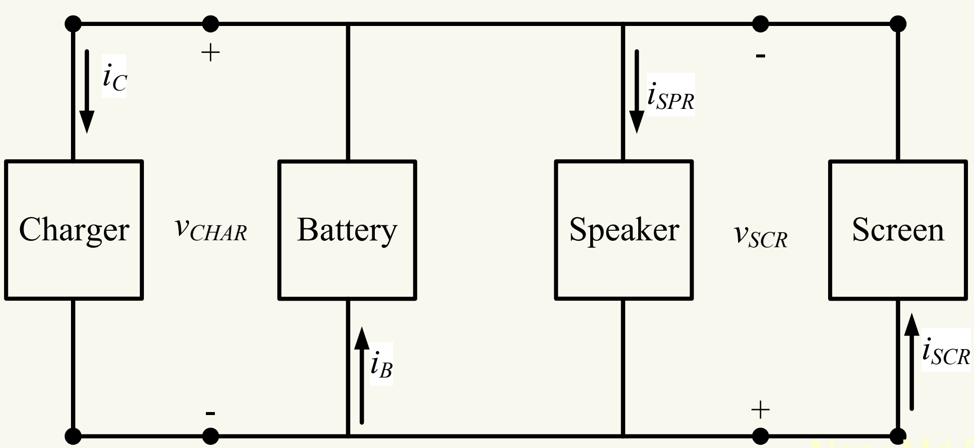


Figure 1

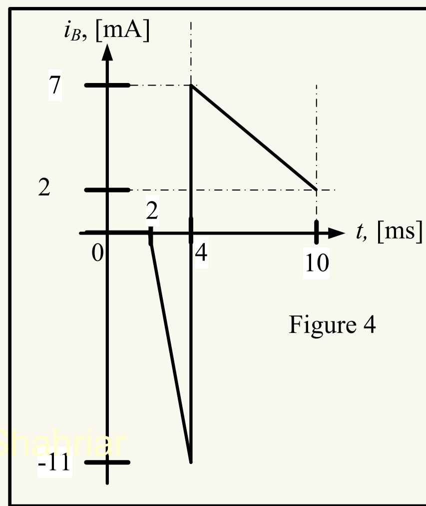


Figure 4

The components of a cell phone are shown in Figure 1. Assume that the charge carriers are electrons.

a) Find the power absorbed by the battery at t = 3[ms].

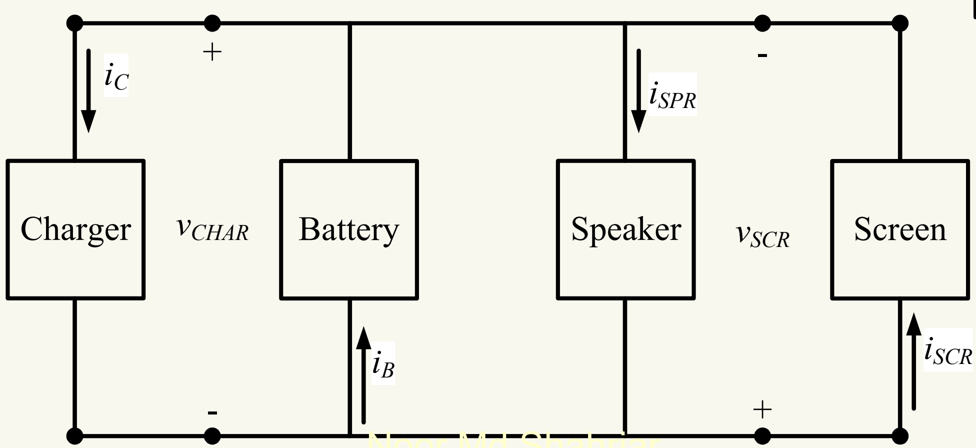


Figure 1

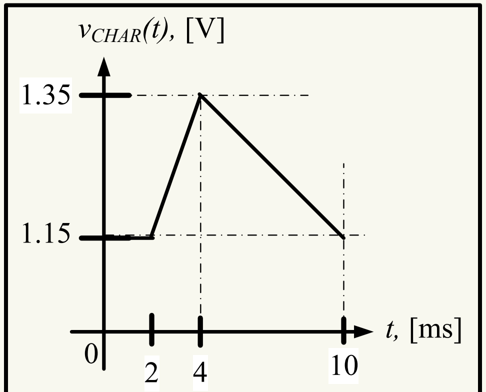


Figure 2

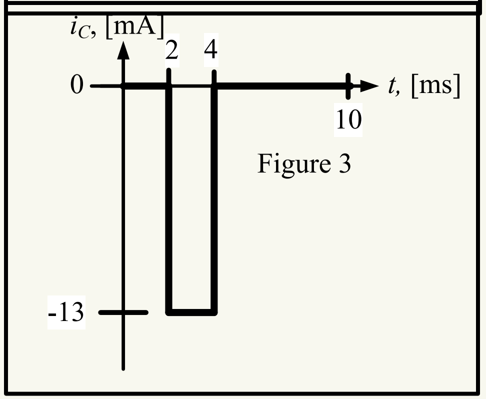


Figure 3

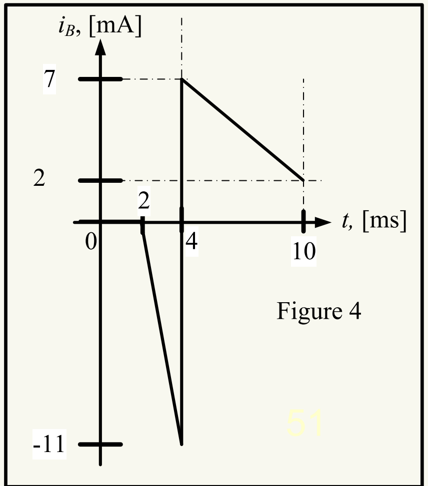


Figure 4

The components of a cell phone are shown in Figure 1. Assume that the charge carriers are electrons.

b) Find the energy delivered by the charger during the third [millisecond], counting [milliseconds] starting at t = 0.

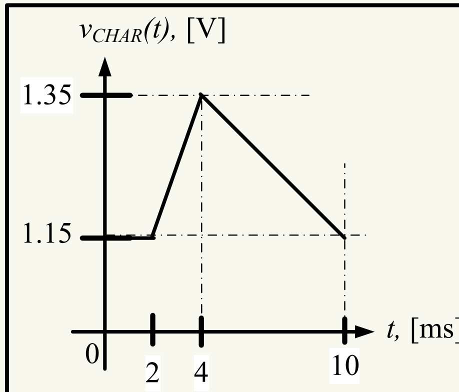


Figure 2

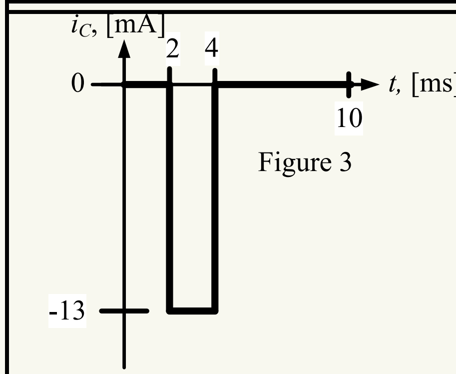


Figure 3

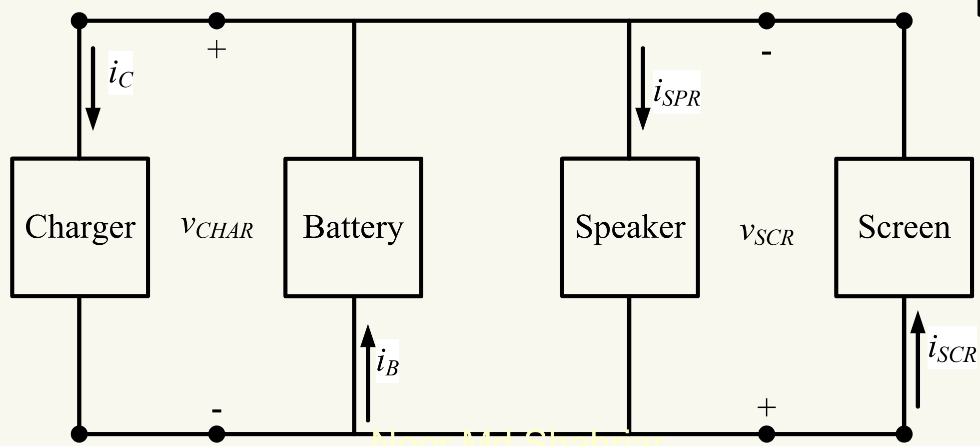


Figure 1

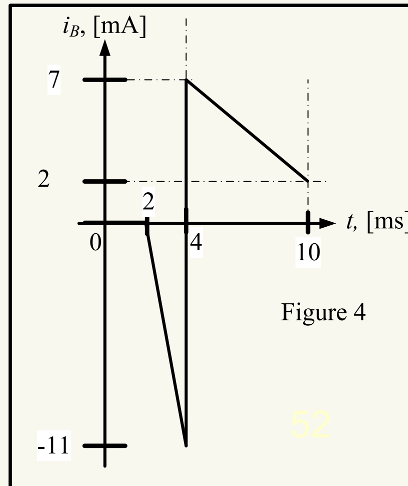


Figure 4

The components of a cell phone are shown in Figure 1. Assume that the charge carriers are electrons.

c) Determine whether the electrons flowing through the charger at *t* = 3[ms] are gaining or losing energy. Explain your answer.

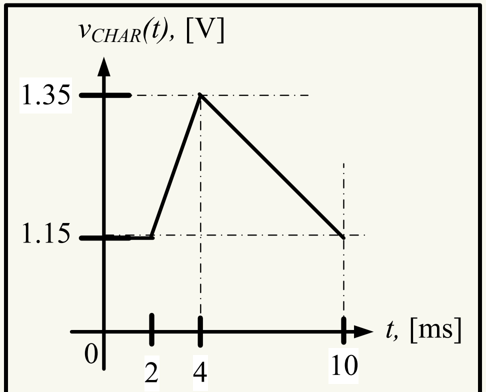


Figure 2

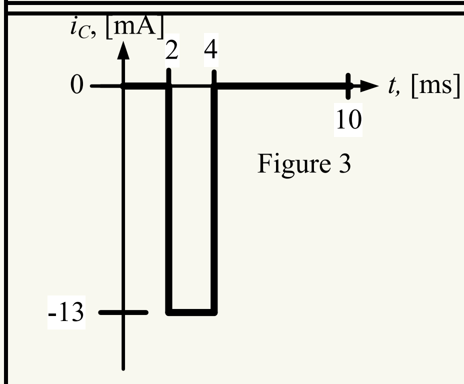


Figure 3

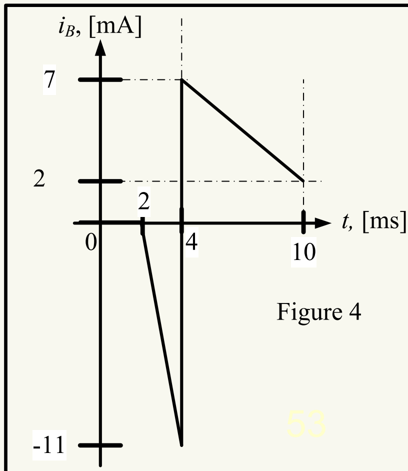


Figure 4

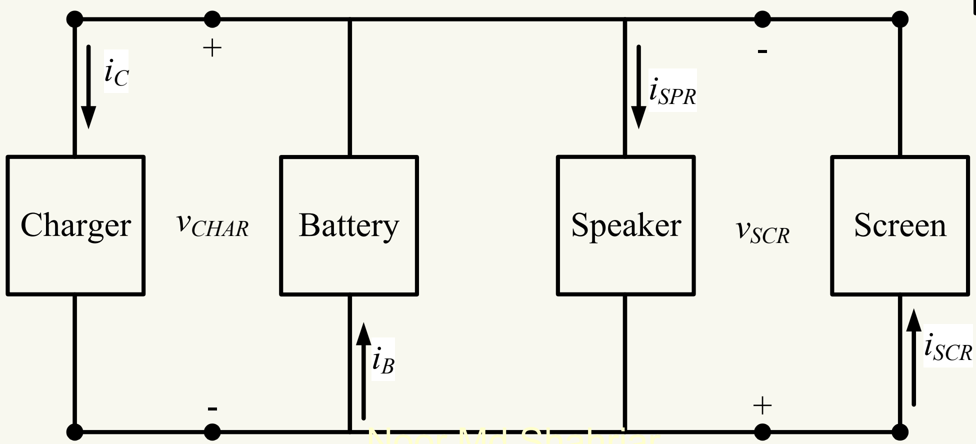
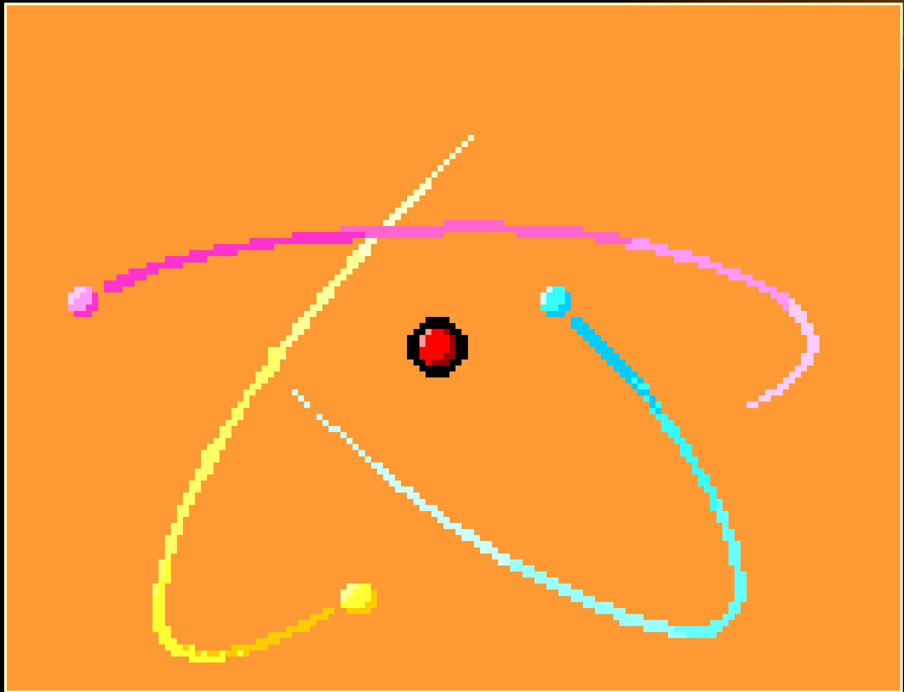


Figure 1

Week -3

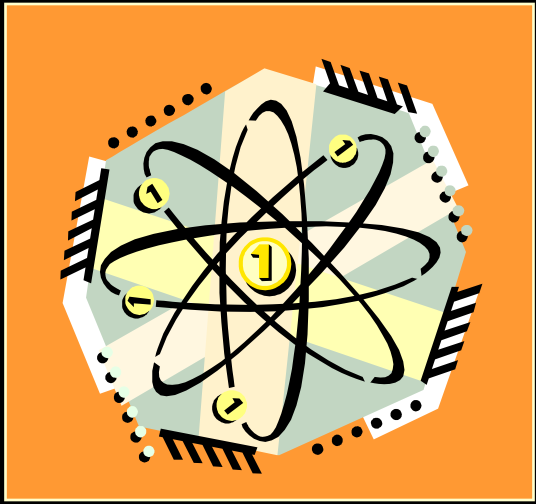
Page- (55-79)

Circuit Elements



Circuit Elements

- In circuits, we think about basic **circuit elements** that are the “building blocks” of our circuits. This is similar to what we do in Chemistry with chemical elements like oxygen or nitrogen.
- A circuit element cannot be broken down or subdivided into other circuit elements.
- A circuit element can be defined in terms of the behavior of the voltage and current at its terminals.



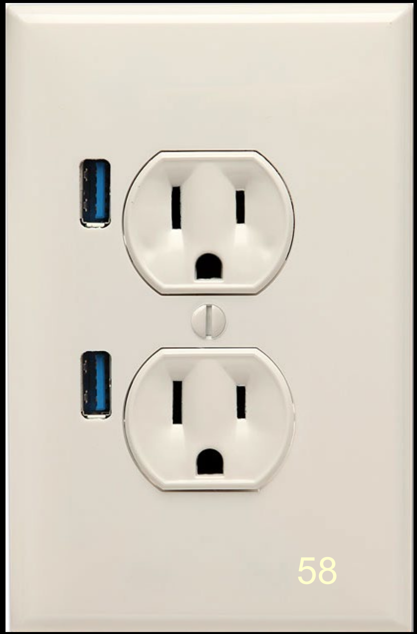
The 5 Basic Circuit Elements

There are 5 basic circuit elements:

1. Voltage sources
2. Current sources
3. Resistors
4. Inductors
5. Capacitors

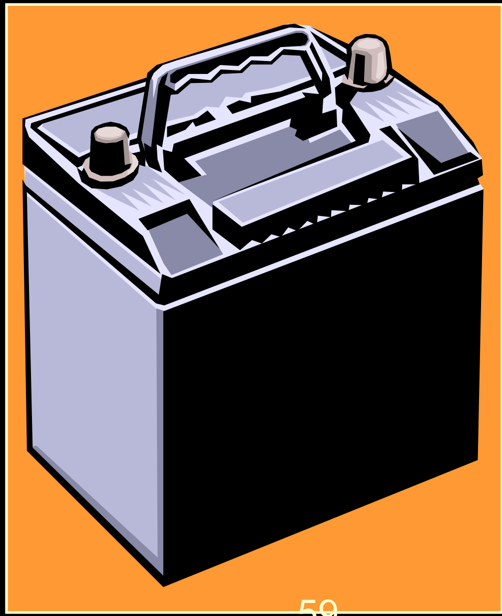
Voltage Sources

- A voltage source is a two-terminal circuit element that maintains a voltage across its terminals.
- The value of the voltage is the defining characteristic of a voltage source.
- Any value of the current can go through the voltage source, in any direction. The current can also be zero. The voltage source does not “care about” current. It “cares” only about voltage.



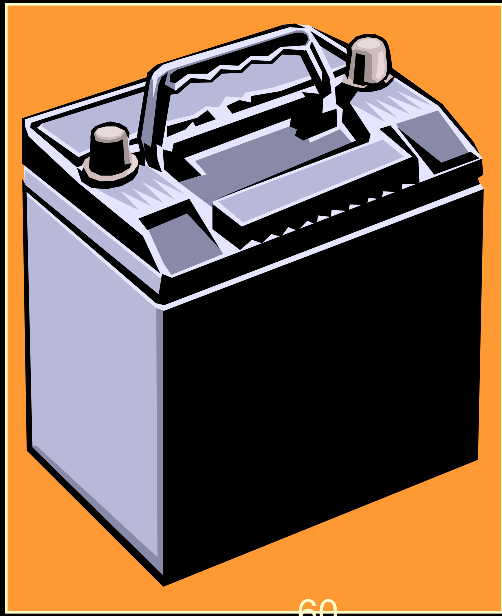
Voltage Sources – Ideal and Practical

- A voltage source maintains that voltage across its terminals no matter what you connect to those terminals.
- We often think of a battery as being a voltage source. For many situations, this is fine. Other times it is not a good model. A real battery will have different voltages across its terminals in some cases, such as when it is supplying a large amount of current. As we have said, a voltage source should not change its voltage as the current changes.



Voltage Sources – Ideal and Practical

- A voltage source maintains that voltage across its terminals no matter what you connect to those terminals.
- We often think of a battery as being a voltage source. For many situations, this is fine. Other times it is not a good model. A real battery will have different voltages across its terminals in some cases, such as when it is supplying a large amount of current. As we have said, a voltage source should not change its voltage as the current changes.
- We sometimes use the term **ideal voltage source** for our circuit elements, and the term **practical voltage source** for things like batteries. We will find that a more accurate model for a battery is an ideal voltage source in series with a resistor. More on that later.



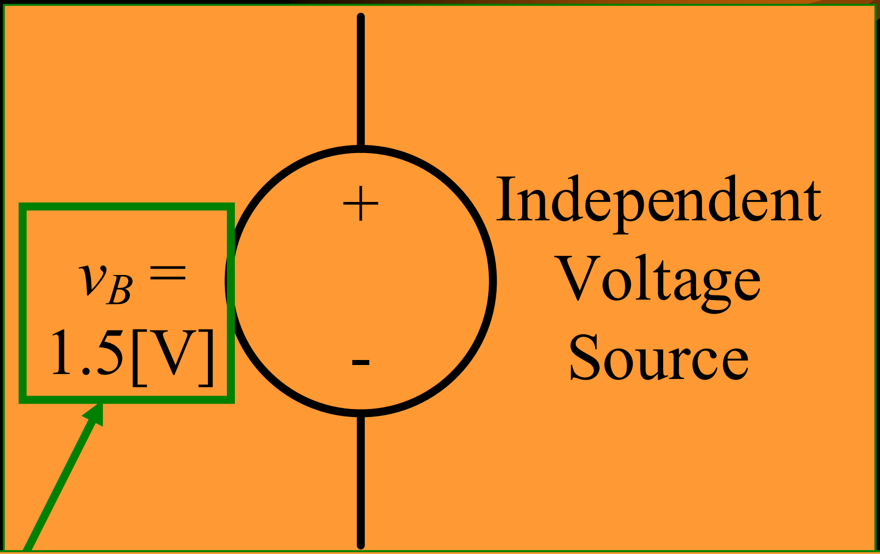
Voltage Sources – 2 kinds

There are 2 kinds of voltage sources:

1. Independent voltage sources
2. Dependent voltage sources, of which there are 2 forms:
   i. Voltage-dependent voltage sources
   ii. Current-dependent voltage sources

Voltage Sources – Schematic Symbol for Independent Sources

The schematic symbol that we use for independent voltage sources is shown here.



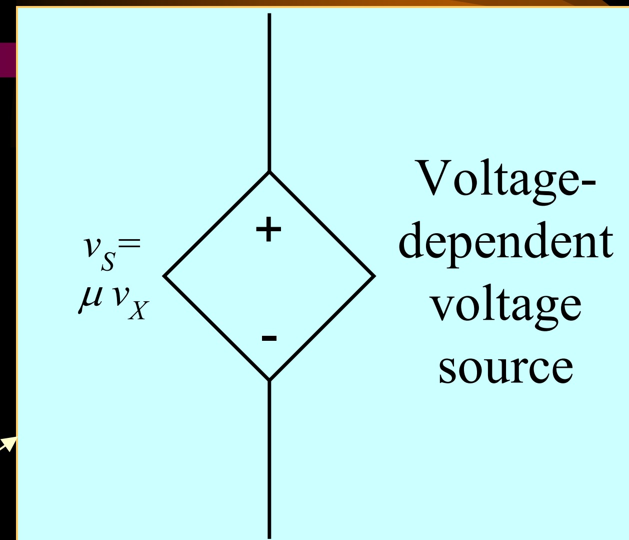
This is intended to indicate that the schematic symbol can be labeled either with a variable, like v_B, or a value, with some number and units. An example might be 1.5[V]. It could also be labeled with both.

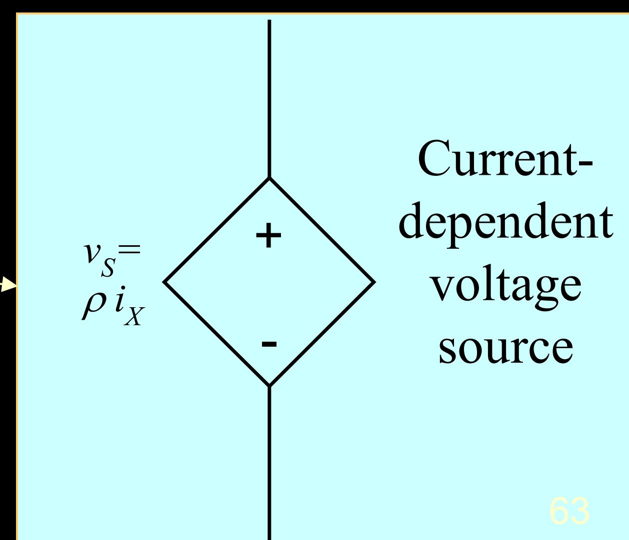
Voltage Sources – Schematic Symbols for Dependent Voltage Sources

The schematic symbols that we use for dependent voltage sources are shown here, of which there are 2 forms:

i. Voltage-dependent voltage sources

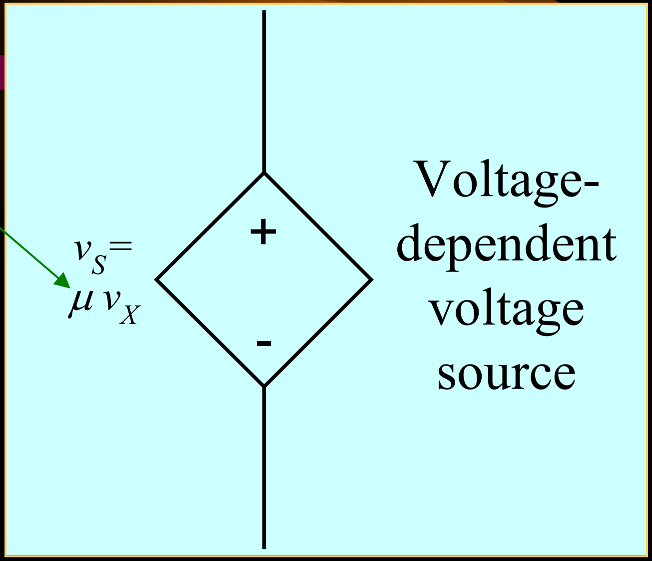
ii. Current-dependent voltage sources





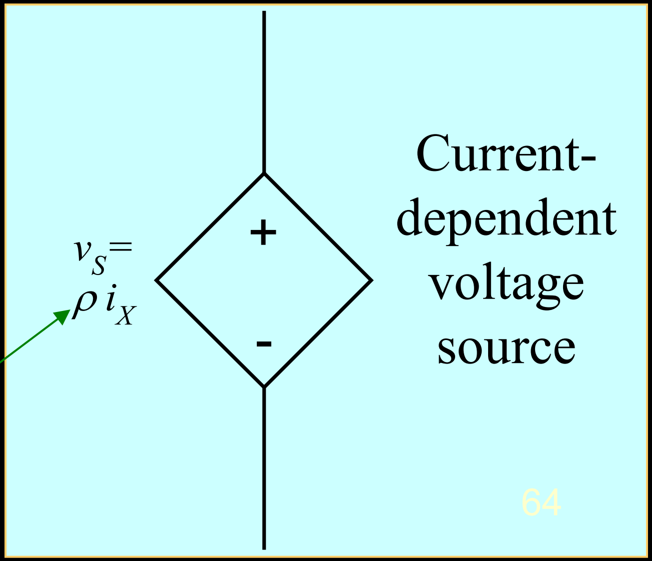
Notes on Schematic Symbols for Dependent Voltage Sources

The symbol μ is the coefficient of the voltage v_X. It is dimensionless. For example, it might be 4.3 v_X. The v_X is a voltage somewhere in the circuit.



The schematic symbols that we use for dependent voltage sources are shown here, of which there are 2 forms:

i. Voltage-dependent voltage sources
ii. Current-dependent voltage sources



The symbol ρ is the coefficient of the current i_X. It has dimensions of [voltage/current]. For example, it might be 4.3[V/A] i_X. The i_X is a current somewhere in the circuit.

Current Sources

- A current source is a two-terminal circuit element that maintains a current through its terminals.
- The value of the current is the defining characteristic of the current source.
- Any voltage can be across the current source, in either polarity. It can also be zero. The current source does not "care about" voltage. It "cares" only about current.

Current Sources - Ideal

- A current source maintains a current through its terminals no matter what you connect to those terminals.
- While there will be devices that reasonably model current sources, these devices are not as familiar as batteries.

Current Sources - Ideal

- A current source maintains a current through its terminals no matter what you connect to those terminals.
- While there will be devices that reasonably model current sources, these devices are not as familiar as batteries.
- We sometimes use the term **ideal current source** for our circuit elements, and the term **practical current source** for actual devices. We will find that a good model for these devices is an ideal current source in parallel with a resistor. More on that later.

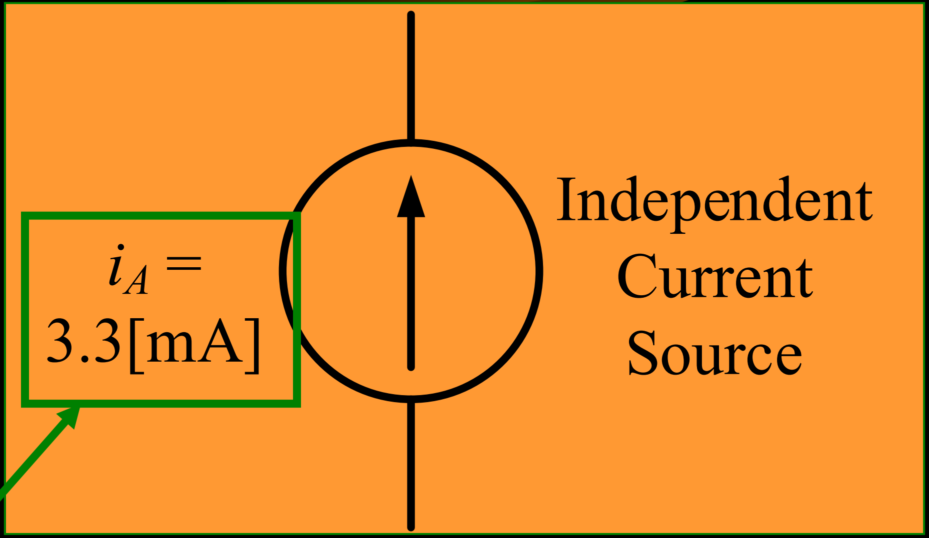
Current Sources – 2 kinds

There are 2 kinds of current sources:

1. Independent current sources
2. Dependent current sources, of which there are 2 forms:
   i. Voltage-dependent current sources
   ii. Current-dependent current sources

Current Sources – Schematic Symbol for Independent Sources

The schematic symbol that we use for independent current sources is shown here.



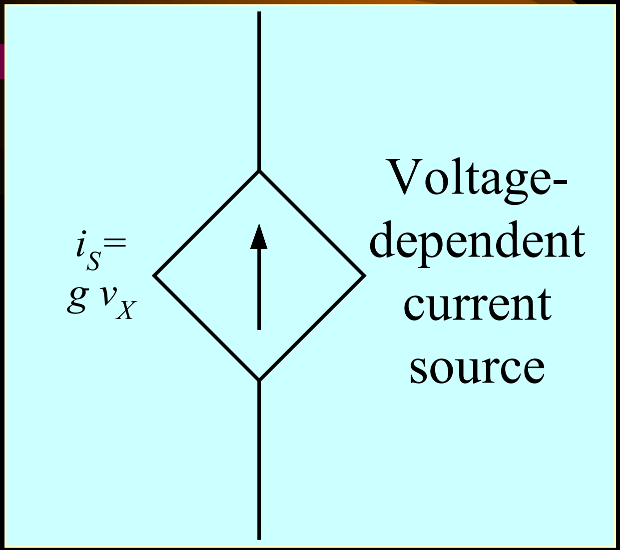
This is intended to indicate that the schematic symbol can be labeled either with a variable, like i_A, or a value, with some number and units. An example might be 3.3[mA]. It could also be labeled with both.

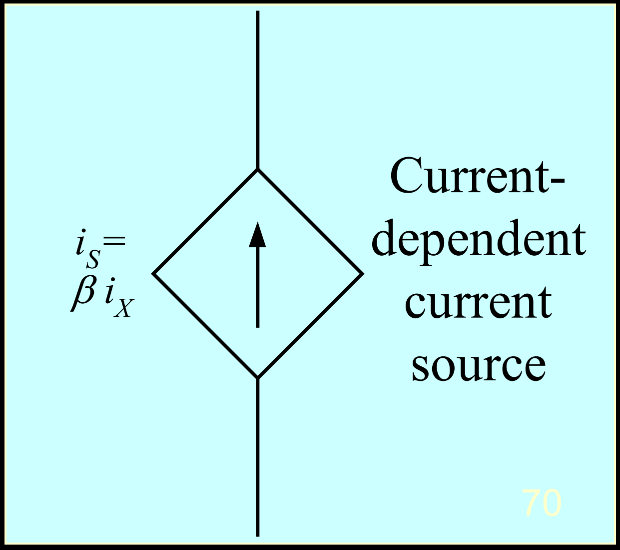
Current Sources – Schematic Symbols for Dependent Current Sources

The schematic symbols that we use for dependent current sources are shown here, of which there are 2 forms:

i. Voltage-dependent current sources

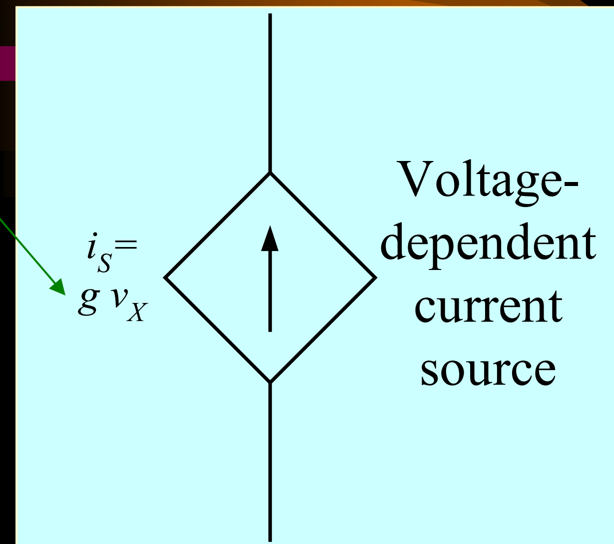
ii. Current-dependent current sources





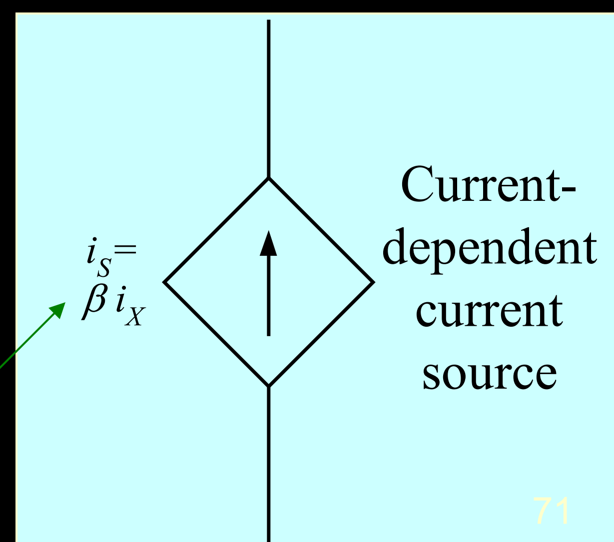
Notes on Schematic Symbols for Dependent Current Sources

The symbol g is the coefficient of the voltage v_X. It has dimensions of [current/voltage]. For example, it might be 16[A/V] v_X. The v_X is a voltage somewhere in the circuit.



The schematic symbols that we use for dependent current sources are shown here, of which there are 2 forms:

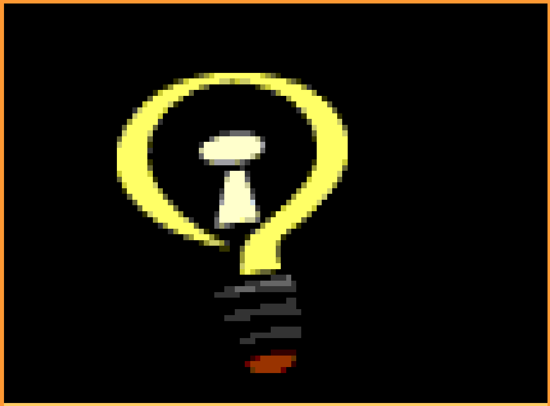
i. Voltage-dependent current sources
ii. Current-dependent current sources



The symbol β is the coefficient of the current i_X. It is dimensionless. For example, it might be 53.7 i_X. The i_X is a current somewhere in the circuit.

Resistors

- A resistor is a two terminal circuit element that has a constant ratio of the voltage across its terminals to the current through its terminals.
- The value of the ratio of voltage to current is the defining characteristic of the resistor.



In many cases a light bulb can be modeled with a resistor.

Resistors – Definition and Units

- A resistor obeys the expression

$$R = \frac{v_R}{i_R}$$

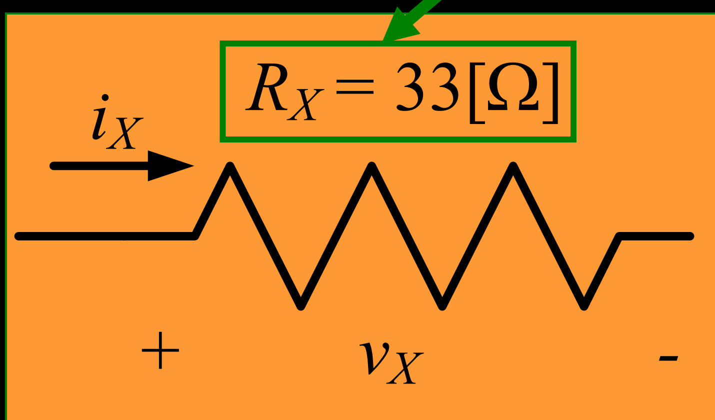
  where R is the *resistance*.
- If something obeys this expression, we can think of it, and model it, as a resistor.
- This expression is called **Ohm's Law**. The unit ([Ohm] or [Ω]) is named for Ohm, and is equal to a [Volt/Ampere].
- IMPORTANT: use Ohm's Law **only** on resistors. It does not hold for sources.

To a first-order approximation, the body can modeled as a resistor. Our goal will be to avoid applying large voltages across our bodies, because it results in large currents through our body. This is not good.

Schematic Symbol for Resistors

The schematic symbol that we use for resistors is shown here.

This is intended to indicate that the schematic symbol can be labeled either with a variable, like R_X, or a value, with some number, and units. An example might be 33[Ω]. It could also be labeled with both.



$$R_X = \frac{v_X}{i_X}$$

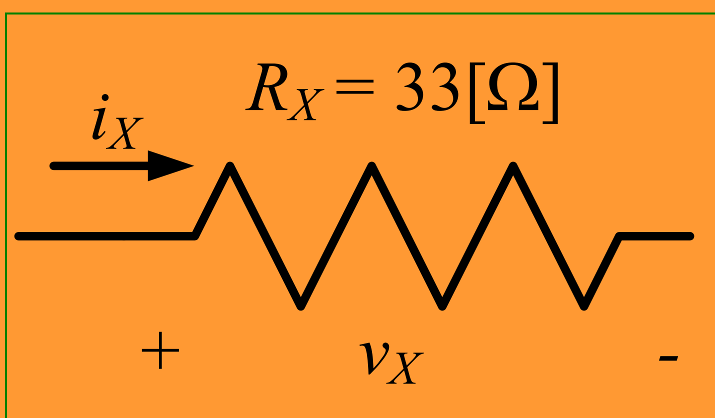
Resistor Polarities

- Previously, we have emphasized the important of reference polarities of current sources and voltages sources. There is no corresponding polarity to a resistor. You can flip it end-for-end, and it will behave the same way.
- However, even in a resistor, direction matters in one sense; we need to have defined the voltage and current in the **passive sign relationship** to use the Ohm's Law equation the way we have it listed here.

Getting the Sign in Ohm's Law from the Sign Relationship

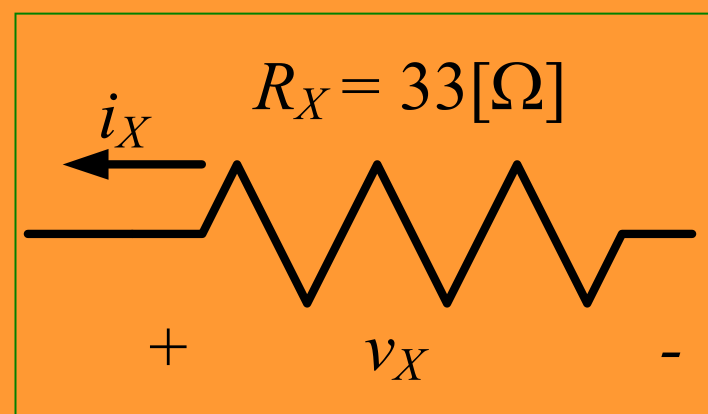
If the reference current is in the direction of the reference voltage drop (Passive Sign Relationship), then…

$$R_X = \frac{v_X}{i_X}$$



If the reference current is in the direction of the reference voltage rise (Active Sign Relationship), then…

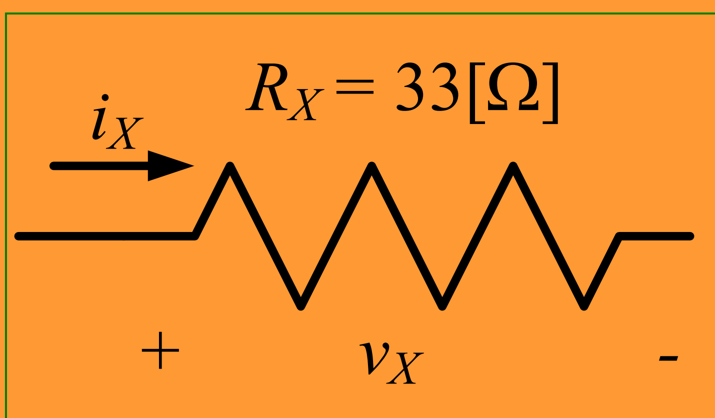
$$R_X = -\frac{v_X}{i_X}$$



The Sign in Ohm's Law Determines the Sign Relationship

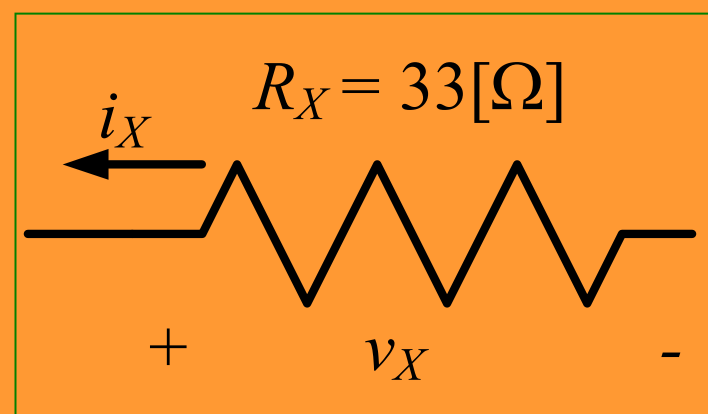
If the Ohm's Law equation has no minus sign, then the voltage and current are in the Passive Sign Relationship.

$$R_X = \frac{v_X}{i_X}$$



If the Ohm's Law equation has a minus sign, then the voltage and current are in the Active Sign Relationship.

$$R_X = -\frac{v_X}{i_X}$$



*Why do we have to worry about the sign in **Everything**?*

- This is one of the central themes in circuit analysis. The polarity, and the sign that goes with that polarity, matters. The key is to find a way to **get the sign correct every time**.
- This is why we need to **define reference polarities** for **every** voltage and current.
- This is why we need to take care about what **relationship** we have used to assign reference polarities (passive sign relationship and active sign relationship).

An analogy: Suppose I was going to give you $10,000. This would probably be fine with you. However, it will matter a great deal which direction the money flows. You will care a great deal about the **sign** of the $10,000 in this transaction. If I give you -$10,000, it means that you are giving $10,000 to me. This would probably **not** be fine with you!

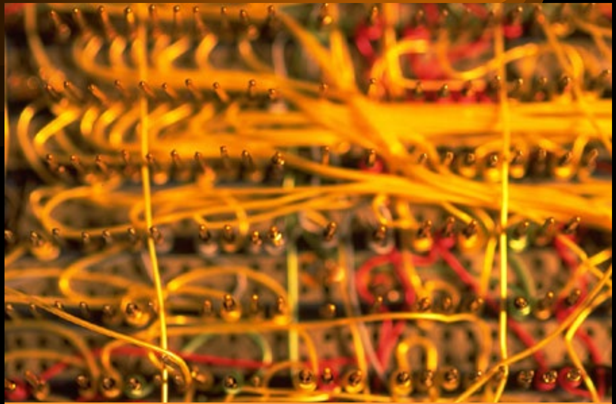
Week -4

Page- (80-119)

Kirchhoff's Laws

Some Fundamental Assumptions – Wires

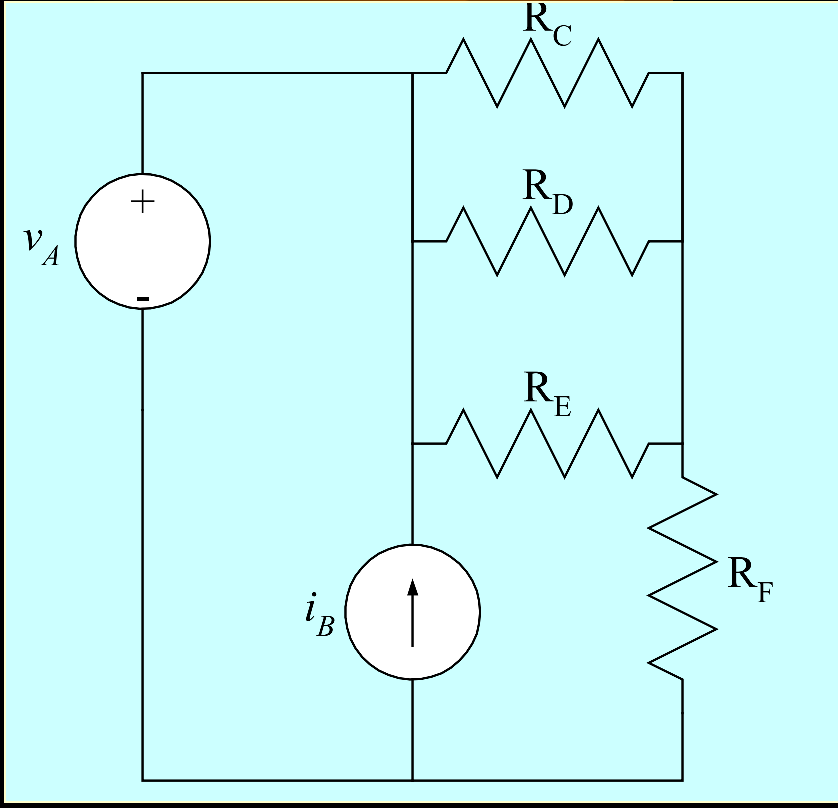
- Although you may not have stated it, or thought about it, when you have drawn circuit schematics, you have connected components or devices with **wires**, and shown this with **lines**.
- Wires can be modeled pretty well as resistors. However, their resistance is usually negligibly small.
- We will think of wires as connections with zero resistance. Note that this is equivalent to having a zero-valued voltage source.



This picture shows wires used to connect electrical components. This particular way of connecting components is called wirewrapping, since the ends of the wires are wrapped around posts.

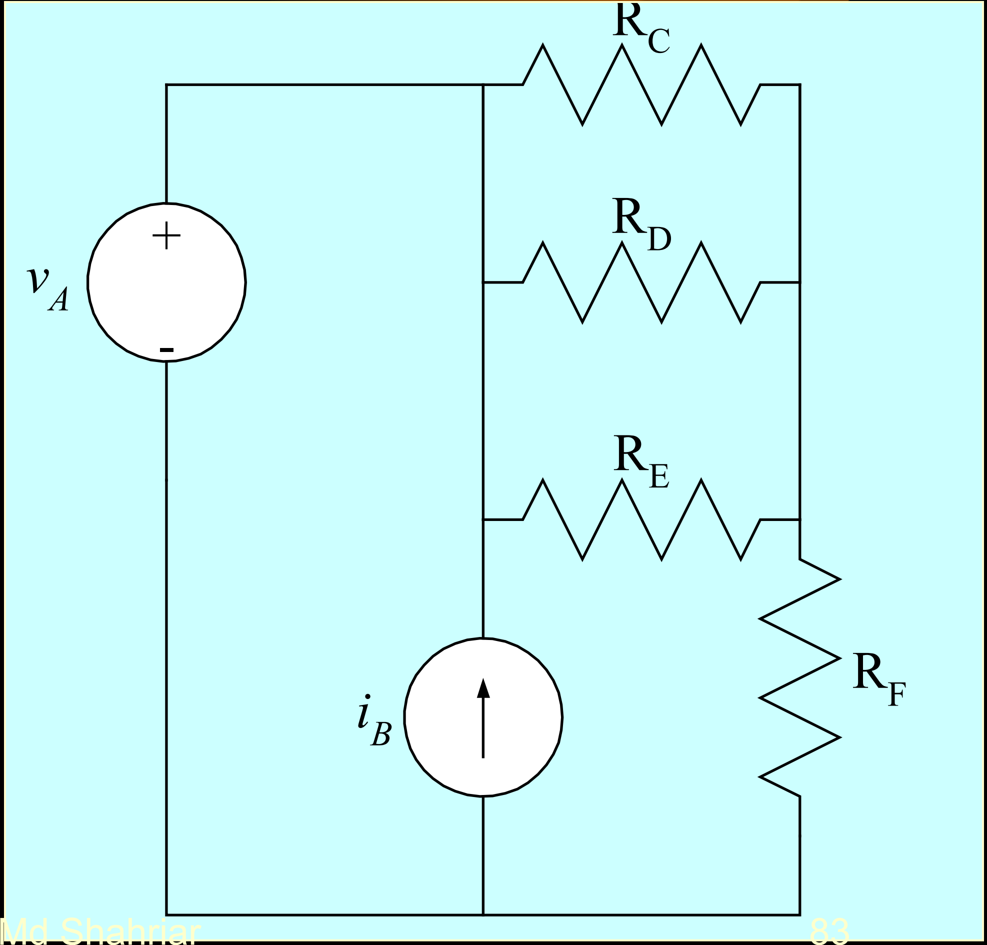
Some Fundamental Assumptions – Nodes

- A node is defined as a place where two or more components are connected.
- The key thing to remember is that we connect components with wires. It doesn't matter how many wires are being used; it only matters how many components are connected together.



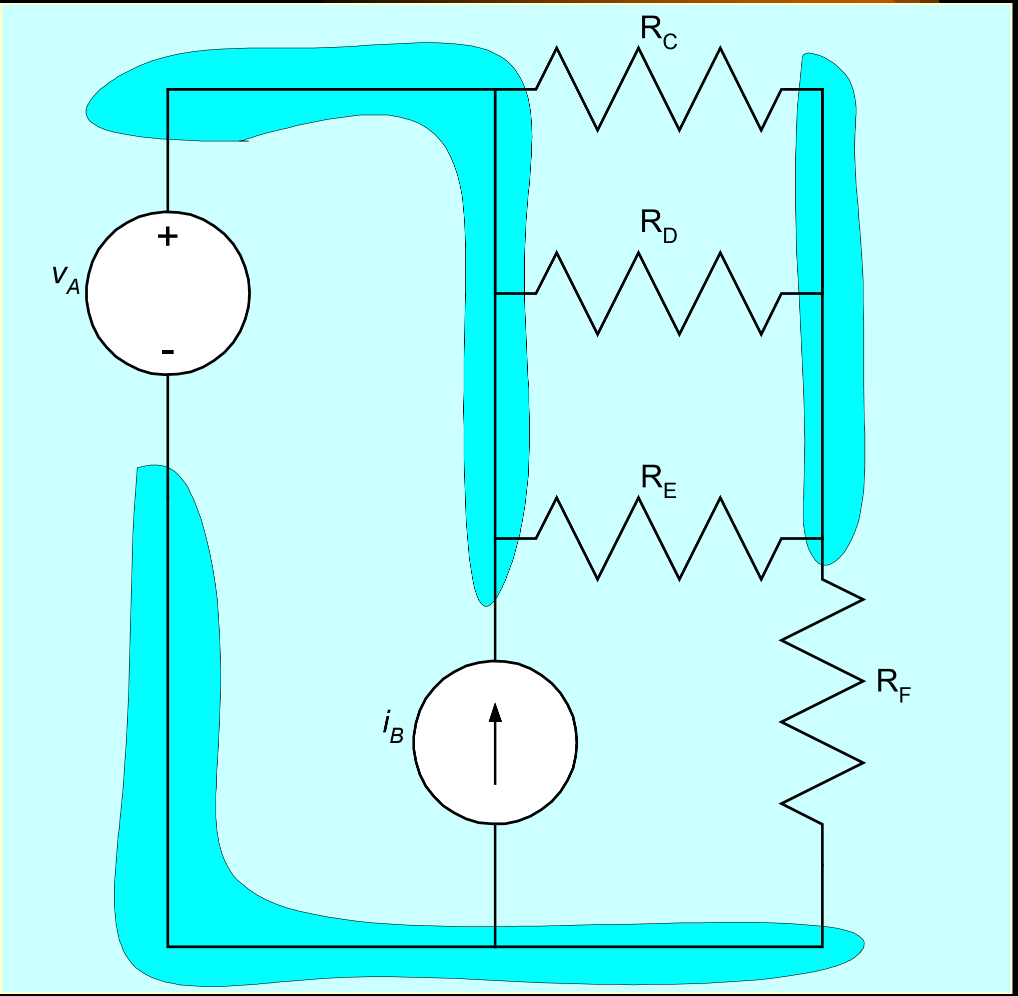
How Many Nodes?

- To test our understanding of nodes, let's look at the example circuit schematic given here.
- How many nodes are there in this circuit?



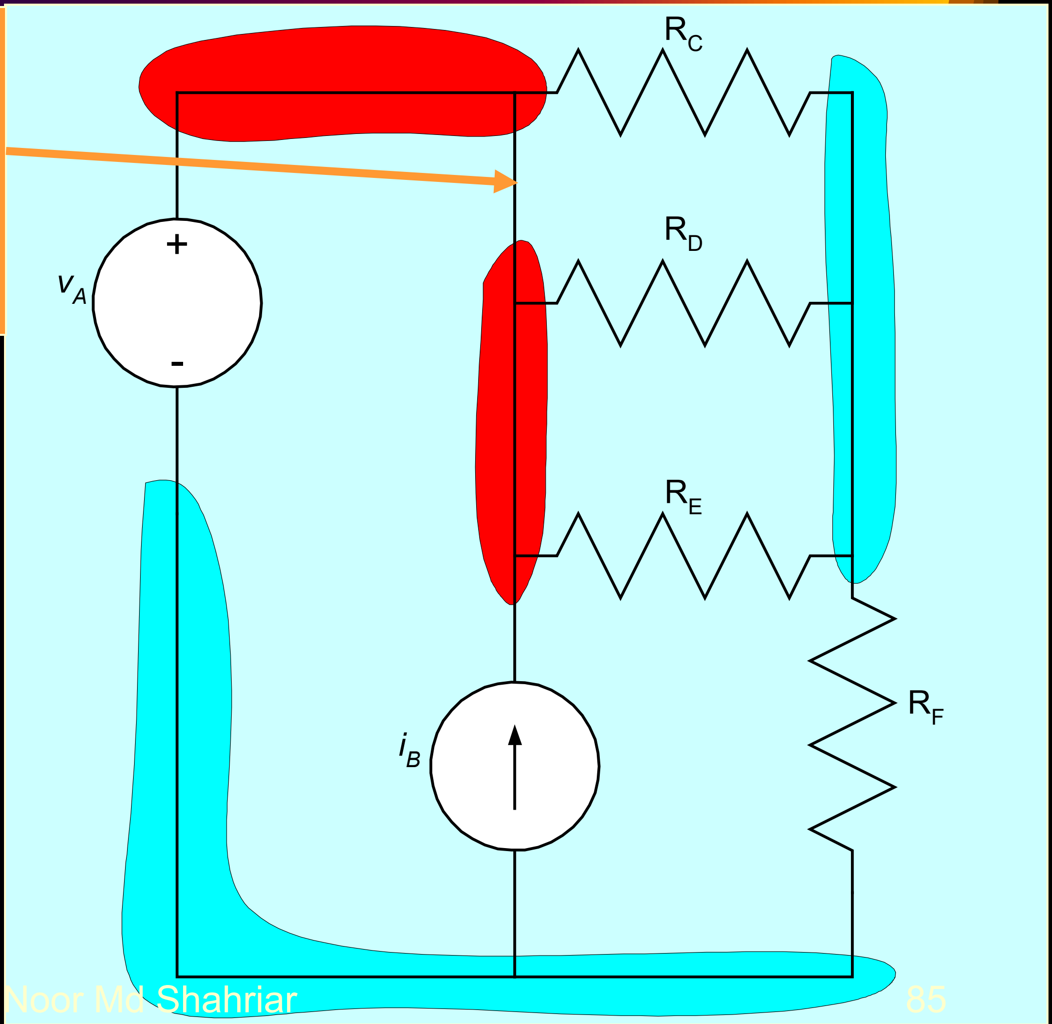
How Many Nodes – Correct Answer

- In this schematic, there are three nodes. These nodes are shown in dark blue here.
- Some students count more than three nodes in a circuit like this. When they do, it is usually because they have considered two points connected by a wire to be two nodes.



How Many Nodes – Wrong Answer

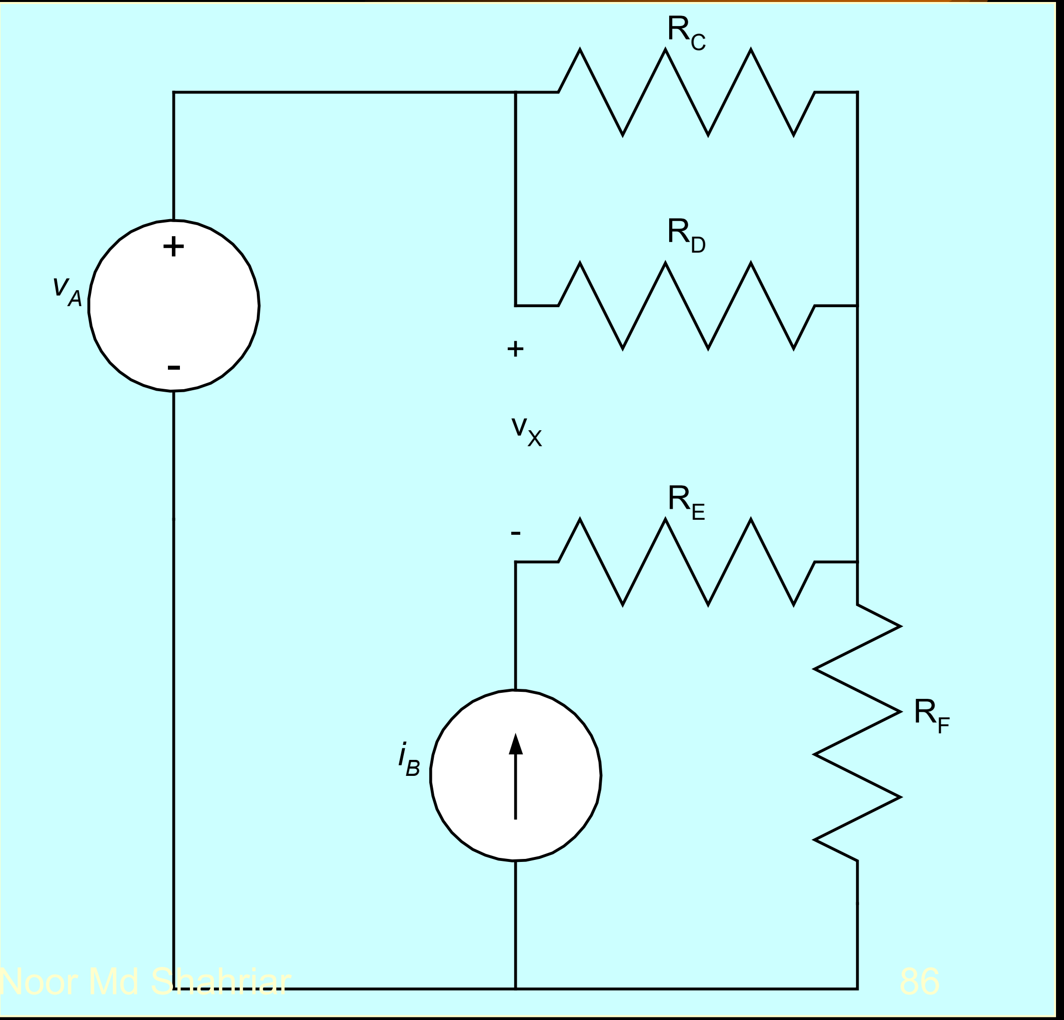
Wire connecting two nodes means that these are really a single node.



- In the example circuit schematic given here, the two red nodes are really the same node. There are not four nodes.
- Remember, two nodes connected by a wire were really only one node in the first place.

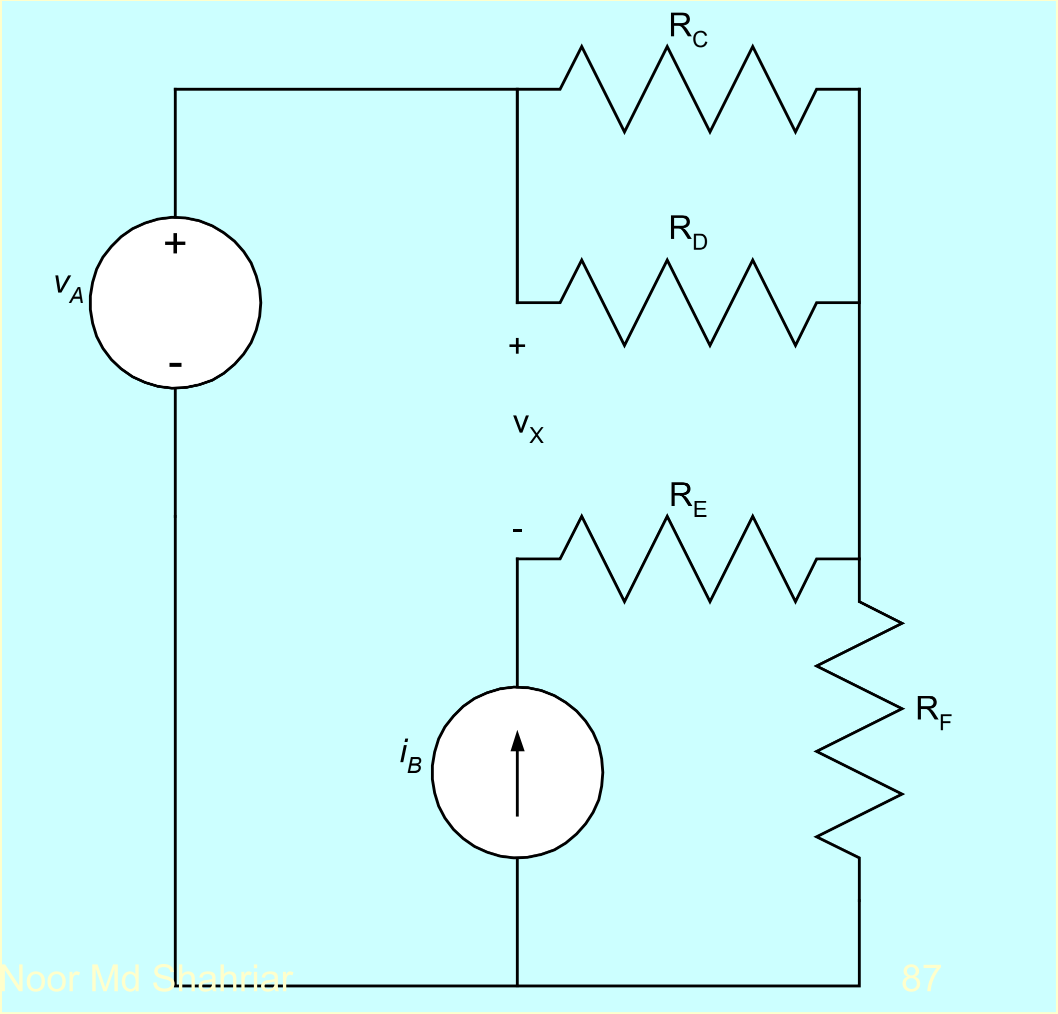
Some Fundamental Assumptions – Closed Loops

- A **closed loop** can be defined in this way: Start at any node and go in any direction and end up where you start. This is a closed loop.
- Note that this loop does not have to follow components. It can jump across open space. Most of the time we will follow components, but we will also have situations where we need to jump between nodes that have no connections.



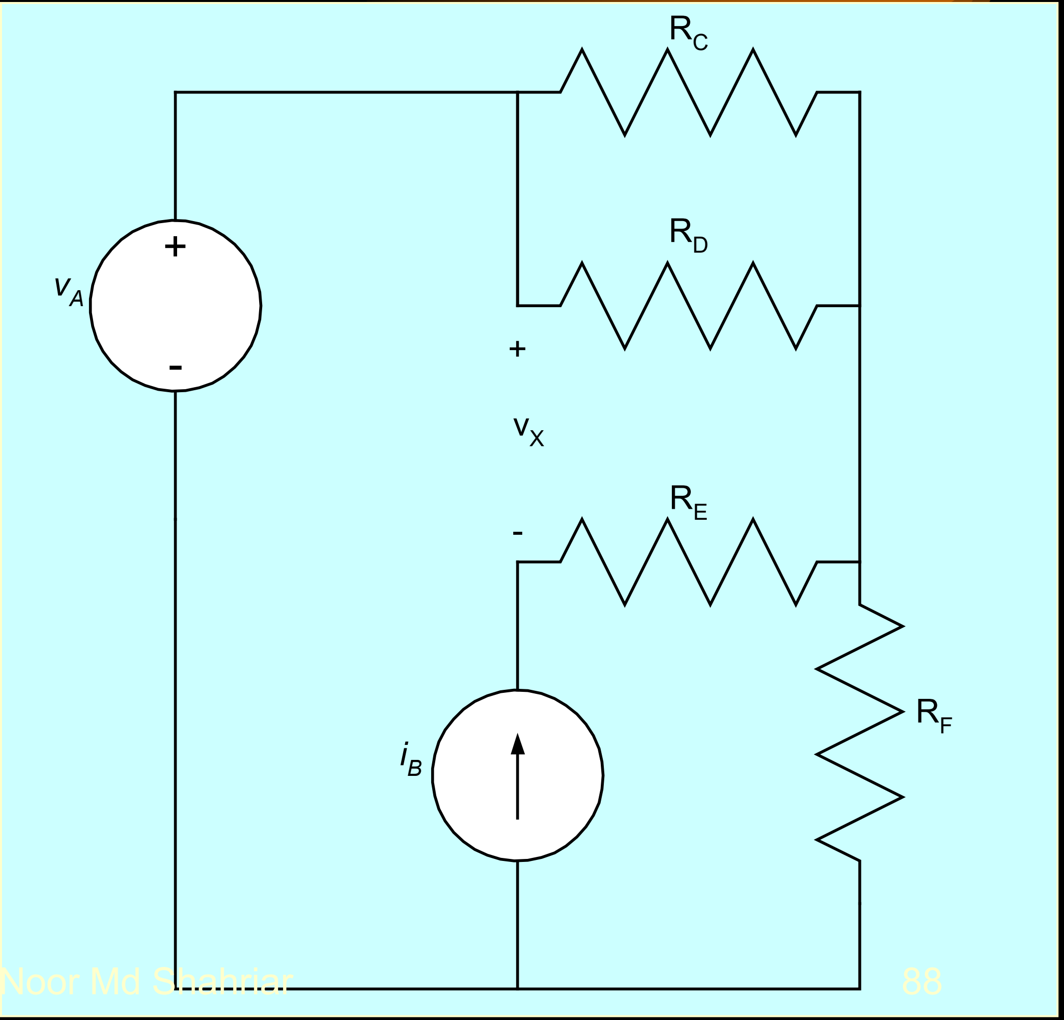
How Many Closed Loops

- To test our understanding of closed loops, let's look at the example circuit schematic given here.
- How many closed loops are there in this circuit?



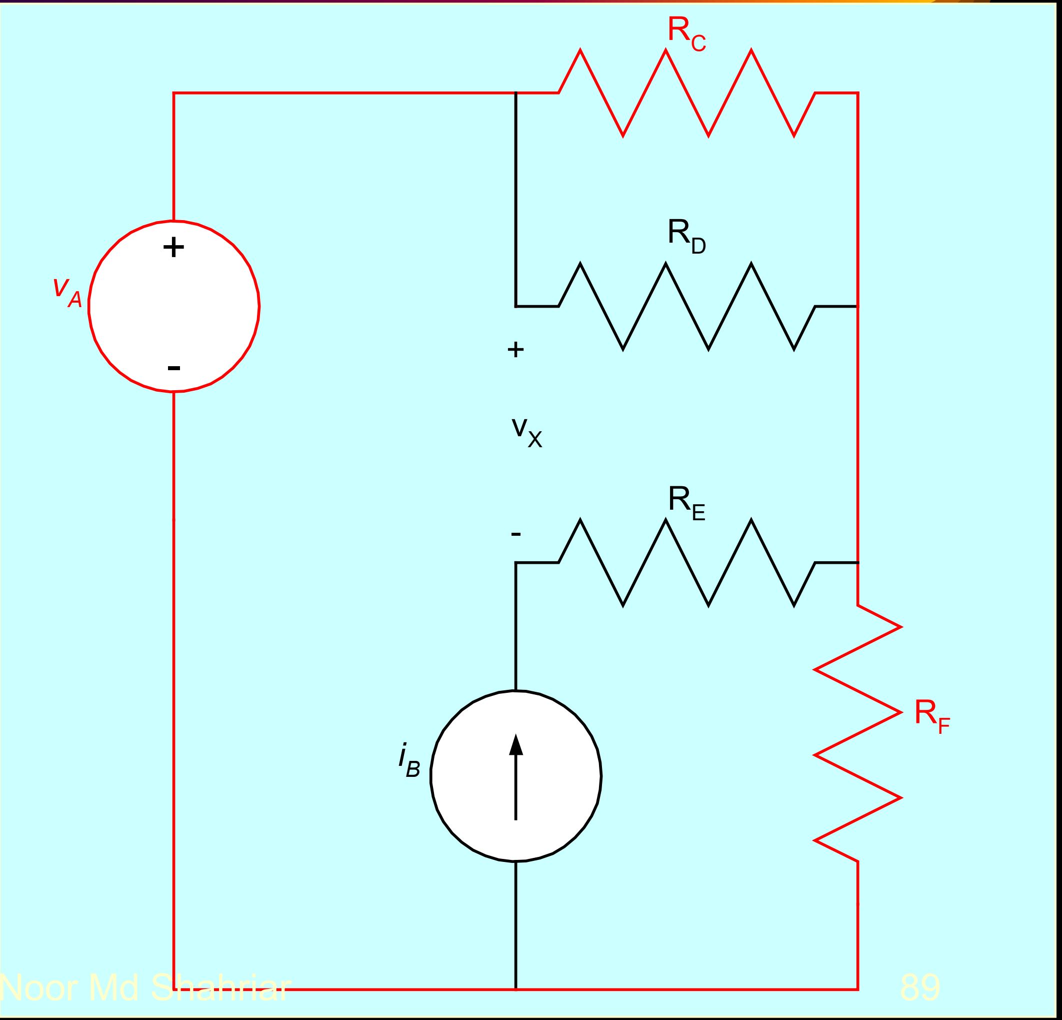
How Many Closed Loops – An Answer

- There are several closed loops that are possible here. We will show a few of them, and allow you to find the others.
- The total number of **closed loops** that follow components and defined voltages in this circuit is 13. The number of **closed loops** as we defined that term, is infinity.
- Finding the number will not turn out to be important. What is important is to recognize closed loops when you see them.



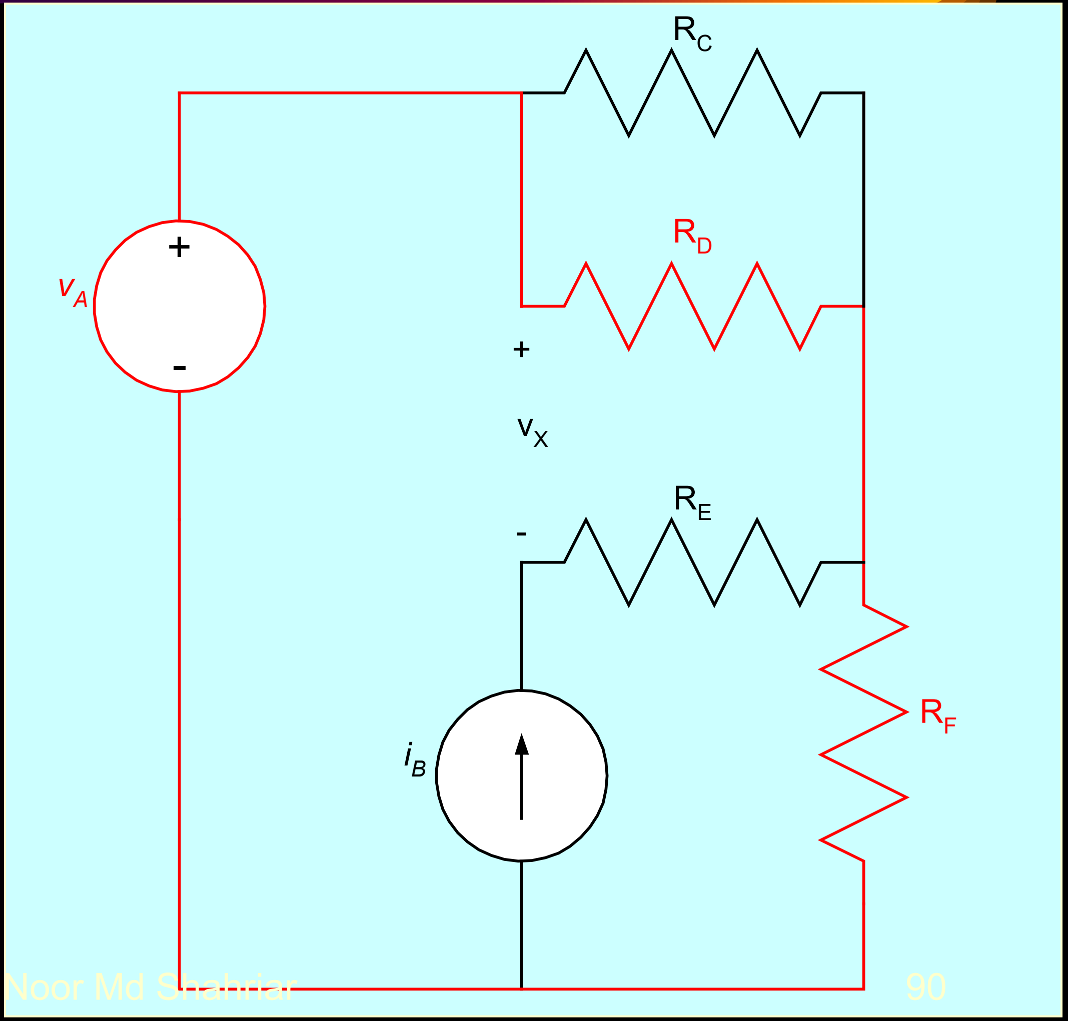
Closed Loops – Loop #1

- Here is a loop we will call Loop #1. The path is shown in red.



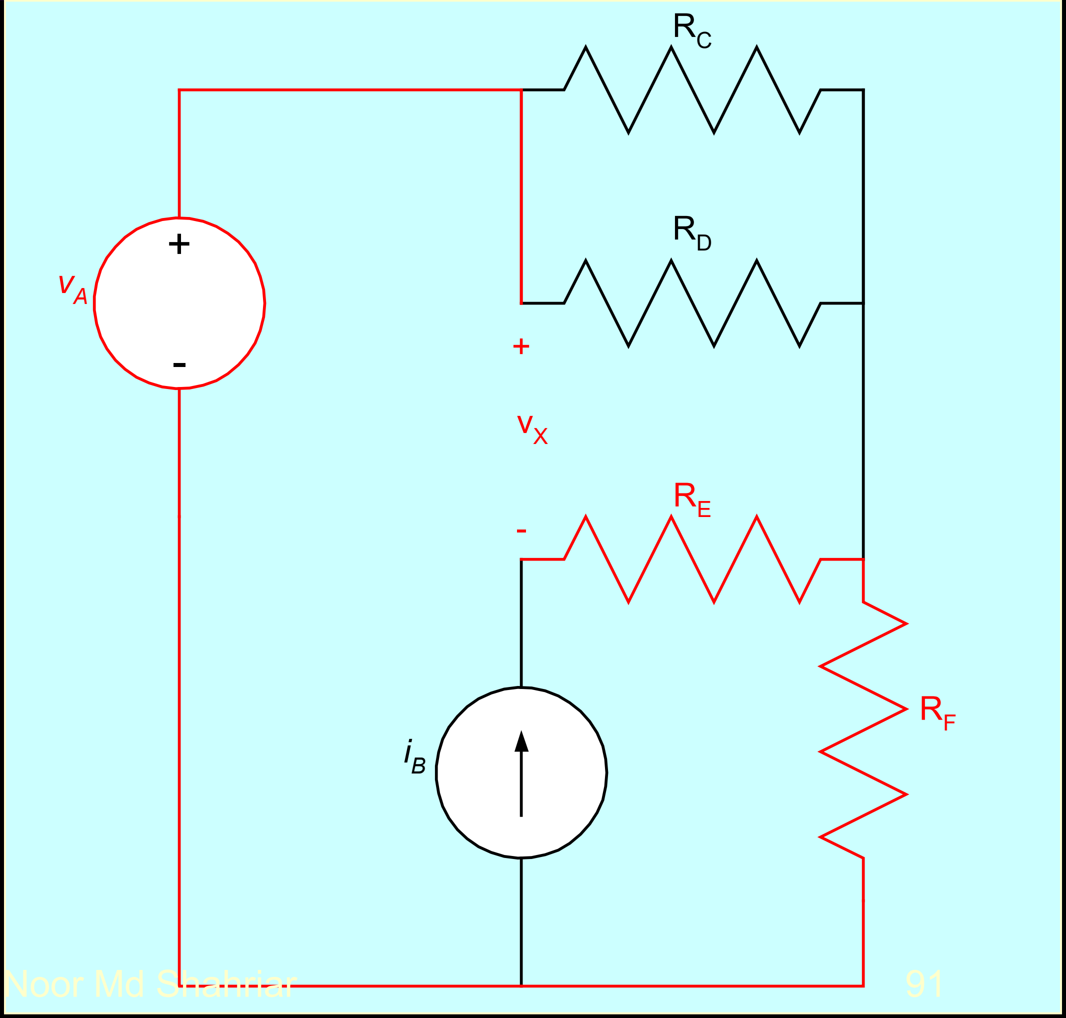
Closed Loops – Loop #2

- Here is Loop #2. The path is shown in red.



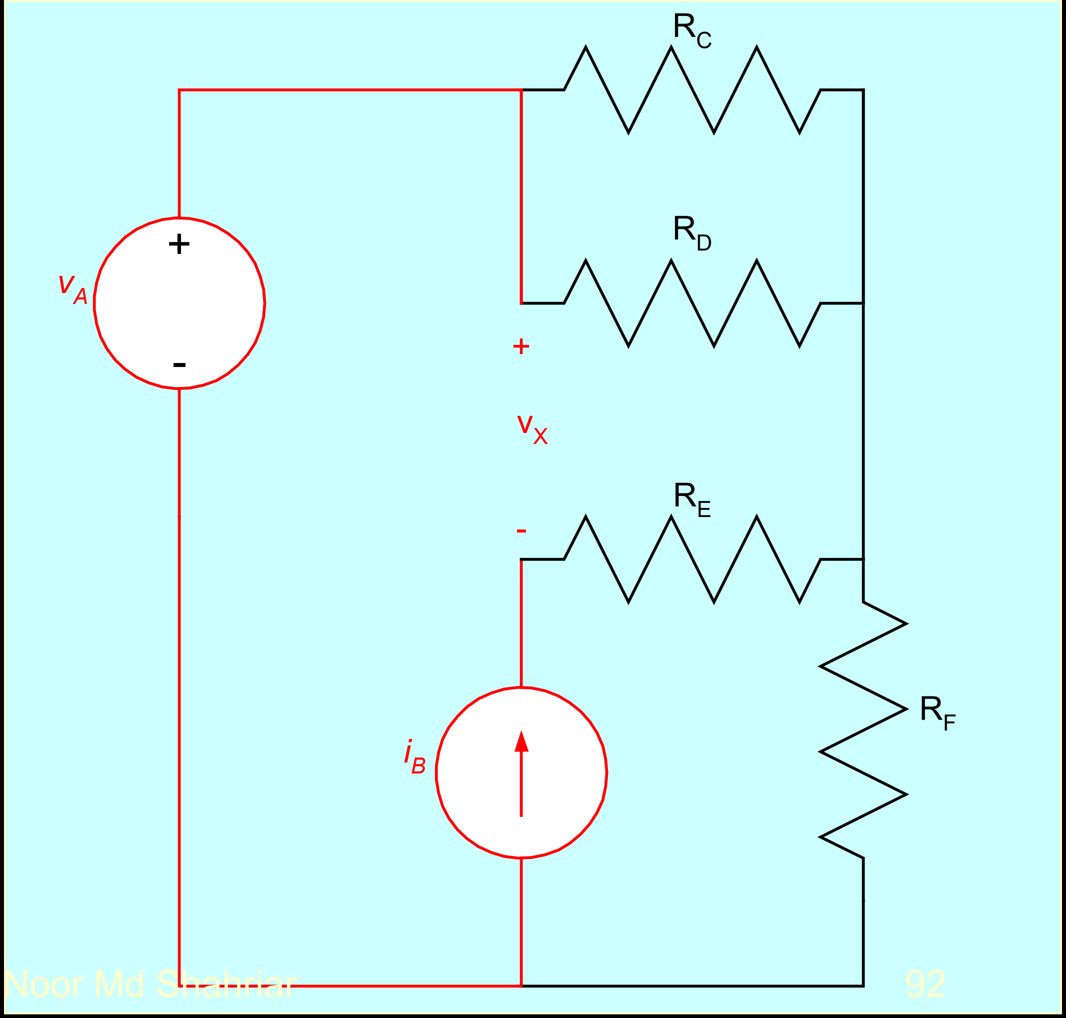
Closed Loops – Loop #3

- Here is Loop #3. The path is shown in red.
- Note that this path is a closed loop that jumps across the voltage labeled v_X. This is still a closed loop.



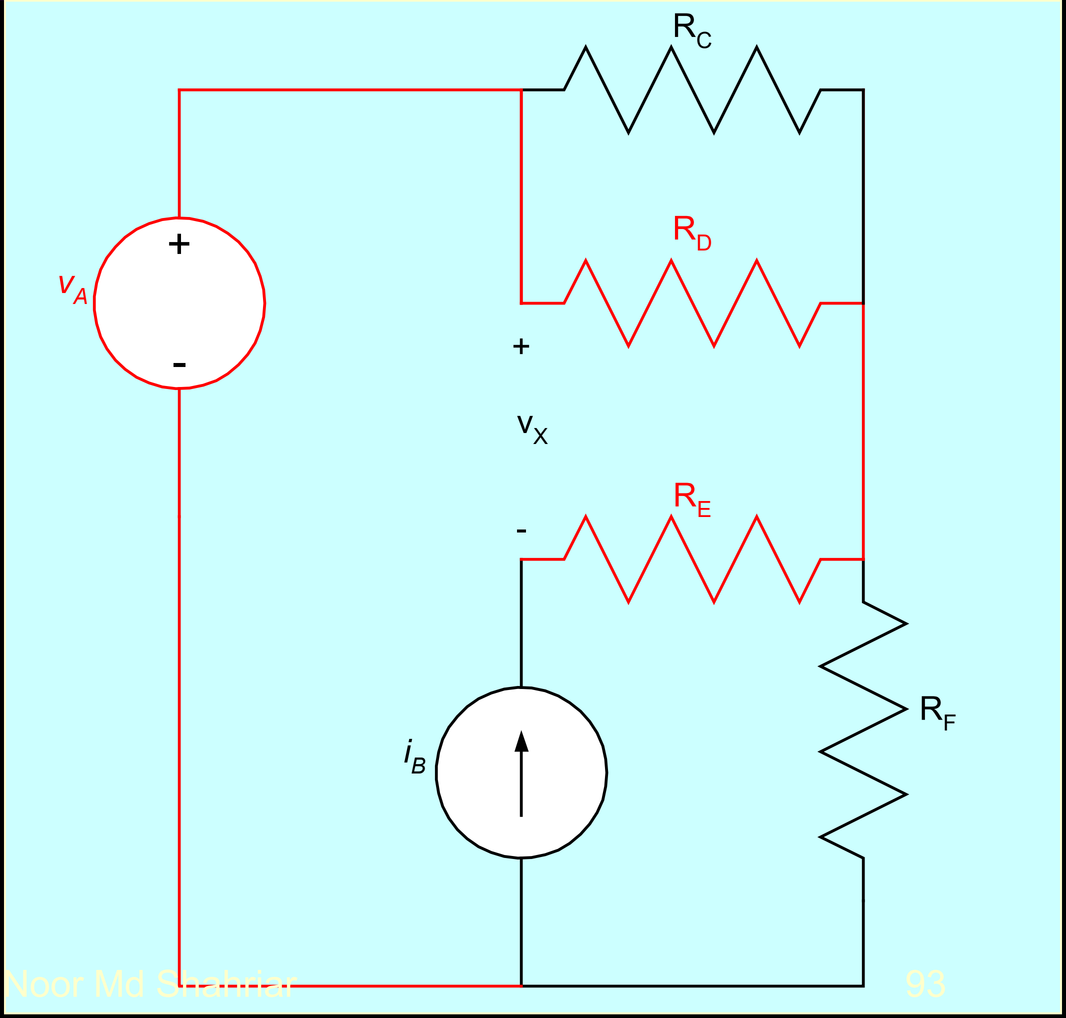
Closed Loops – Loop #4

- Here is Loop #4. The path is shown in red.
- Note that this path is a closed loop that jumps across the voltage labeled v_X. This is still a closed loop. The loop also crossed the current source. Remember that a current source can have a voltage across it.



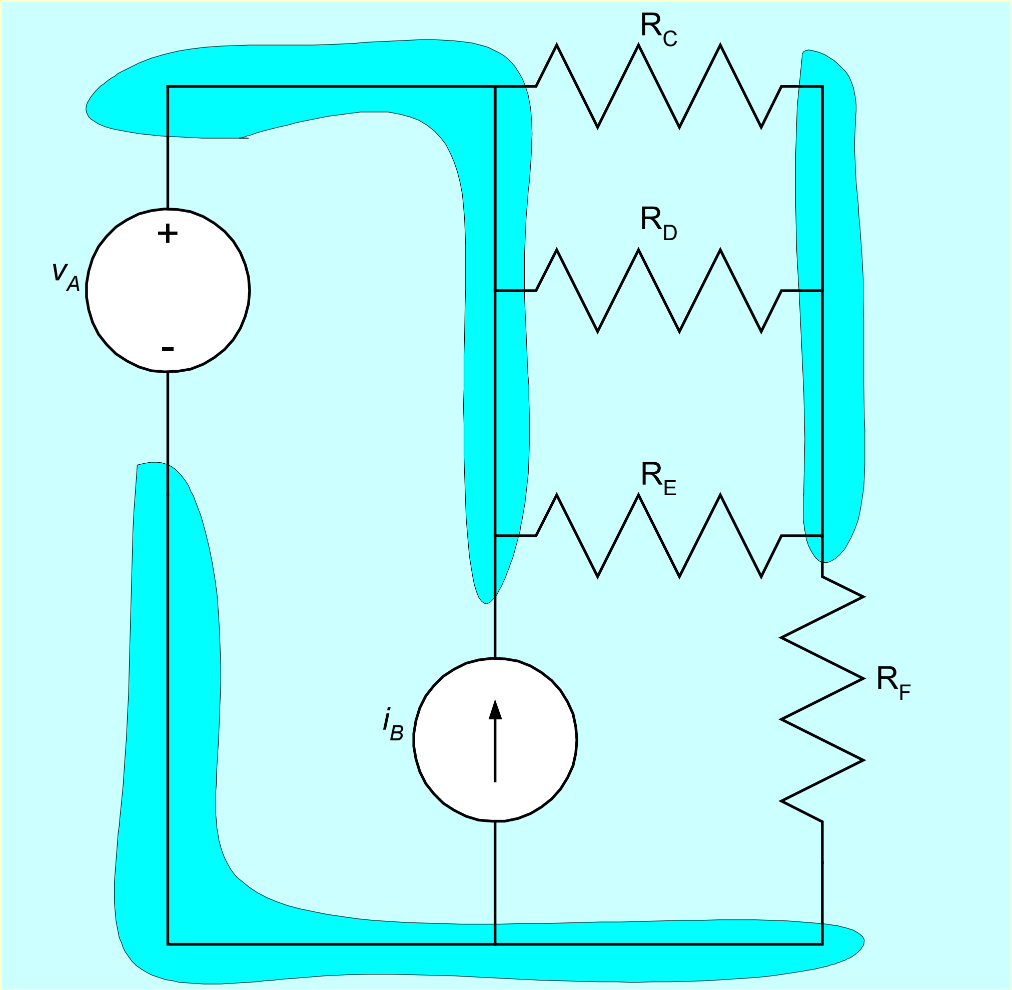
A Not-Closed Loop

- The path is shown in red here is not closed.
- Note that this path does not end where it started.



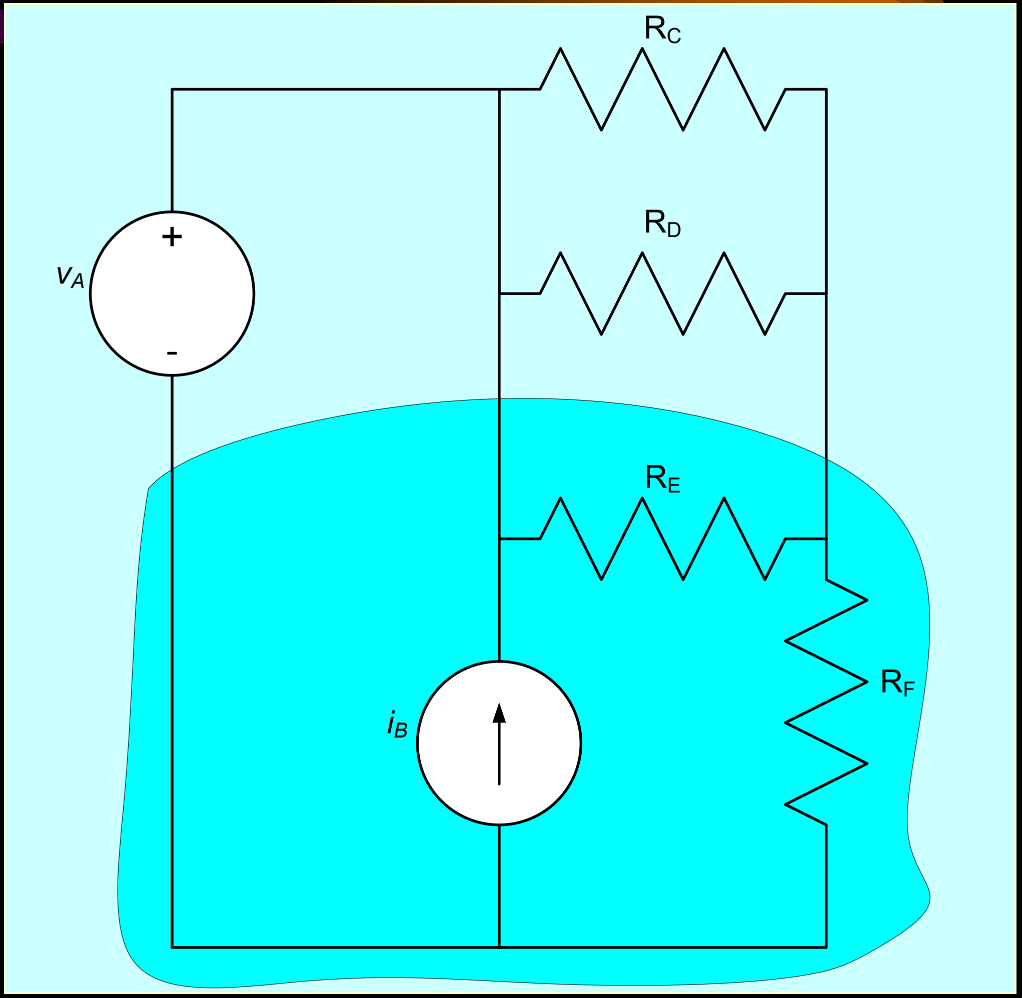
Some Fundamental Assumptions -Closed Surfaces

- A **closed surface** can be defined in this way: Start drawing a line at any place, move in any direction and end up where you start. This boundary thus drawn will be called a closed surface.
- We will note that the nodes we defined earlier are closed surfaces. All nodes are closed surfaces, but not all closed surfaces are nodes.



Other Closed Surfaces

- A **closed surface** can be defined in this way: Start drawing a line at any place, move in any direction and end up where you start. This boundary thus drawn will be called a closed surface.
- The dark blue shape in the diagram at the right is a closed surface, but it is not a node. Closed surfaces can enclose components, devices, or elements.



Kirchhoff's Current Law (KCL)

- With these definitions, we are prepared to state Kirchhoff's Current Law:

  The algebraic (or signed) summation of currents through any closed surface must equal zero.

Kirchhoff's Current Law (KCL) – Some notes.

The algebraic (or signed) summation of currents through any closed surface must equal zero.

This law essentially means that charge does not build up at a connection point, and that charge is conserved.

This law is often stated as applying to nodes. It applies to any closed surface. For any closed surface, the charge that enters must leave somewhere else. A node is just a **small** closed surface. A node is the closed surface that we use most often. But, we can use any closed surface, and sometimes it is really necessary to use closed surfaces that are not nodes.

Current Polarities

Again, the issue of the sign, or polarity, or direction, of the current arises. When we write a Kirchhoff Current Law equation, we attach a sign to each reference current polarity, depending on whether the reference current is entering or leaving the closed surface. This can be done in different ways.

Kirchhoff's Current Law (KCL) – a Systematic Approach

The algebraic (or signed) summation of currents through any closed surface must equal zero.

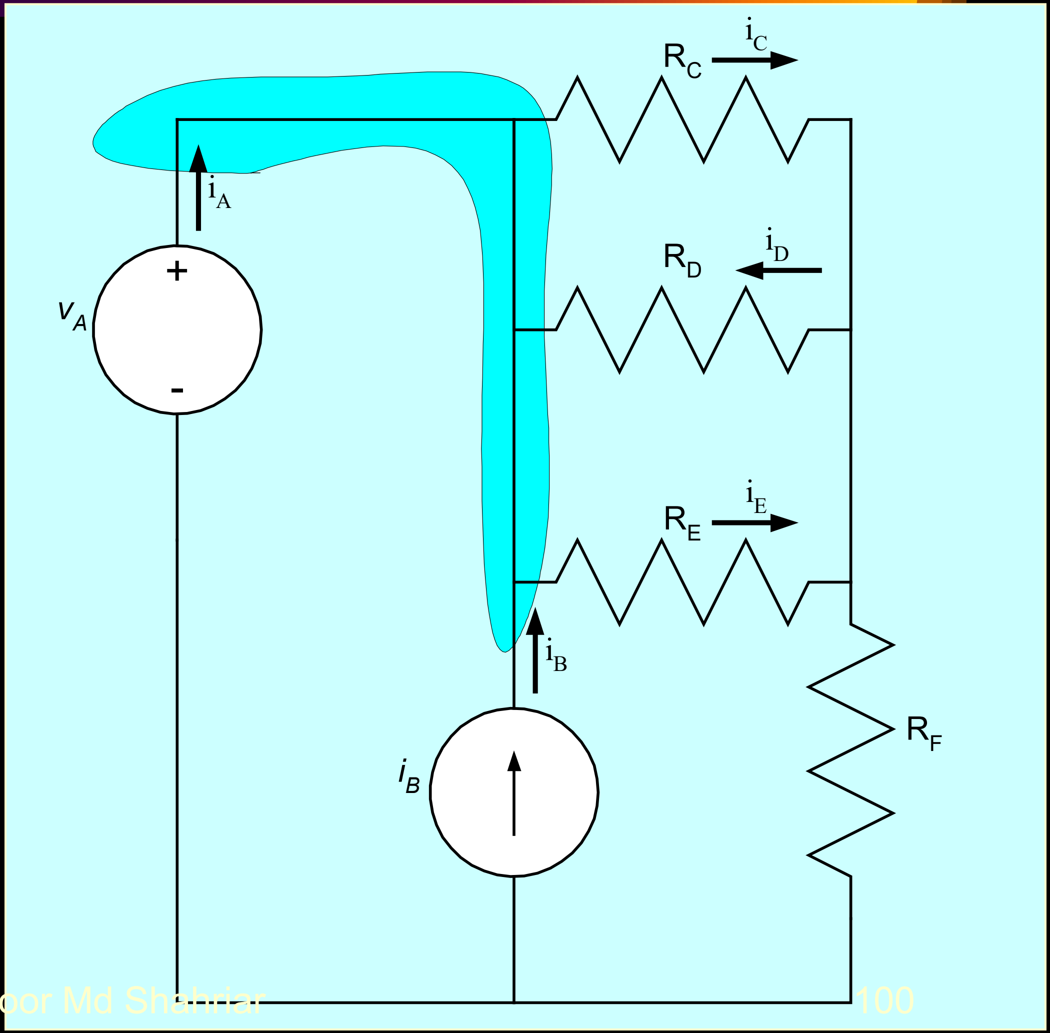
For most students, it is a good idea to choose one way to write KCL equations, and just do it that way every time. The idea is this; if you always do it the same way, you are less likely to get confused about which way you were doing it in a certain equation.

For this set of material, we will always assign a positive sign to a term that refers to a reference current that leaves a closed surface, and a negative sign to a term that refers to a reference current that enters a closed surface.

Kirchhoff's Current Law (KCL) – an Example

- For this set of material, we will always assign a positive sign to a term that refers to a current that leaves a closed surface, and a negative sign to a term that refers to a current that enters a closed surface.
- In this example, we have already assigned reference polarities for all of the currents for the nodes indicated in darker blue.
- For this circuit, and using my rule, we have the following equation:

$$-i_A + i_C - i_D + i_E - i_B = 0$$



Kirchhoff's Current Law (KCL) – Example Done Another Way

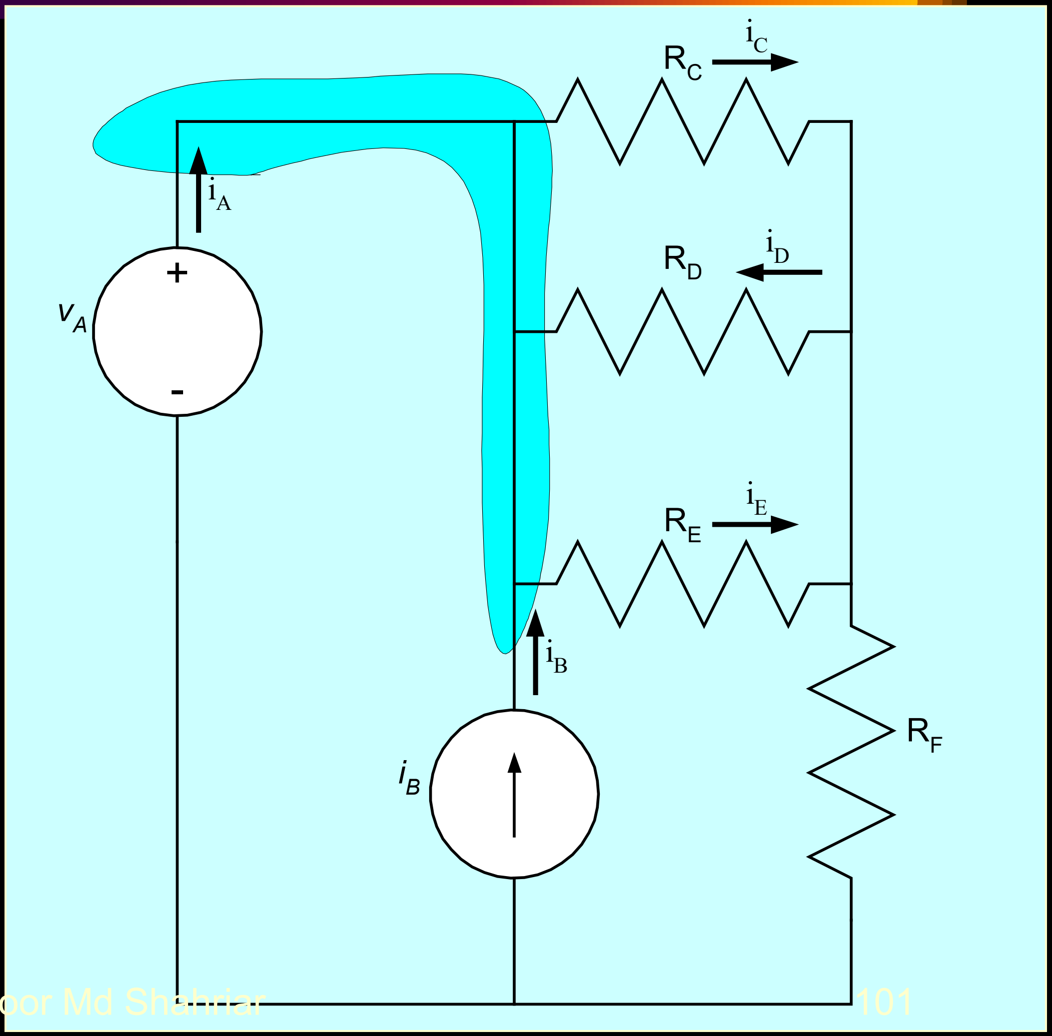
- Some prefer to write this same equation in a different way; they say that the current entering the closed surface must equal the current leaving the closed surface. Thus, they write :

$$i_A + i_D + i_B = i_C + i_E$$

- Compare this to the equation that we wrote in the last slide:

$$-i_A + i_C - i_D + i_E - i_B = 0$$

- These are the same equation. Use either method.



Kirchhoff's Voltage Law (KVL)

- Now, we are prepared to state Kirchhoff's Voltage Law:

  The algebraic (or signed) summation of voltages around any closed loop must equal zero.

Kirchhoff's Voltage Law (KVL) – Some notes.

The algebraic (or signed) summation of voltages around any closed loop must equal zero.

This law essentially means that energy is conserved. If we move around, wherever we move, if we end up in the place we started, we cannot have changed the potential at that point.

This applies to all closed loops. While we usually write equations for closed loops that follow components, we do not need to. The only thing that we need to do is end up where we started.

Kirchhoff's Voltage Law (KVL) – a Systematic Approach

The algebraic (or signed) summation of voltages around a closed loop must equal zero.

For most students, it is a good idea to choose one way to write KVL equations, and just do it that way every time. The idea is this: If you always do it the same way, you are less likely to get confused about which way you were doing it in a certain equation.

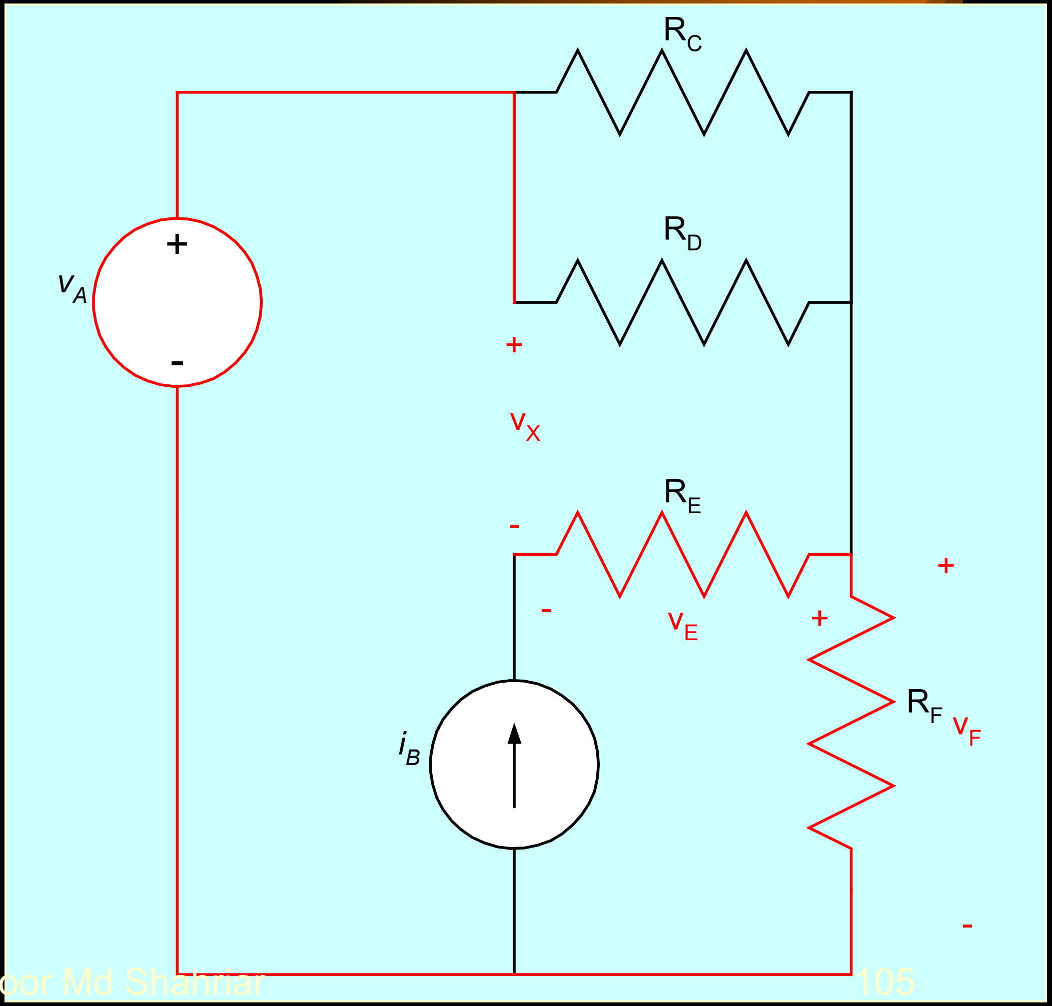
(At least we will do this for planar circuits. For nonplanar circuits, clockwise does not mean anything. If this is confusing, ignore it for now.)

For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a reference voltage drop, and a negative sign to a term that refers to a reference voltage rise.

Kirchhoff's Voltage Law (KVL) – an Example

- For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a voltage drop, and a negative sign to a term that refers to a voltage rise.
- In this example, we have already assigned reference polarities for all of the voltages for the loop indicated in red.
- For this circuit, and using our rule, starting at the bottom, we have the following equation:

$$-v_A + v_X - v_E + v_F = 0.$$

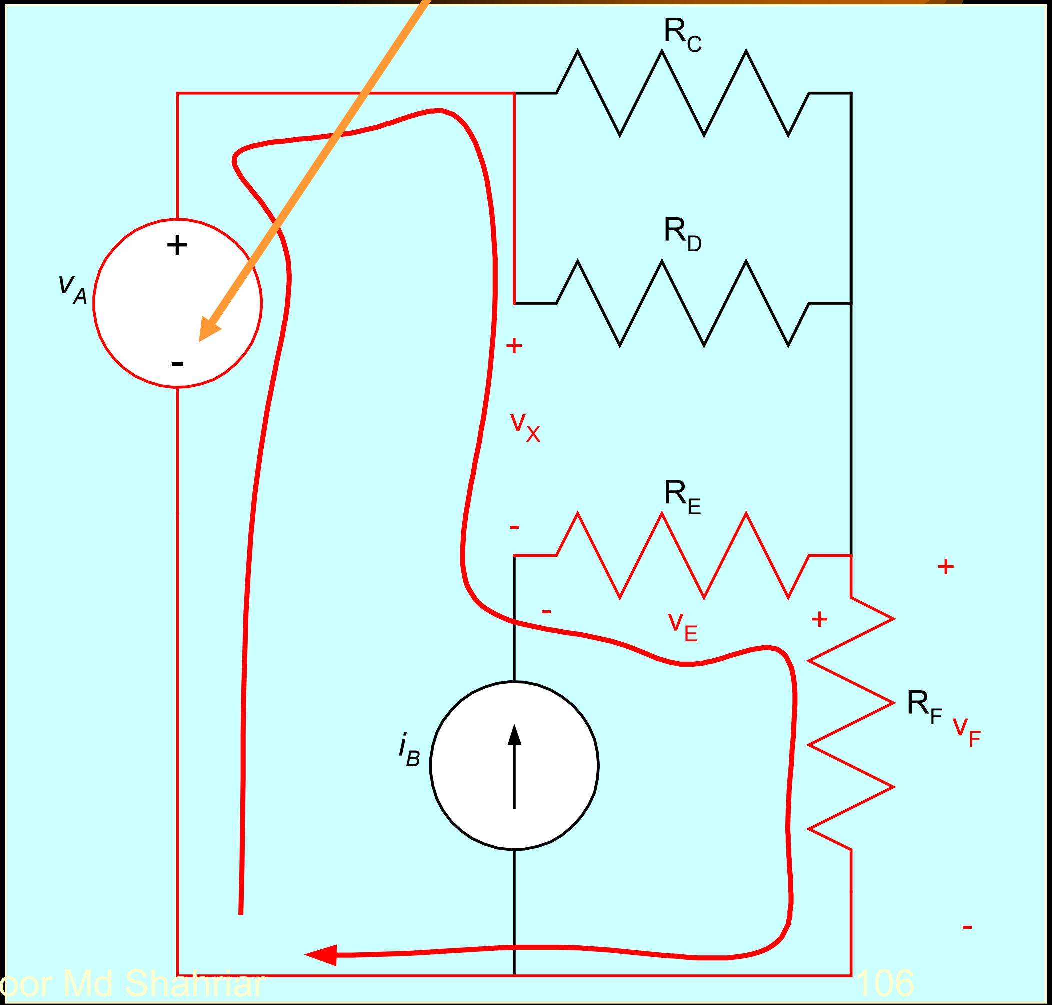


Kirchhoff's Voltage Law (KVL) – Notes

As we go up through the voltage source, we enter the negative sign first. Thus, v_A has a negative sign in the equation.

- For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a voltage drop, and a negative sign to a term that refers to a voltage rise.
- Some students like to use the following handy mnemonic device: Use the sign of the voltage that is on the side of the voltage that you enter. This amounts to the same thing.

$$-v_A + v_X - v_E + v_F = 0$$



Kirchhoff's Voltage Law (KVL) – Example Done Another Way

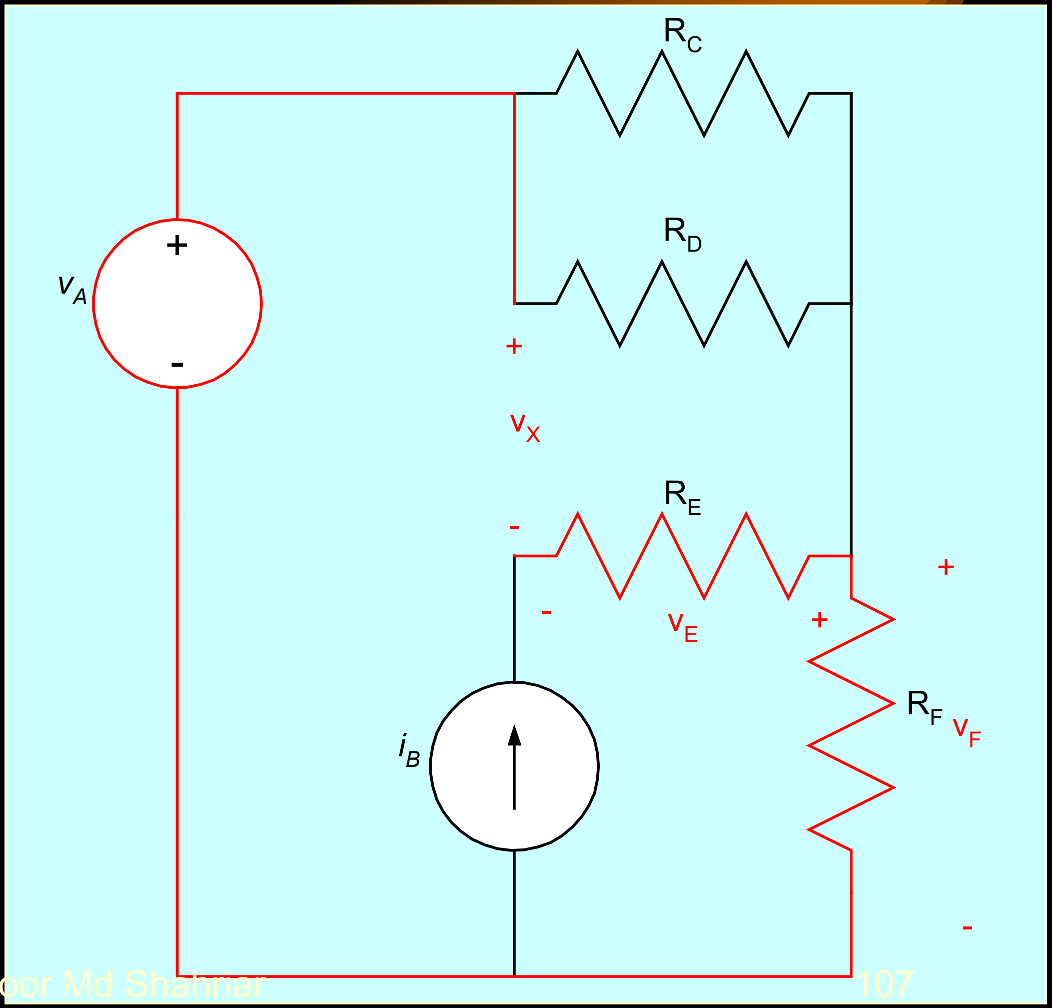
- Some textbooks, and some students, prefer to write this same equation in a different way; they say that the voltage drops must equal the voltage rises. Thus, they write the following equation:

$$v_X + v_F = v_A + v_E.$$

Compare this to the equation that we wrote in the last slide:

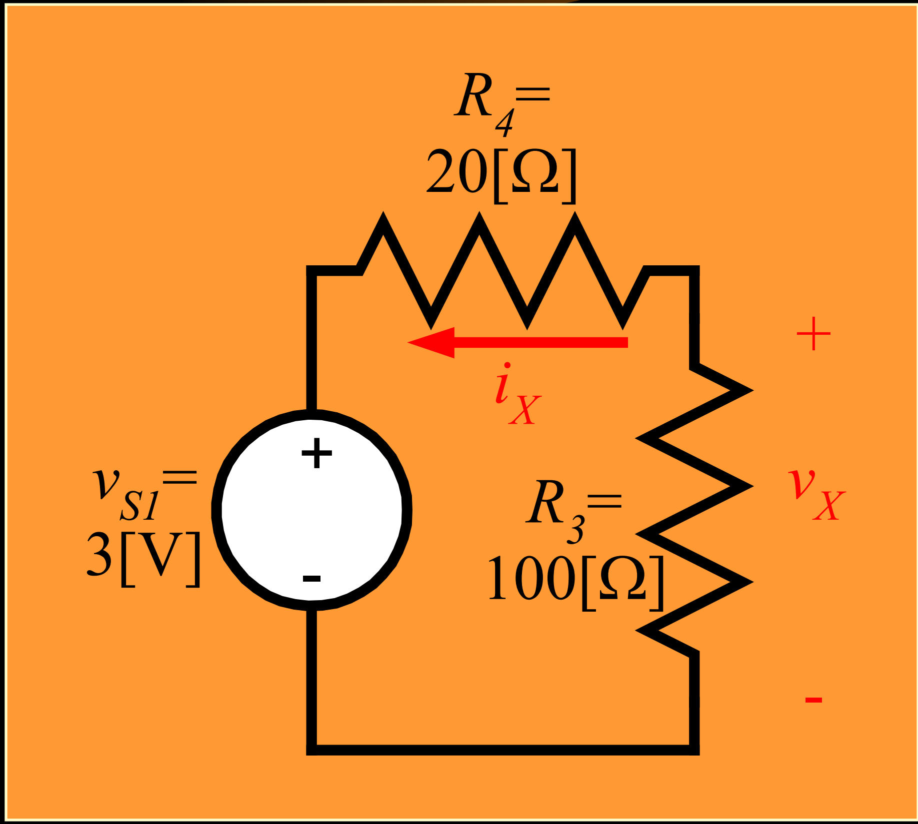
$$-v_A + v_X - v_E + v_F = 0.$$

These are the same equation. Use either method.



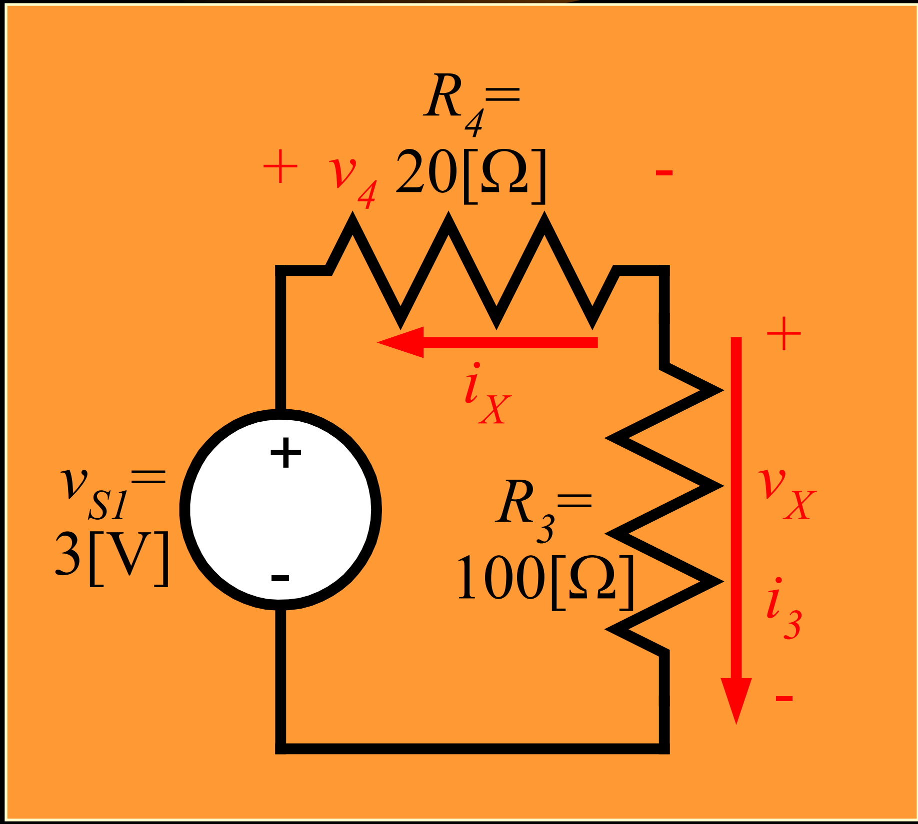
Example #1

- Let us do an example to test out our new found skills.
- In the circuit shown here, find the voltage v_X and the current i_X.



Example #1 – Step 1

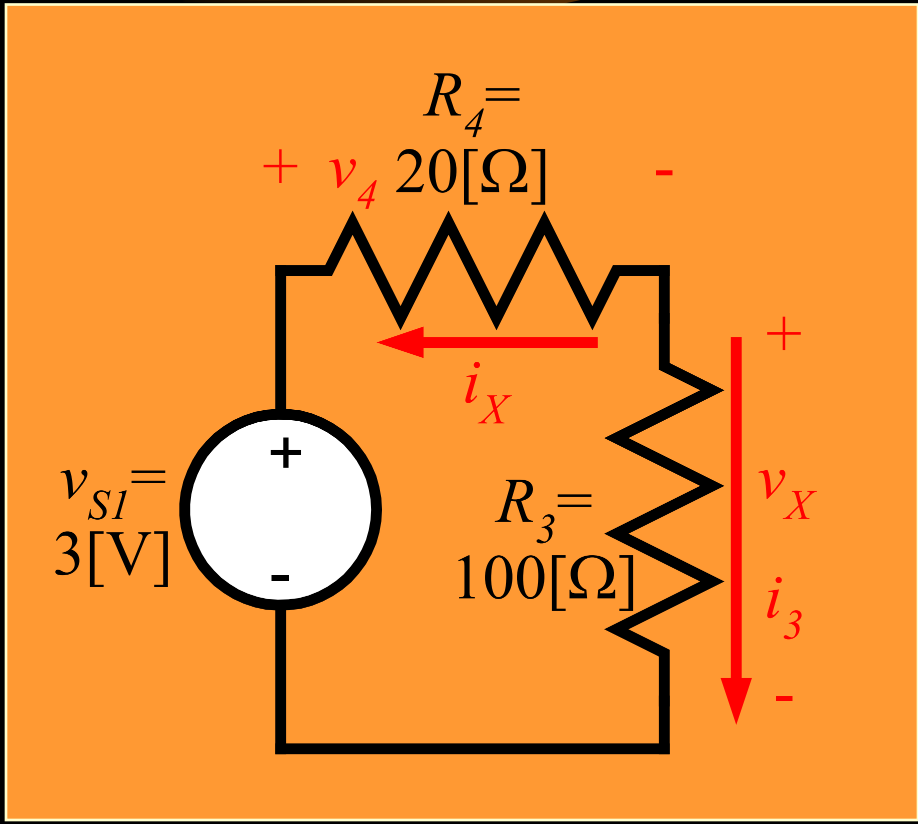
- The first step in solving is to define variables we need.
- In the circuit shown here, we will define v_4 and i_3.



Example #1 – Step 2

- The second step in solving is to write some equations. Let's start with KVL.

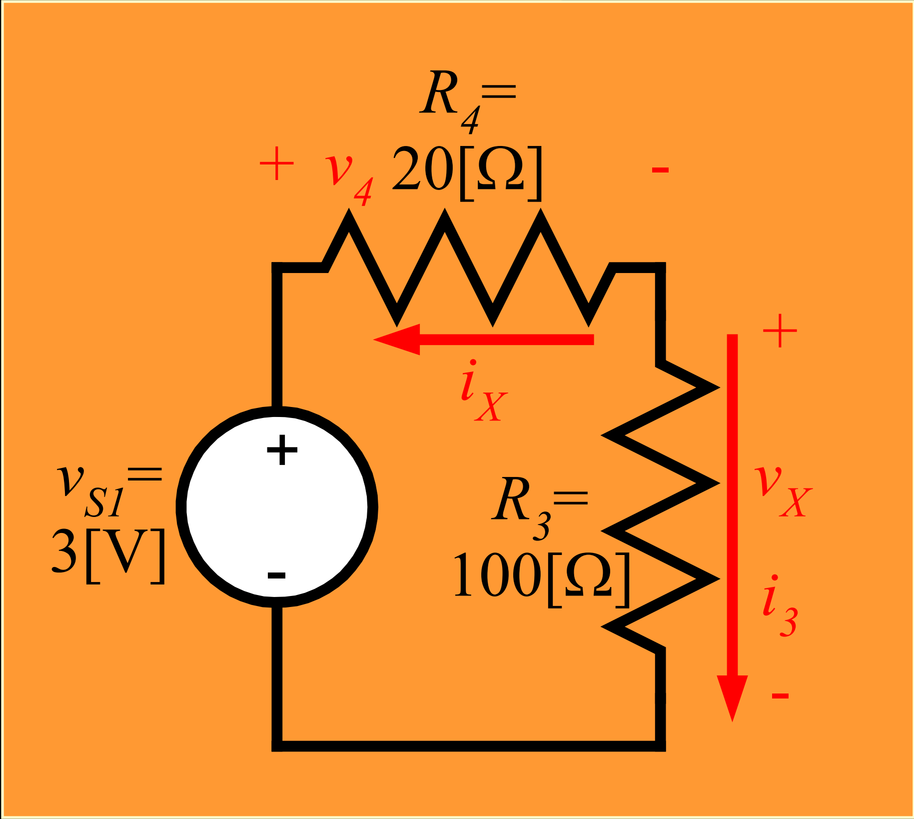
$$-v_{S1} + v_4 + v_X = 0, \text{ or}$$
$$-3[\text{V}] + v_4 + v_X = 0.$$



Example #1 – Step 3

- Now let's write Ohm's Law for the resistors.

$$v_4 = -i_X R_4, \text{ and}$$
$$v_X = i_3 R_3.$$

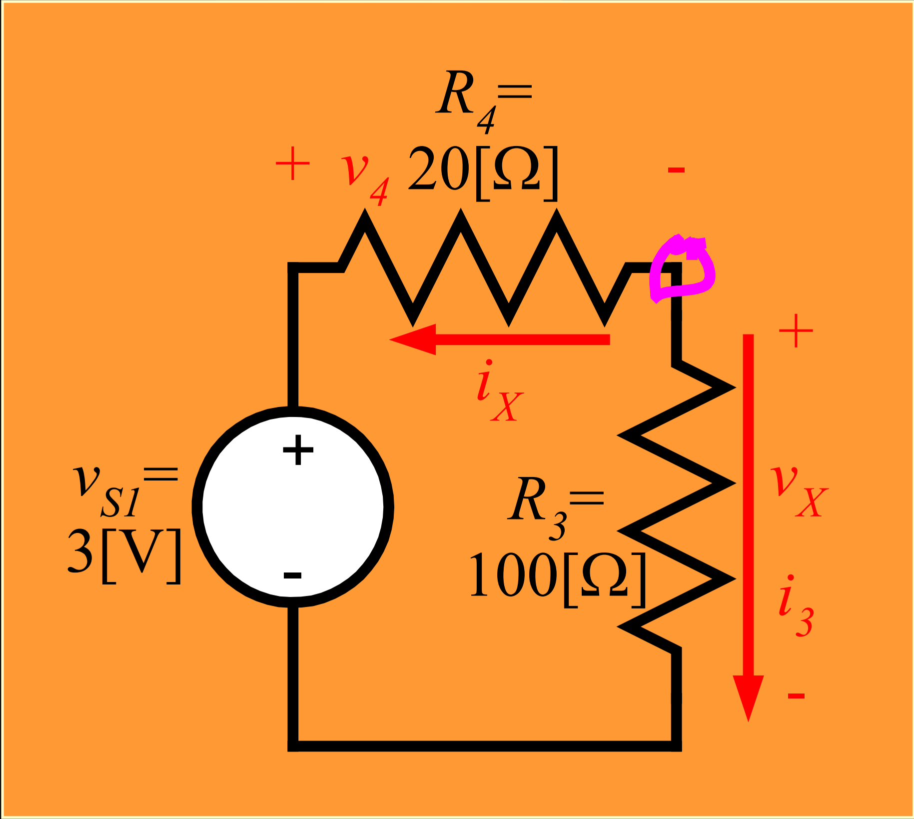


Notice that there is a **sign** in Ohm's Law.

Example #1 – Step 4

- Next, let's write KCL for the node marked in violet.

$$i_X + i_3 = 0, \text{ or}$$
$$i_3 = -i_X.$$



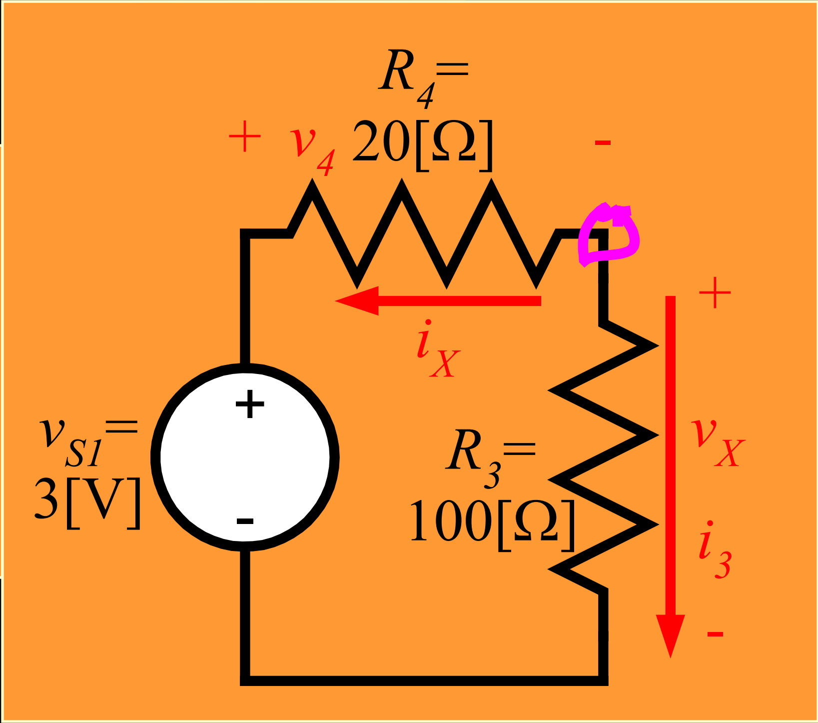
Notice that we can write KCL for a node, or any other closed surface.

Example #1 – Step 5

- We are ready to solve.

$$-3[\text{V}] - i_X 20[\Omega] - i_X 100[\Omega] = 0, \text{ or}$$

$$i_X = \frac{-3[\text{V}]}{120[\Omega]} = -25[\text{mA}].$$



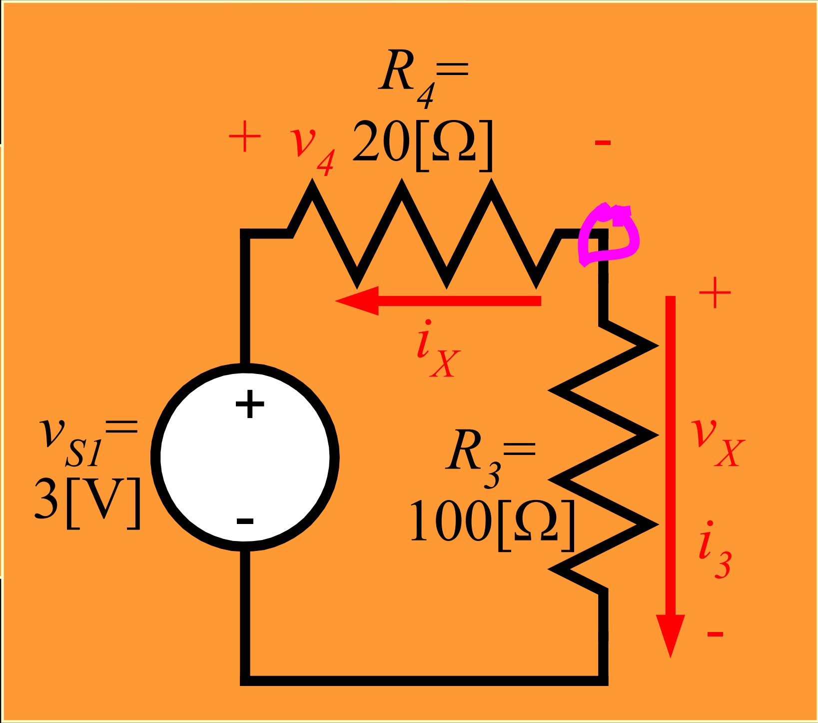
We have substituted into our KVL equation from other equations.

Example #1 – Step 6

- Next, for the other requested solution.

$$v_X = i_3 R_3 = -i_X R_3, \text{ or}$$

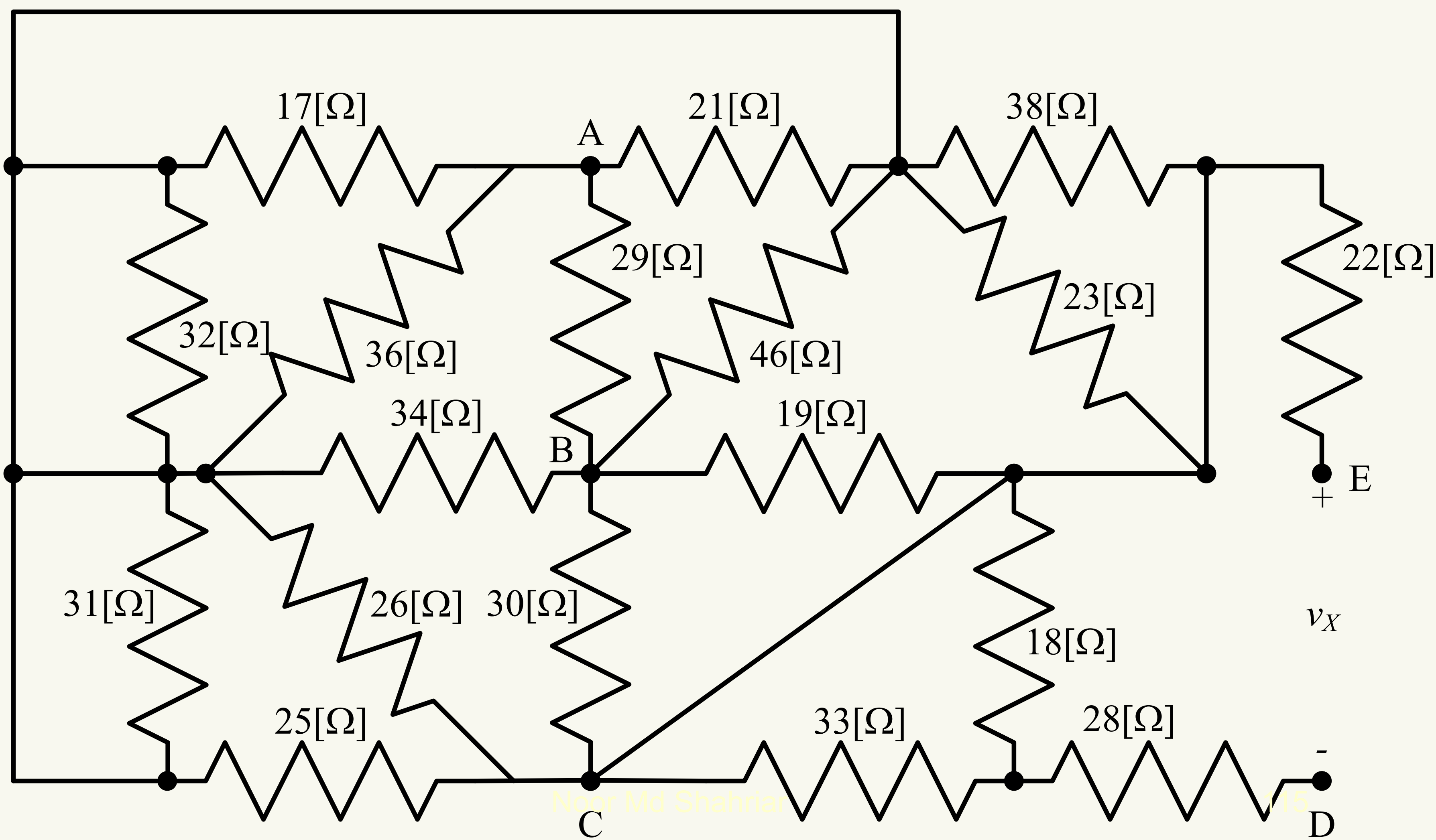
$$v_X = -(-25[\text{mA}])100[\Omega] = 2.5[\text{V}].$$



We have substituted into Ohm's Law, using our solution for i_X.

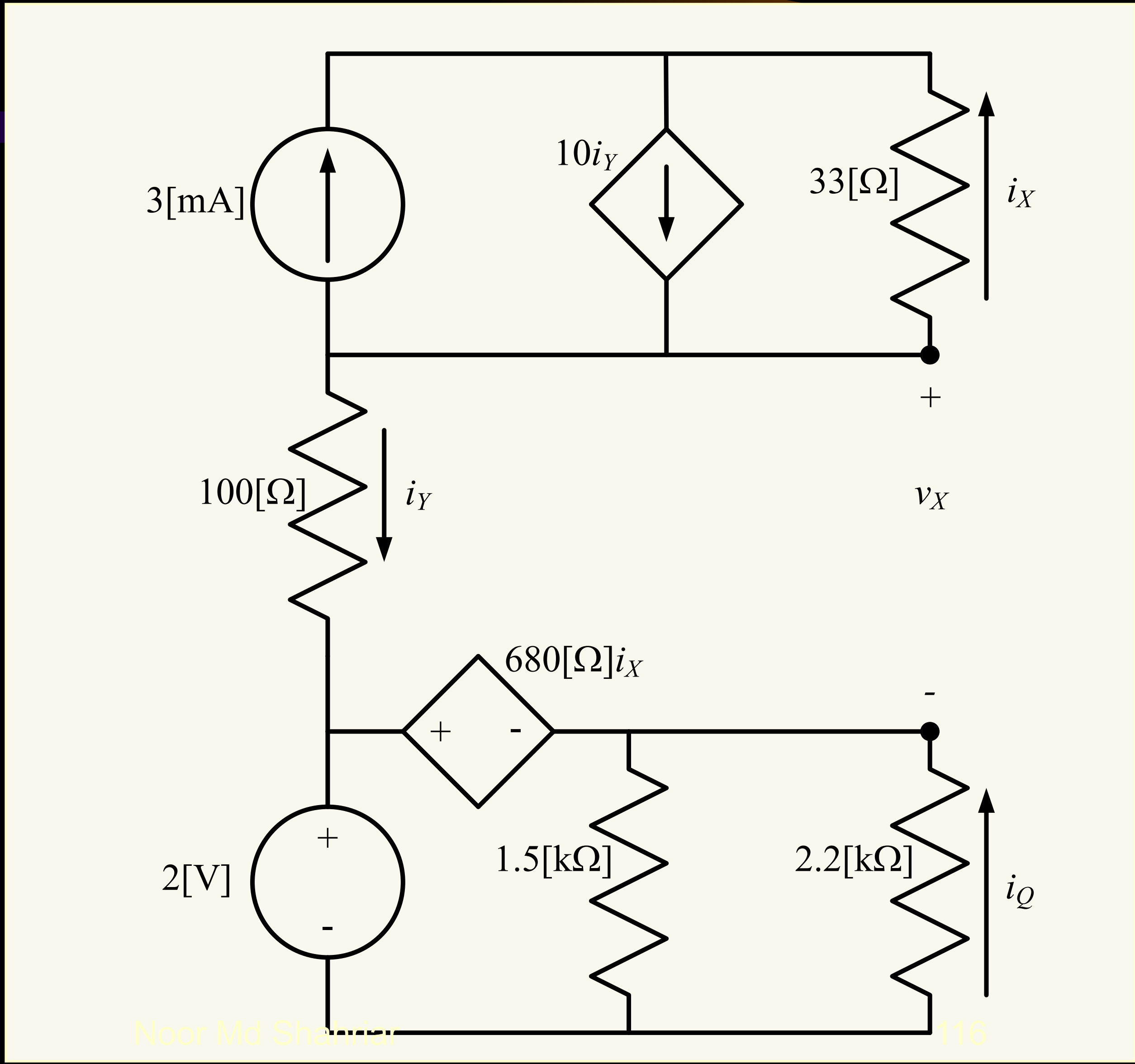
Example Problem #2

How many nodes are there in this circuit?



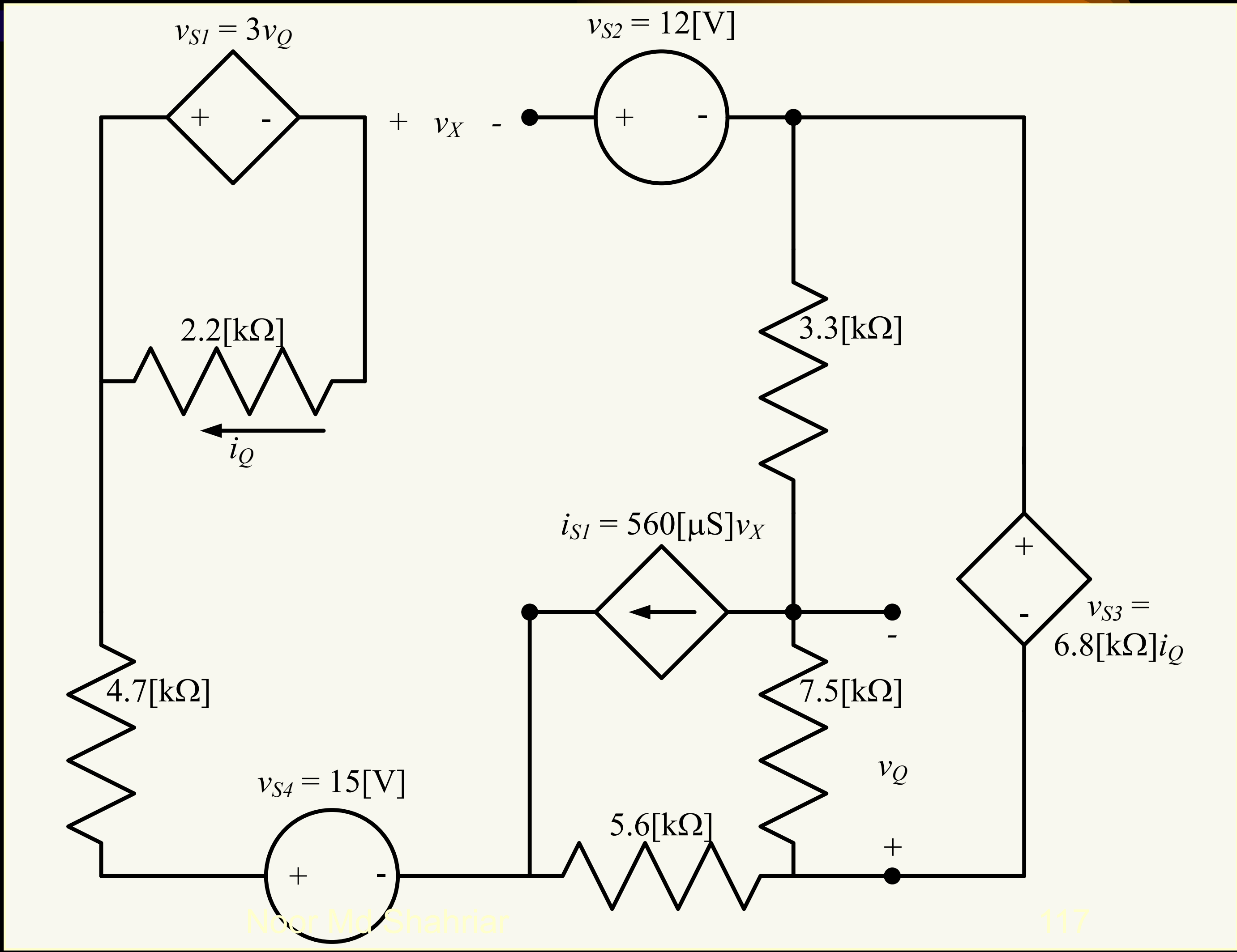
Example Problem #3

- Let's do another example. Find the voltage v_X, the currents i_X and i_Q, and the power absorbed by each of the dependent sources.



Example Problem #4

- Let's do another example. Find the voltage v_X.



This problem is taken from one edition of the Nilsson and Reidel text, "Electric Circuits".

Example #5 – Problem 2.20

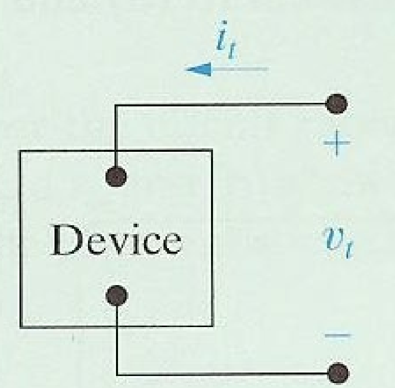
The voltage and current were measured at the terminals of the device shown in Fig. P2.20(a). The results are tabulated in Fig. P2.20(b).

a) Construct a circuit model for this device using an ideal current source and a resistor.

b) Use the model to predict the value of i_t when a 20 Ω resistor is connected across the terminals of the device.

For part a), they mean a current source in parallel with a resistance.

Figure P2.20



(a)

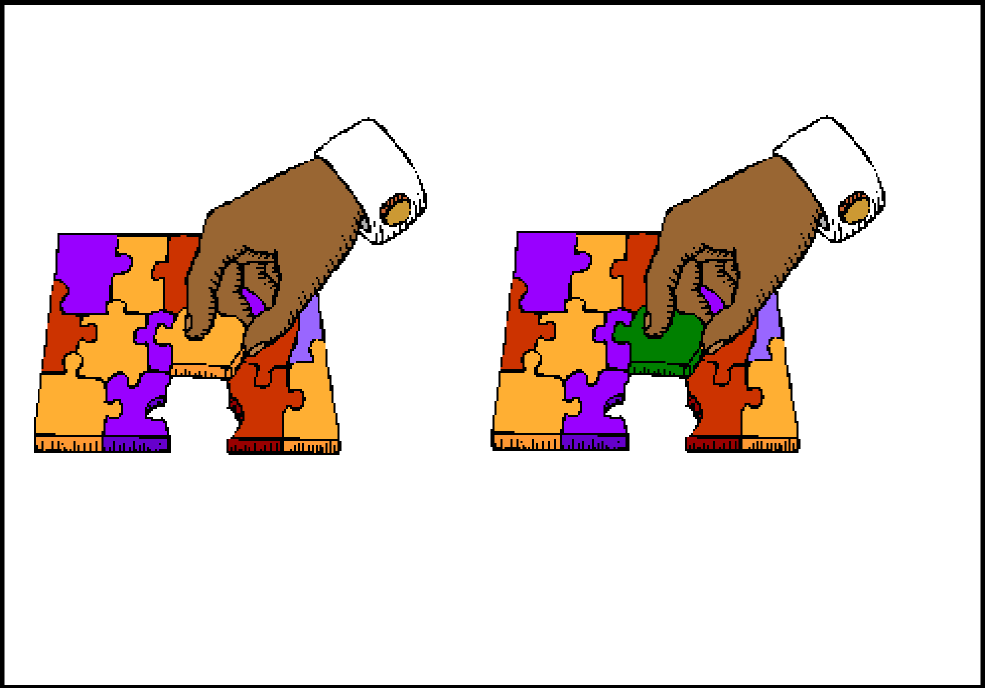
v_t (V)	i_t (A)
50	0
65	3
80	6
95	9
110	12
125	15

(b)

Week -5

Page- (120-134)

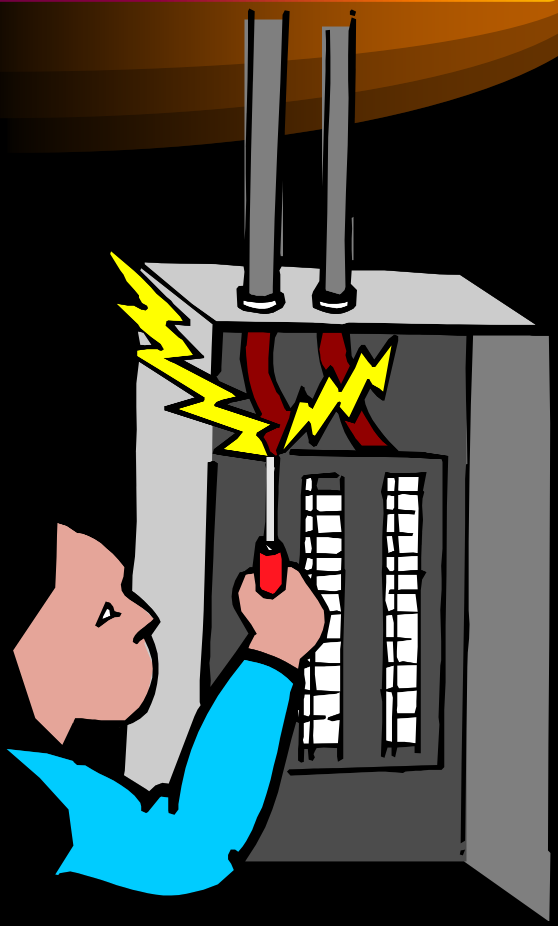
Series, Parallel, and other Resistance Equivalent Circuits



Equivalent Circuits – The Concept

Equivalent circuits are ways of looking at or solving circuits. The idea is that if we can make a circuit simpler, we can make it easier to solve, and easier to understand.

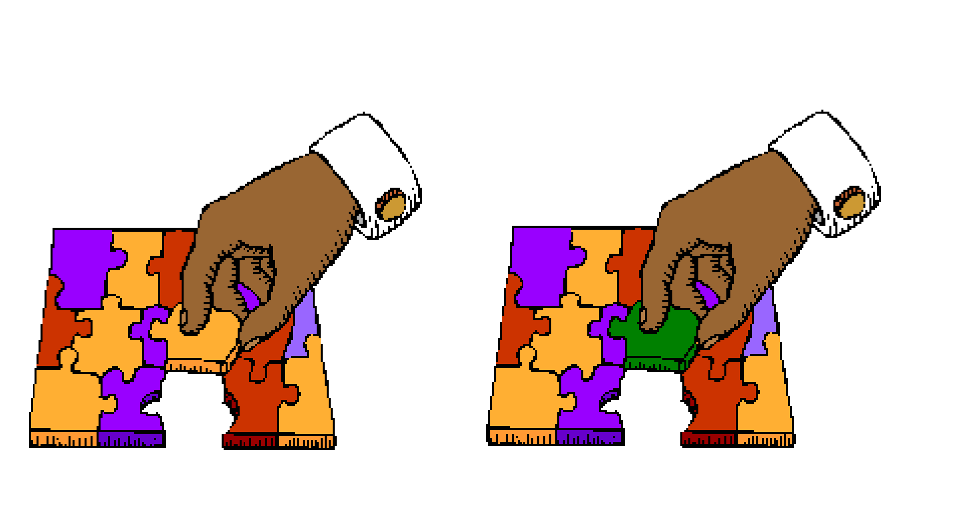
The key is to use equivalent circuits properly. After defining equivalent circuits, we will start with the simplest equivalent circuits, series and parallel combinations of resistors.



Equivalent Circuits: A Definition

Imagine that we have a circuit, and a portion of the circuit can be identified, made up of one or more parts. That portion can be replaced with another set of components, if we do it properly. We call these portions equivalent circuits.

Two circuits are considered to be equivalent if they behave the same with respect to the things to which they are connected. One can replace one circuit with another circuit, and everything else cannot tell the difference.

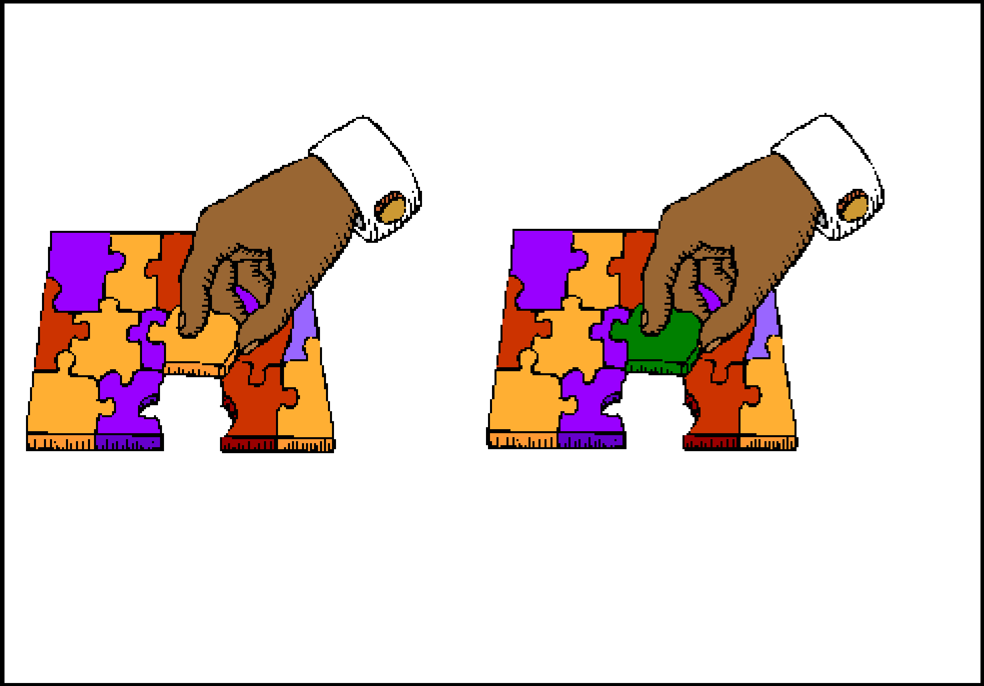


We will use an analogy for equivalent circuits here. This analogy is that of jigsaw puzzle pieces. The idea is that two different jigsaw puzzle pieces with the same shape can be thought of as equivalent, even though they are different. The rest of the puzzle does not “notice” a difference. This is analogous to the case with equivalent circuits.

Equivalent Circuits: A Definition Considered

Two circuits are considered to be equivalent if they behave the same with respect to the things to which they are connected. One can replace one circuit with another circuit, and everything else cannot tell the difference.

In this jigsaw puzzle, the rest of the puzzle cannot tell whether the yellow or the green piece is inserted. This is analogous to what happens with equivalent circuits.

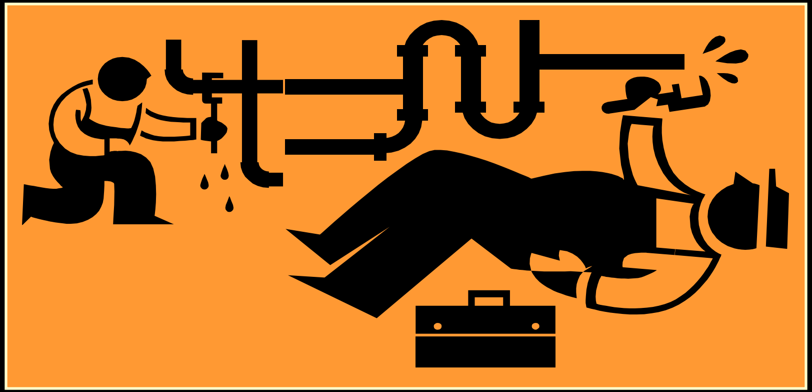


Series Combination: A Structural Definition

A Definition:

Two parts of a circuit are in series if the same current flows through both of them.

Note: It must be more than just the same value of current in the two parts. The same exact charge carriers need to go through one, and then the other, part of the circuit.

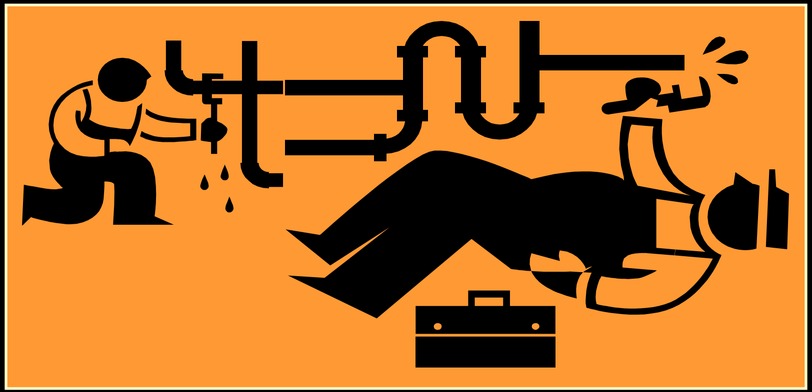


Series Combination: Hydraulic Version of the Definition

A Definition:

Two parts of a circuit are in series if the same current flows through both of them.

A hydraulic analogy: Two water pipes are in series if every drop of water that goes through one pipe, then goes through the other pipe.

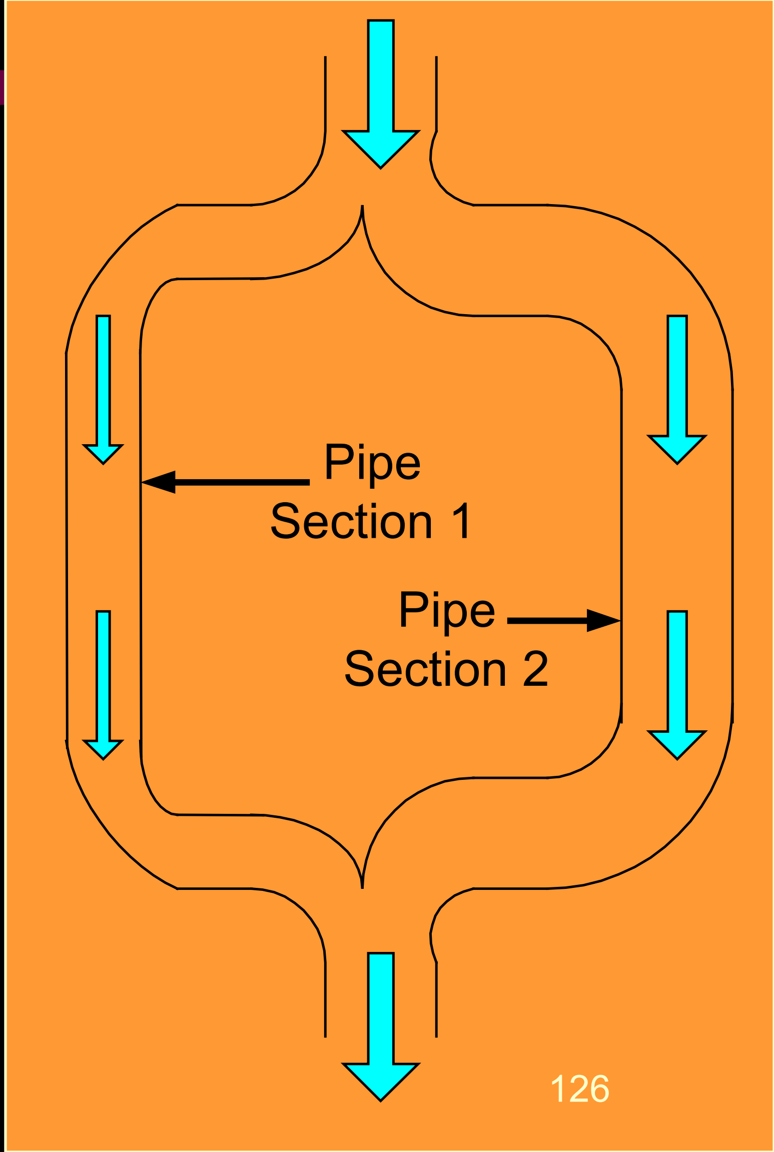


Parallel Combination: A Structural Definition

A Definition:

Two parts of a circuit are in parallel if the same voltage is across both of them.

Note: It must be more than just the same value of the voltage in the two parts. The same exact voltage must be across each part of the circuit. In other words, the two end points must be connected together.

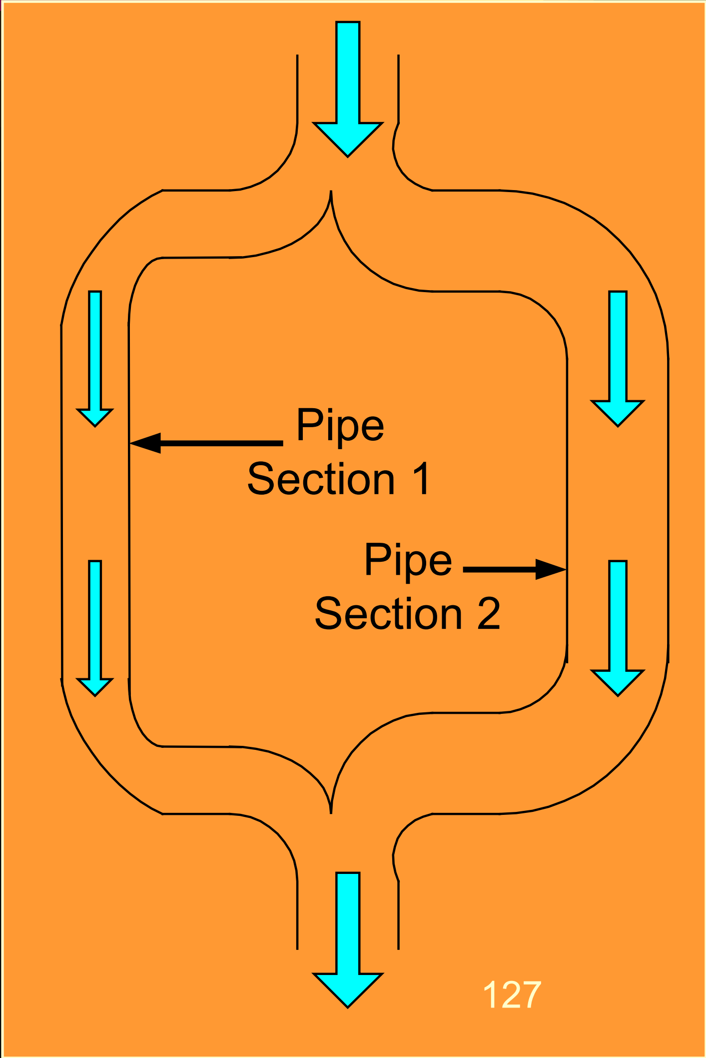


Parallel Combination: Hydraulic Version of the Definition

A Definition:

Two parts of a circuit are in parallel if the same voltage is across both of them.

A hydraulic analogy: Two water pipes are in parallel the two pipes have their ends connected together. The analogy here is between voltage and height. The difference between the height of two ends of a pipe, must be the same as that between the two ends of another pipe, if the two pipes are connected together.

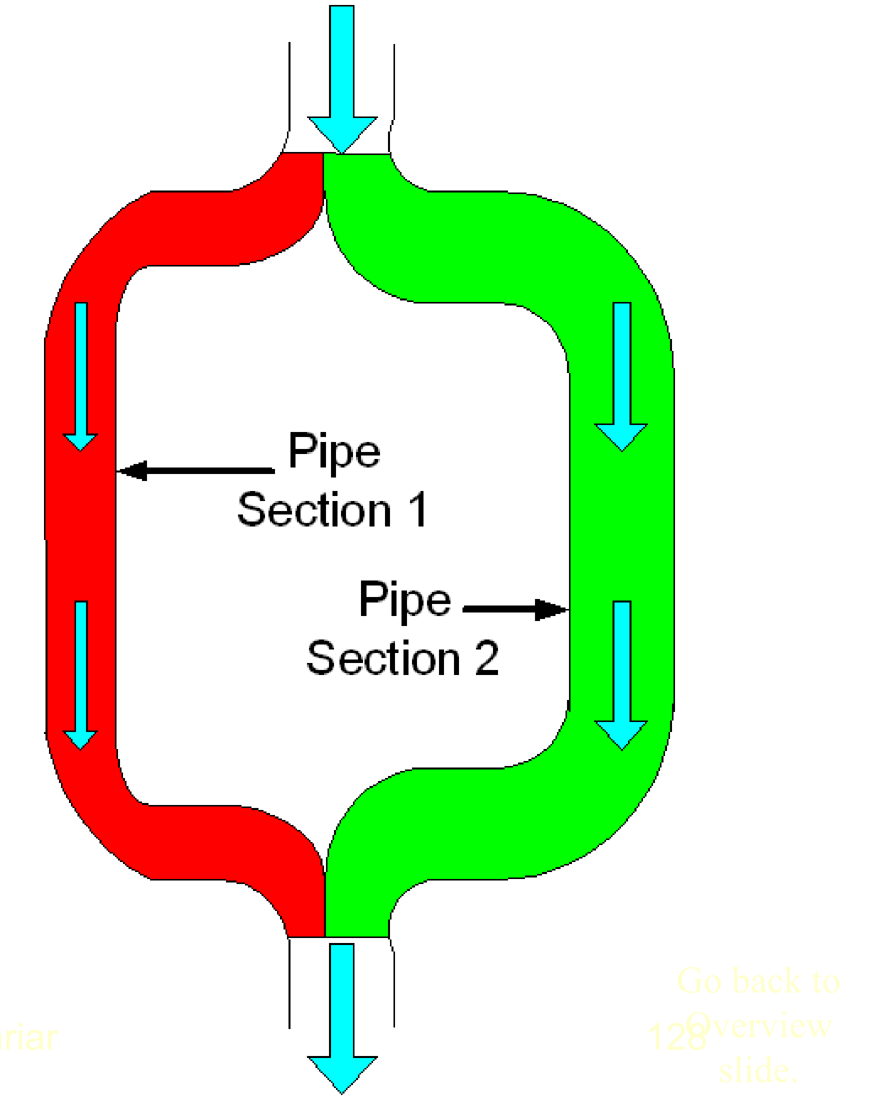


Parallel Combination: A Hydraulic Example

A Definition:

Two parts of a circuit are in parallel if the same voltage is across both of them.

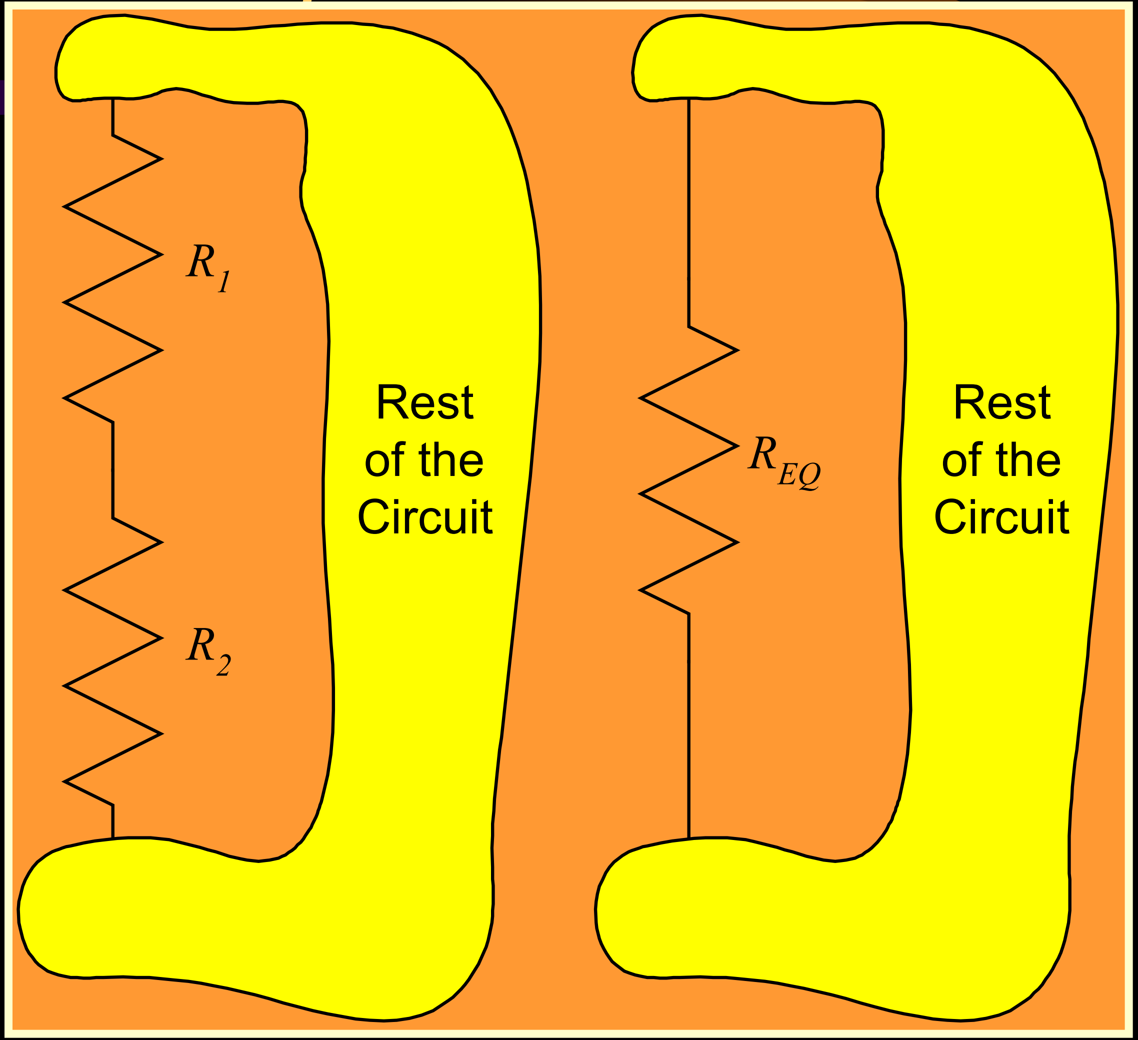
A hydraulic analogy: Two water pipes are in parallel if the two pipes have their ends connected together. The Pipe Section 1 (in red) and Pipe Section 2 (in green) in this set of water pipes are in parallel. Their ends are connected together.



Series Resistors Equivalent Circuits

Two series resistors, R_1 and R_2, can be replaced with an equivalent circuit with a single resistor R_{EQ}, as long as

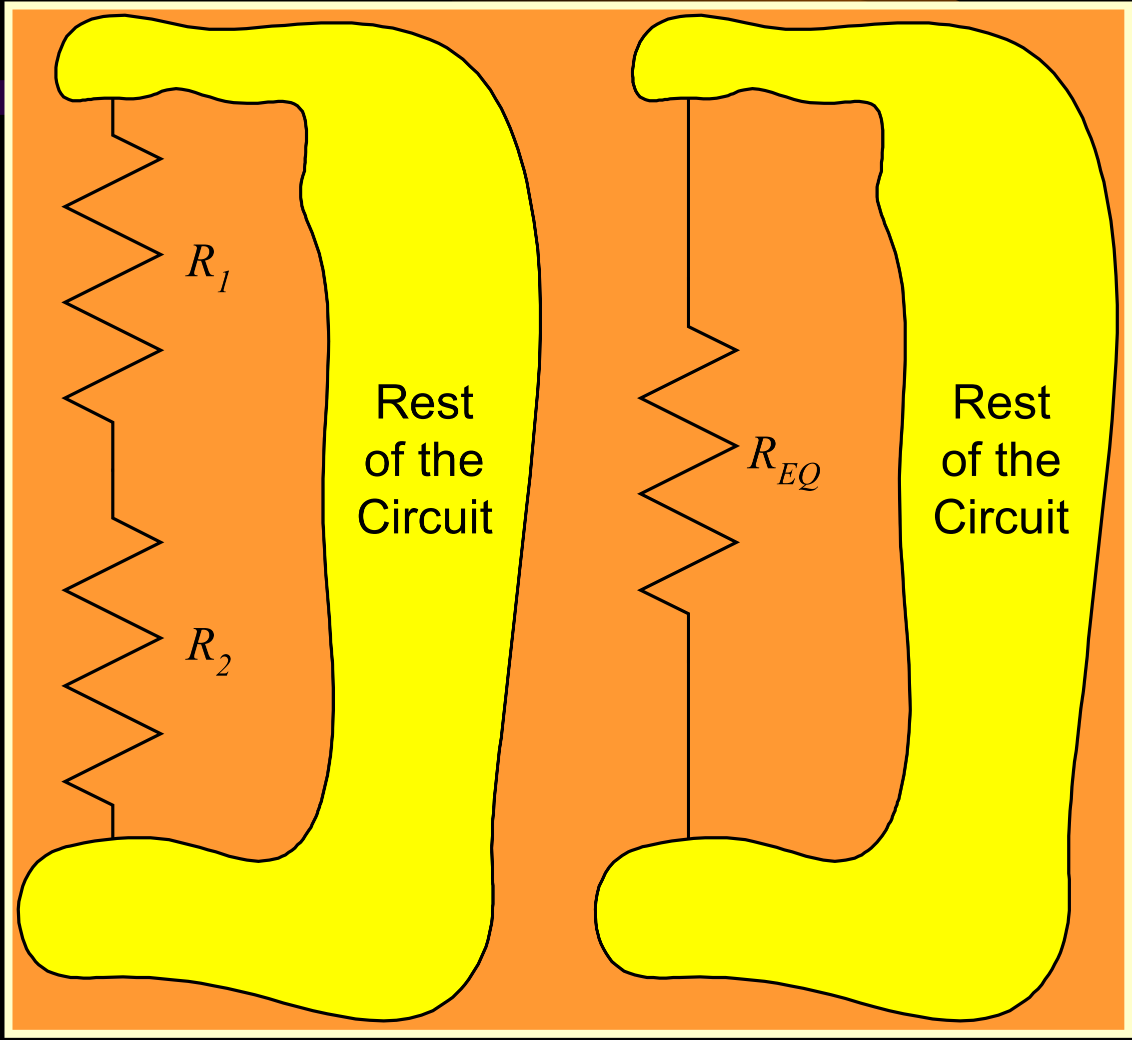
$$R_{EQ} = R_1 + R_2.$$



More than 2 Series Resistors

This rule can be extended to more than two series resistors. In this case, for N series resistors, we have

$$R_{EQ} = R_1 + R_2 + \ldots + R_N .$$

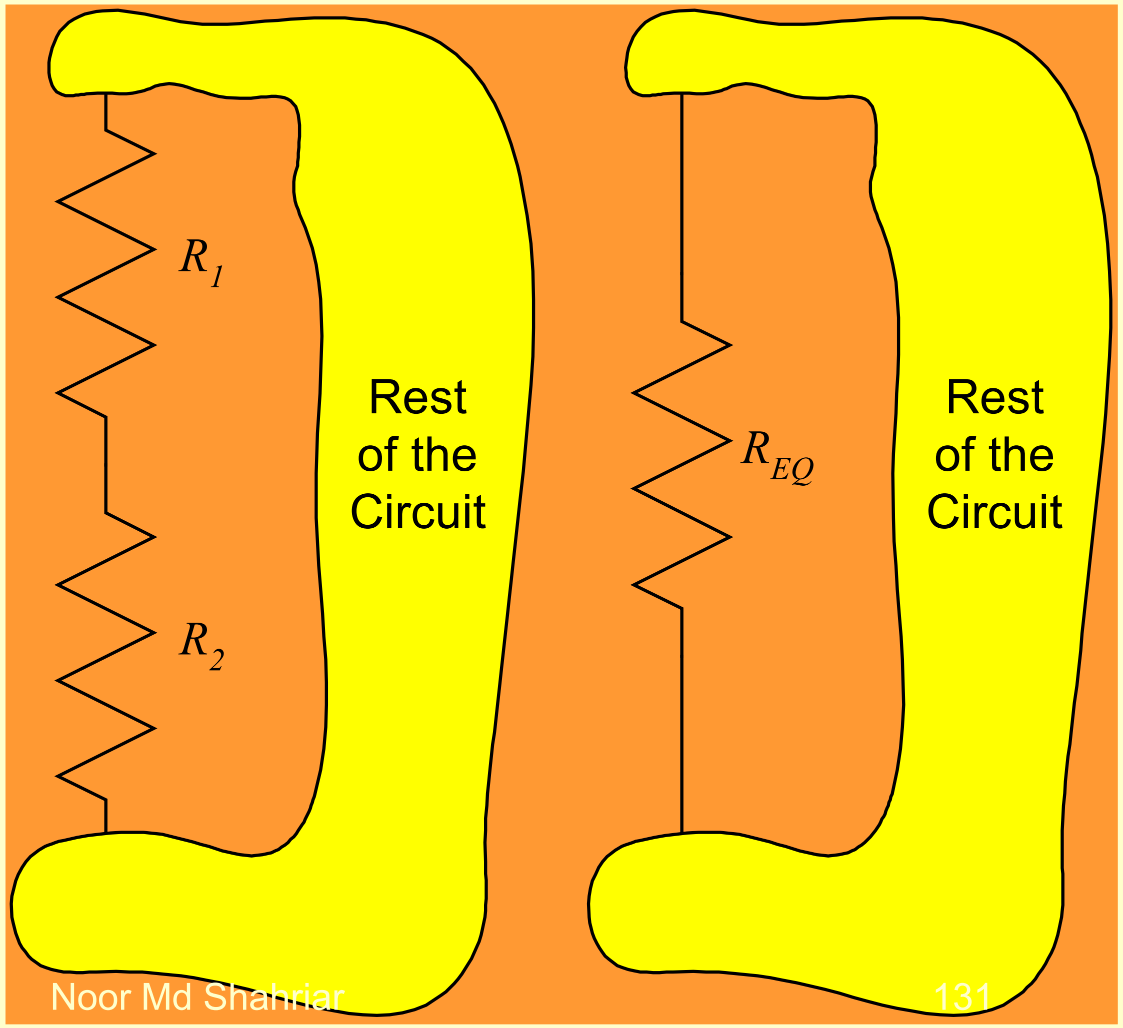


Series Resistors Equivalent Circuits: A Reminder

Two series resistors, R_1 and R_2, can be replaced with an equivalent circuit with a single resistor R_{EQ}, as long as

$$R_{EQ} = R_1 + R_2.$$

Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)

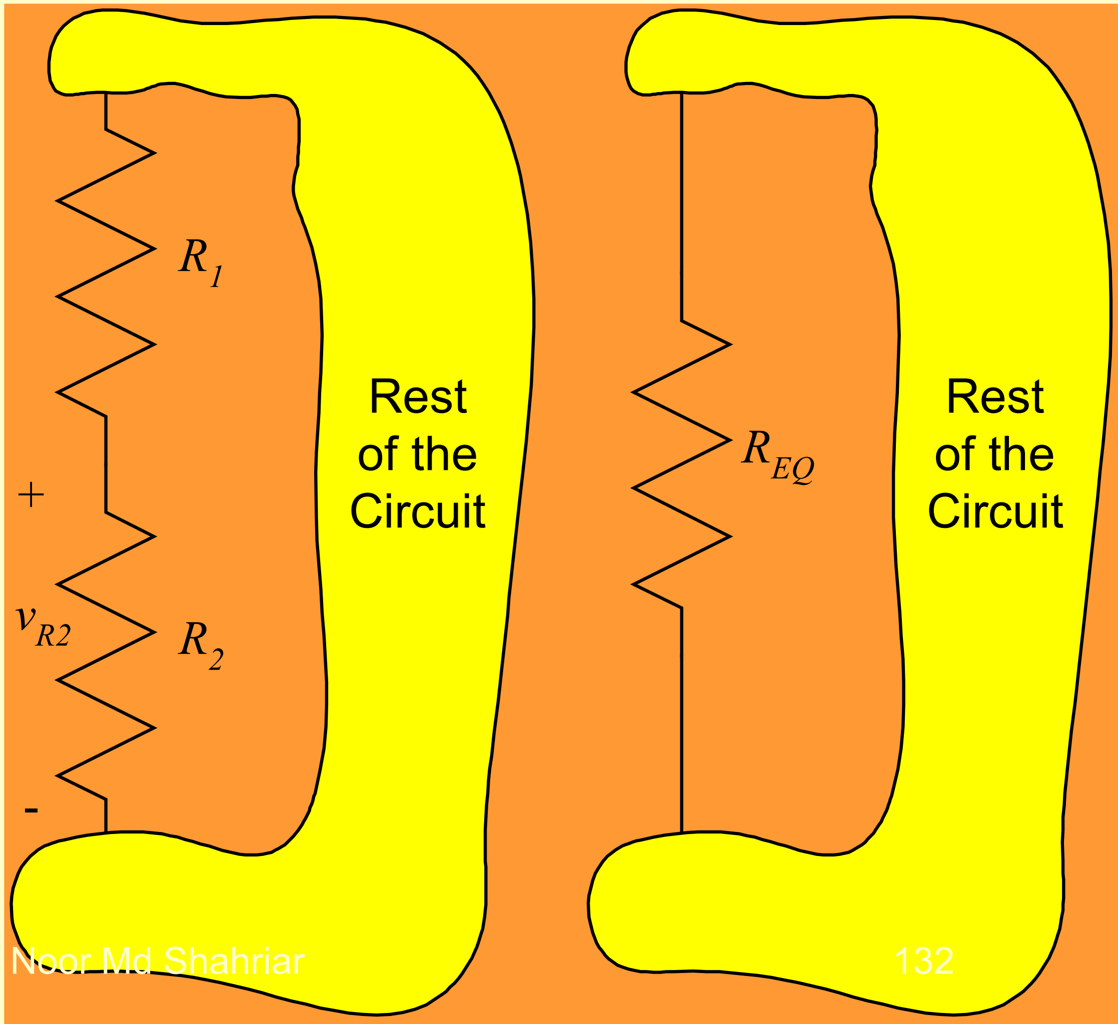


Series Resistors Equivalent Circuits: Another Reminder

Resistors R_1 and R_2 can be replaced with a single resistor R_{EQ}, as long as

$$R_{EQ} = R_1 + R_2.$$

Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.) ***The voltage v_{R2} does not exist in the right hand equivalent.***



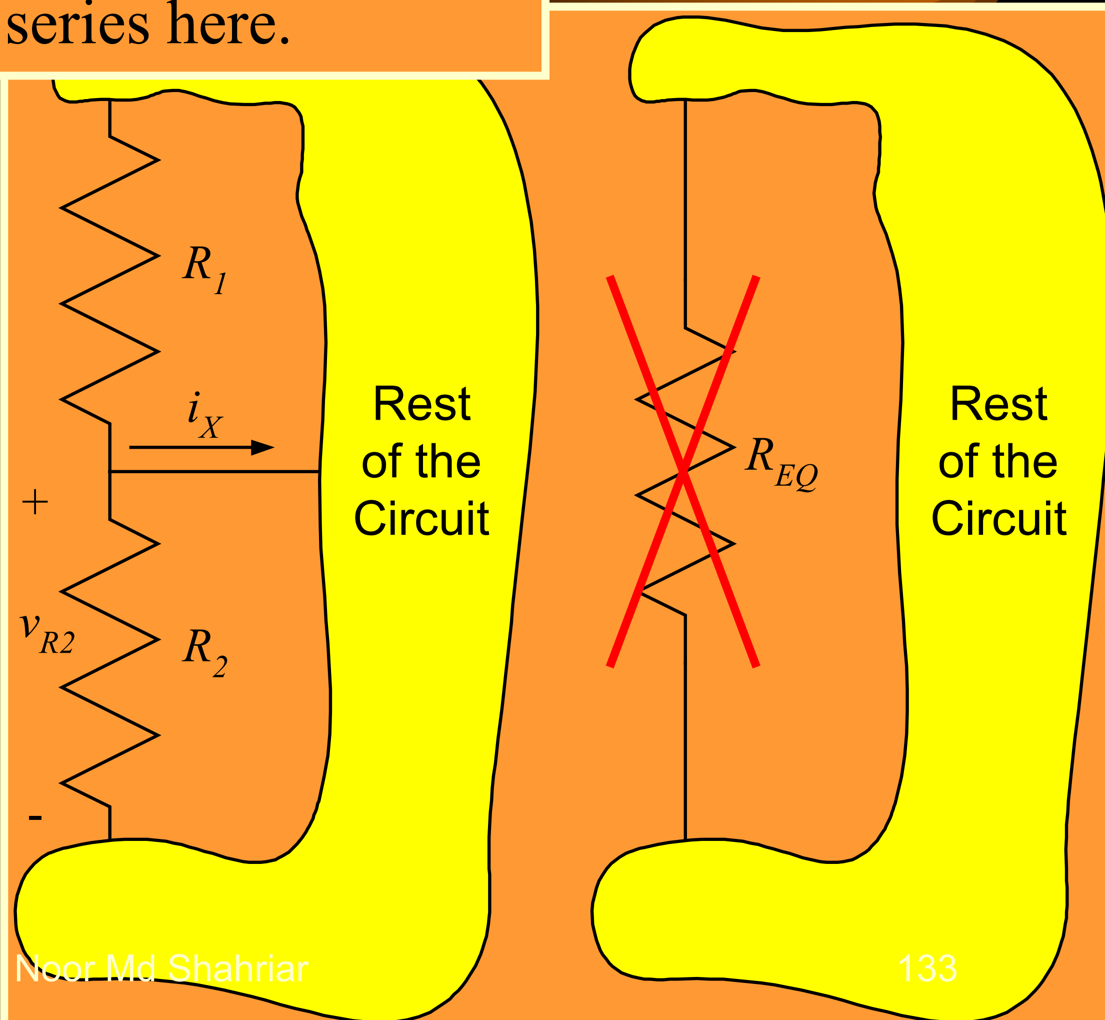
The Resistors Must be in Series

Resistors R_1 and R_2 can be replaced with a single resistor R_{EQ}, as long as

$$R_{EQ} = R_1 + R_2.$$

Remember also that these two equivalent circuits are equivalent ***only when R_1 and R_2 are in series***. If there is something connected to the node between them, and it carries current, ($i_X \neq 0$) then this does not work.

R_1 and R_2 are not in series here.



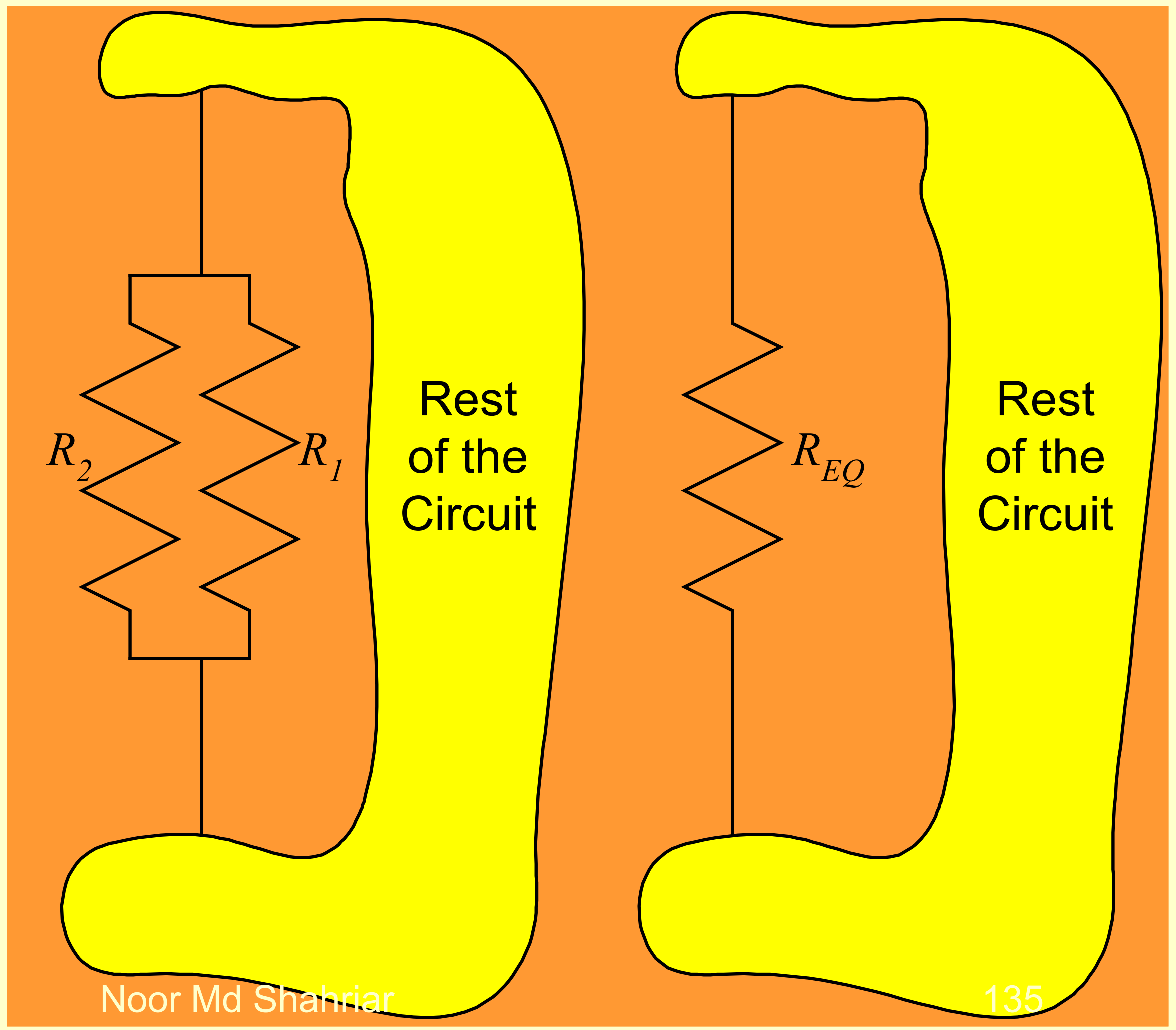
Week -6

Page- (135-147)

Parallel Resistors Equivalent Circuits

Two parallel resistors, R_1 and R_2, can be replaced with an equivalent circuit with a single resistor R_{EQ}, as long as

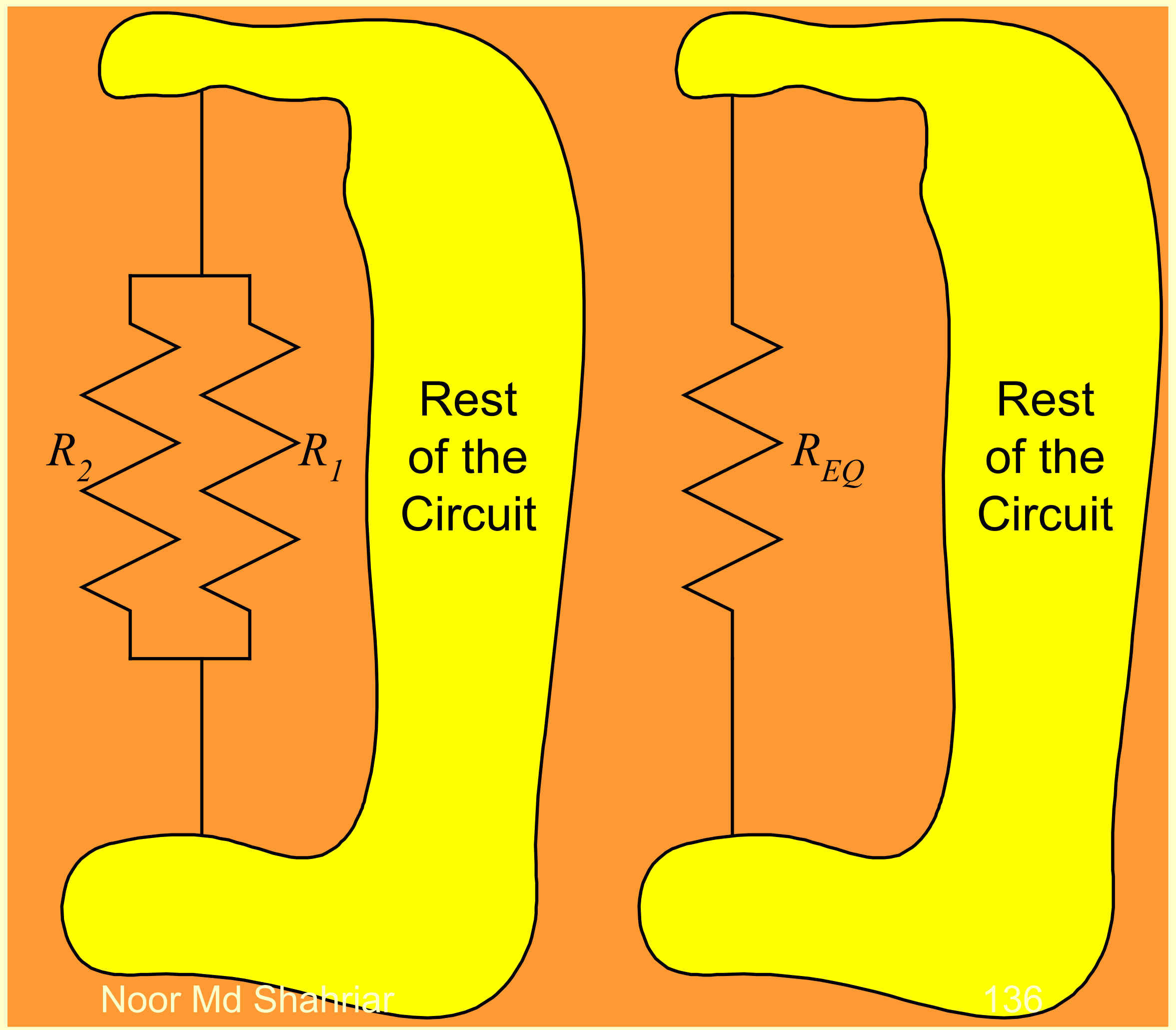
$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}.$$



More than 2 Parallel Resistors

This rule can be extended to more than two parallel resistors. In this case, for *N* parallel resistors, we have

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}.$$

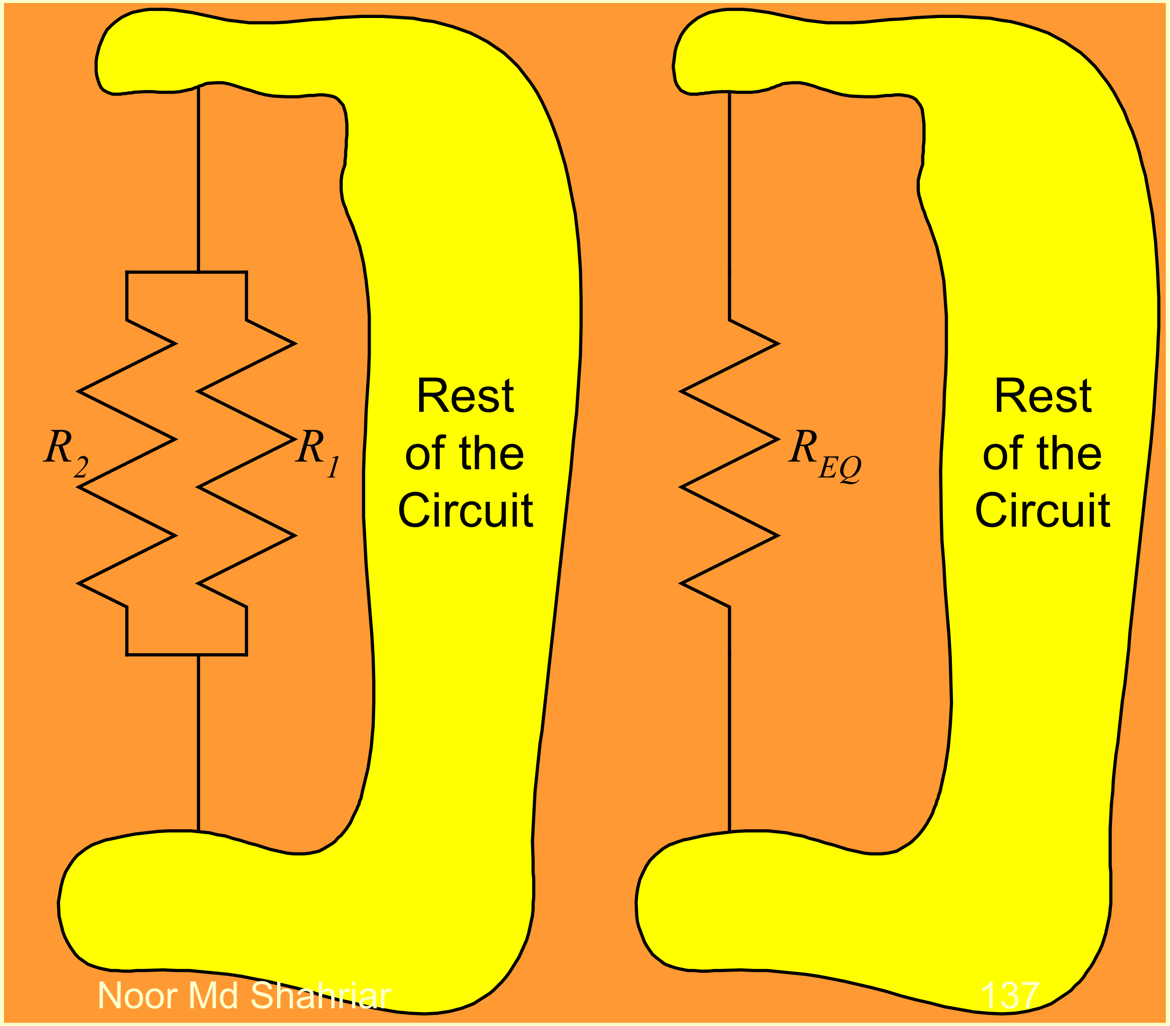


Parallel Resistors Notation

We have a special notation for this operation. When two things, Thing1 and Thing2, are in parallel, we write Thing1||Thing2 to indicate this. So, we can say that

$$\text{if } \frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2},$$

$$\text{then } R_{EQ} = R_1 \parallel R_2.$$

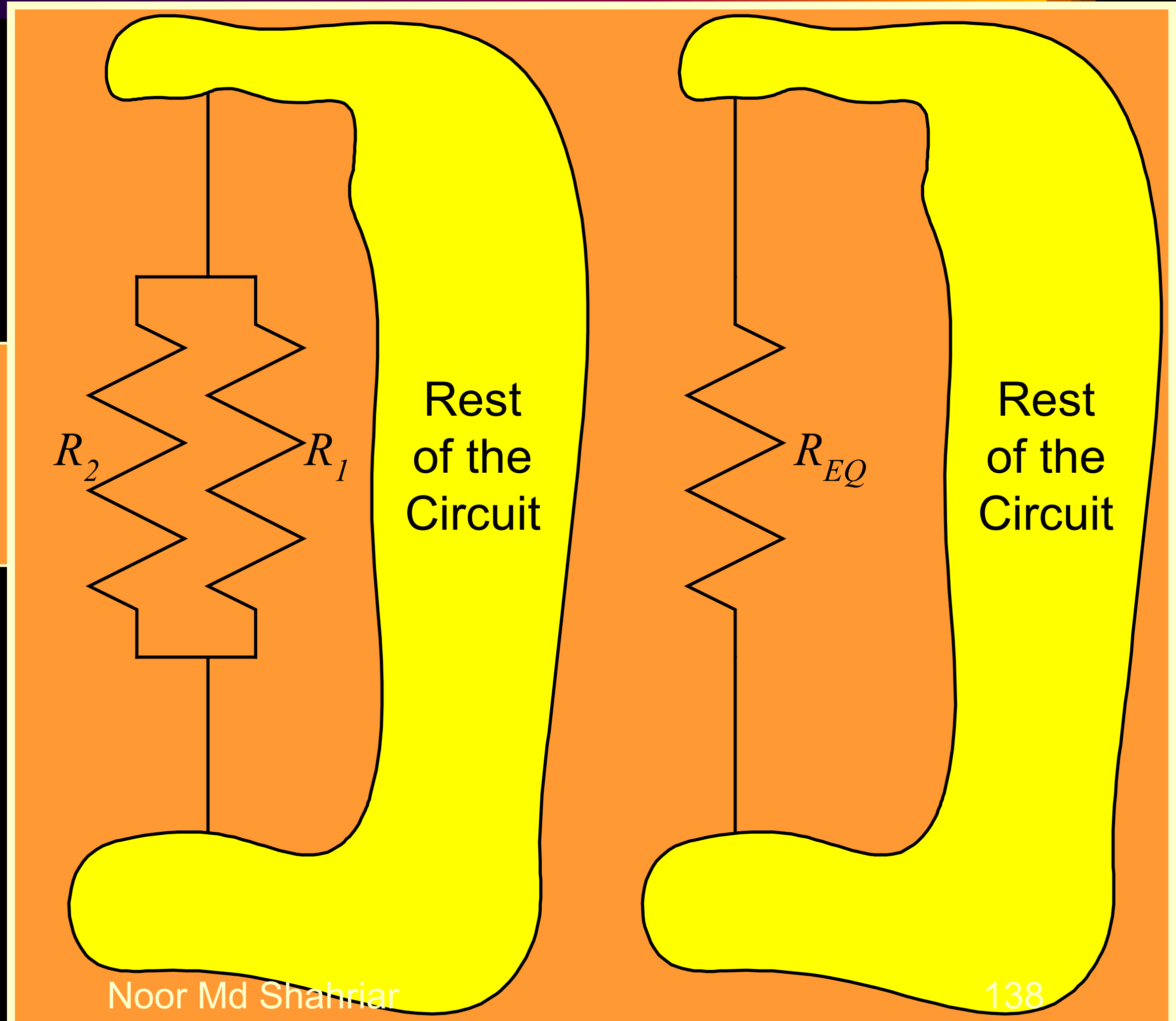


Parallel Resistor Rule for 2 Resistors

When there are only two resistors, then you can perform the algebra, and find that

$$R_{EQ} = R_1 \,||\, R_2 = \frac{R_1 R_2}{R_1 + R_2}.$$

This is called the product-over-sum rule for parallel resistors. Remember that the product-over-sum rule ***only works for two resistors***, not for three or more.

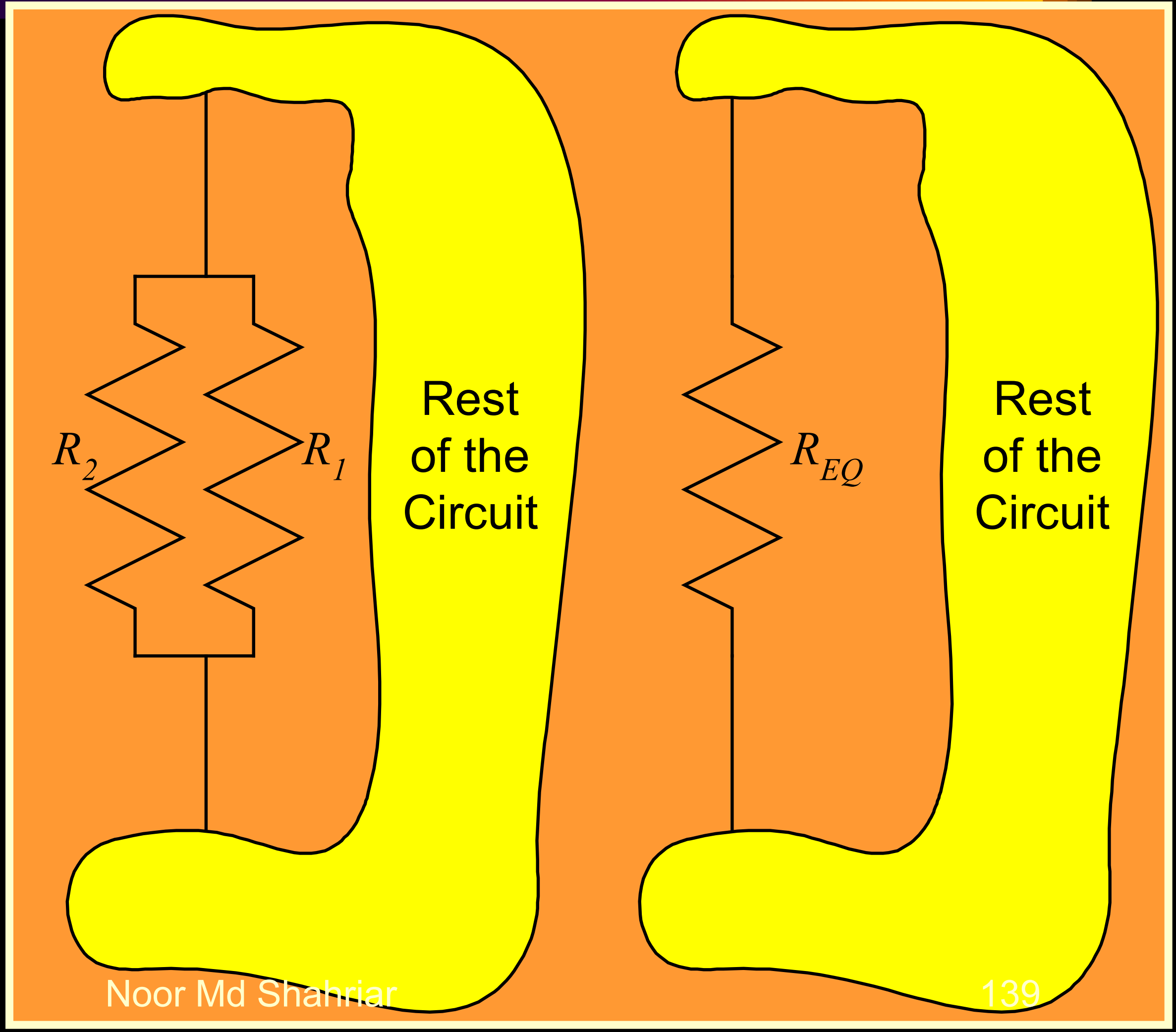


Parallel Resistors Equivalent Circuits: A Reminder

Two parallel resistors, R_1 and R_2, can be replaced with a single resistor R_{EQ}, as long as

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

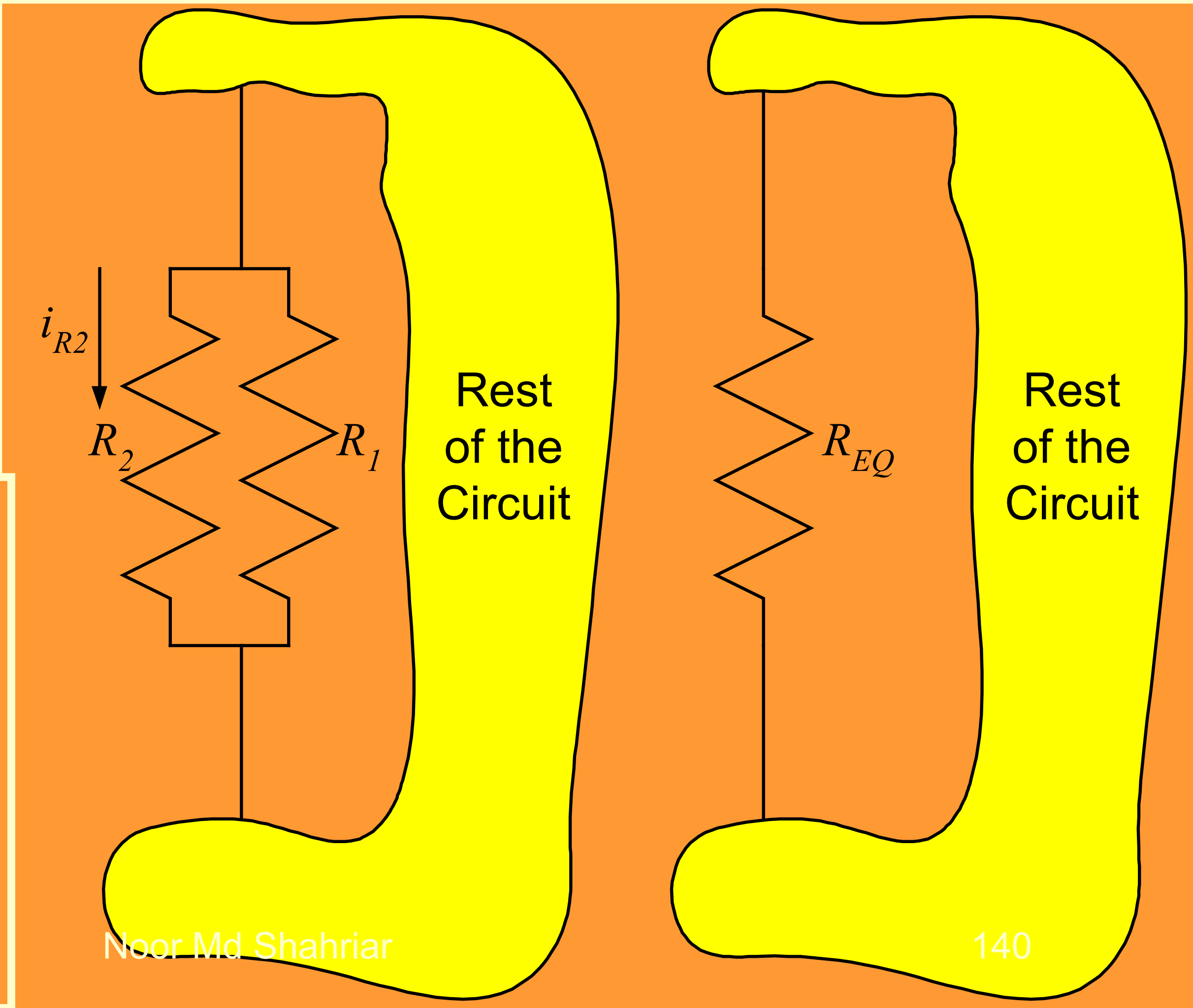
Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)



Parallel Resistors
Equivalent Circuits: Another Reminder

Two parallel resistors, R_1 and R_2, can be replaced with R_{EQ}, as long as

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}.$$



Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.) ***The current i_{R2} does not exist in the right hand equivalent.***

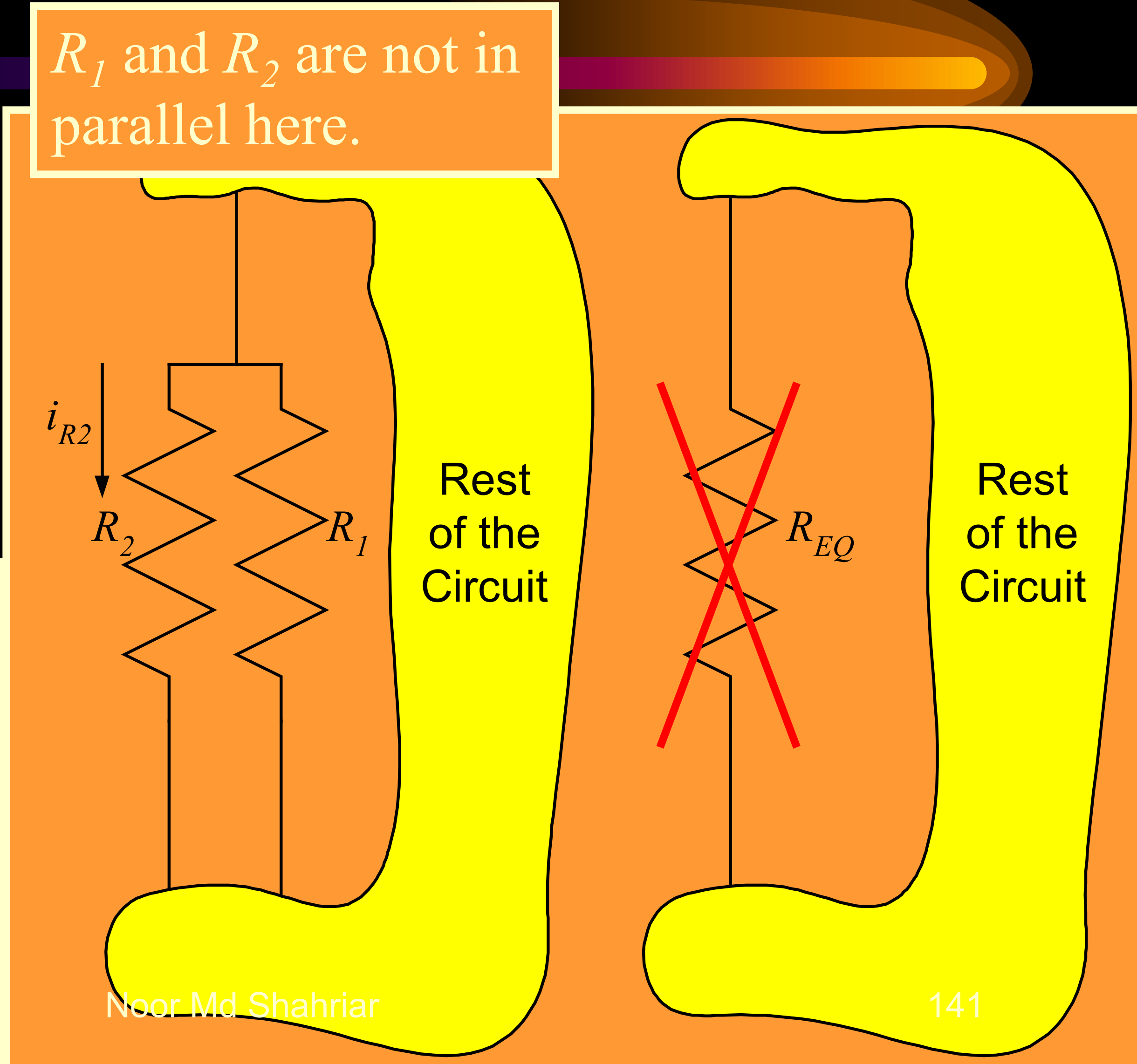
The Resistors Must be in Parallel

Go back to Overview slide.

Two parallel resistors, R_1 and R_2, can be replaced with R_{EQ}, as long as

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Remember also that these two equivalent circuits are equivalent ***only when R_1 and R_2 are in parallel***. If the two terminals of the resistors are not connected together, then this does not work.



Why are we doing this? Isn't all this obvious?

- This is a good question.
- Indeed, most students come to the study of engineering circuit analysis with a little background in circuits. Among the things that they believe that they do know is the concept of series and parallel.
- However, once complicated circuits are encountered, the simple rules that some students have used to identify series and parallel combinations can fail. We need rules that will always work.

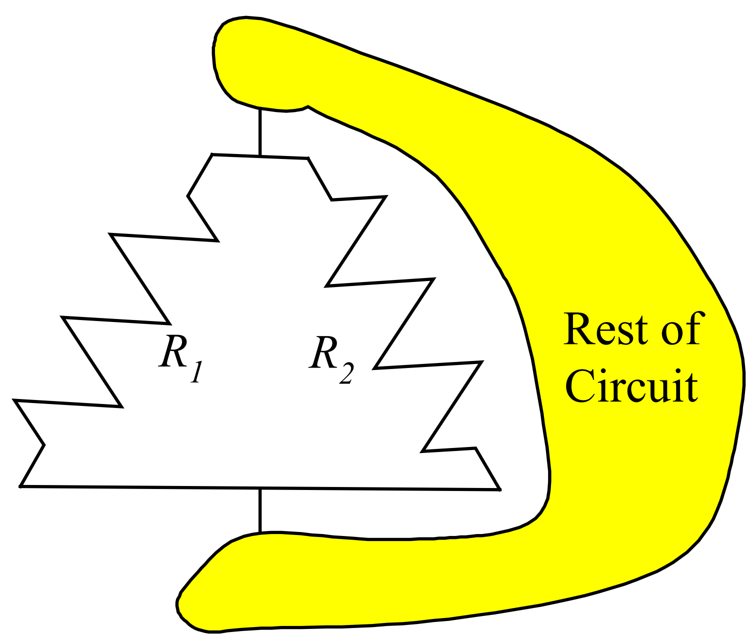
Go back to Overview slide.

Why It Isn't Obvious

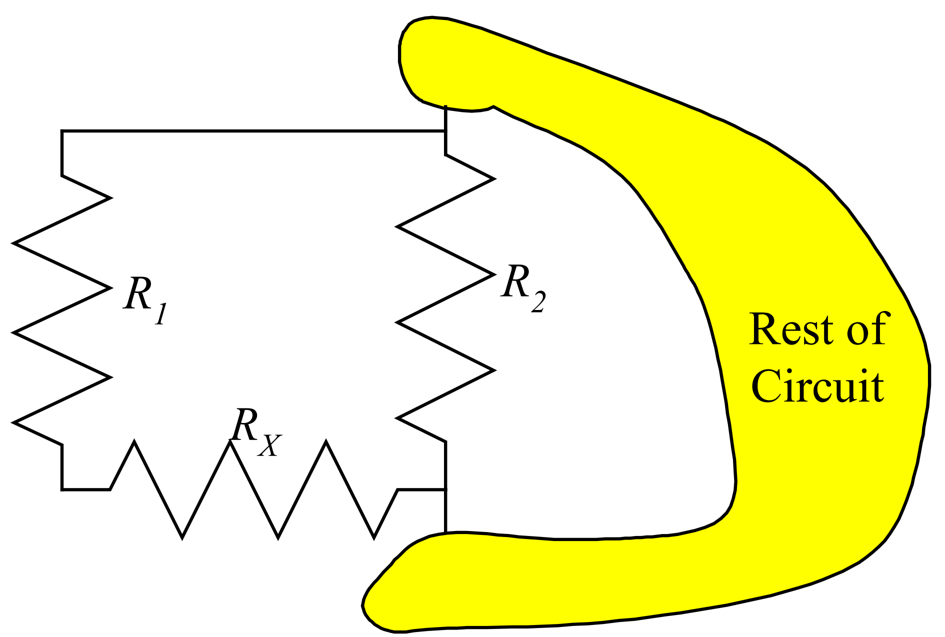
- The problem for students in many cases is that they identify series and parallel by the orientation and position of the resistors, and not by the way they are connected.
- In the case of parallel resistors, the resistors do not have to be drawn "parallel", that is, along lines with the same slope. The angle does not matter. Only the nature of the connection matters.
- In the case of series resistors, they do not have to be drawn along a single line. The alignment does not matter. Only the nature of the connection matters.

Examples (Parallel)

- Some examples are given here.



R_1 and R_2 are in parallel

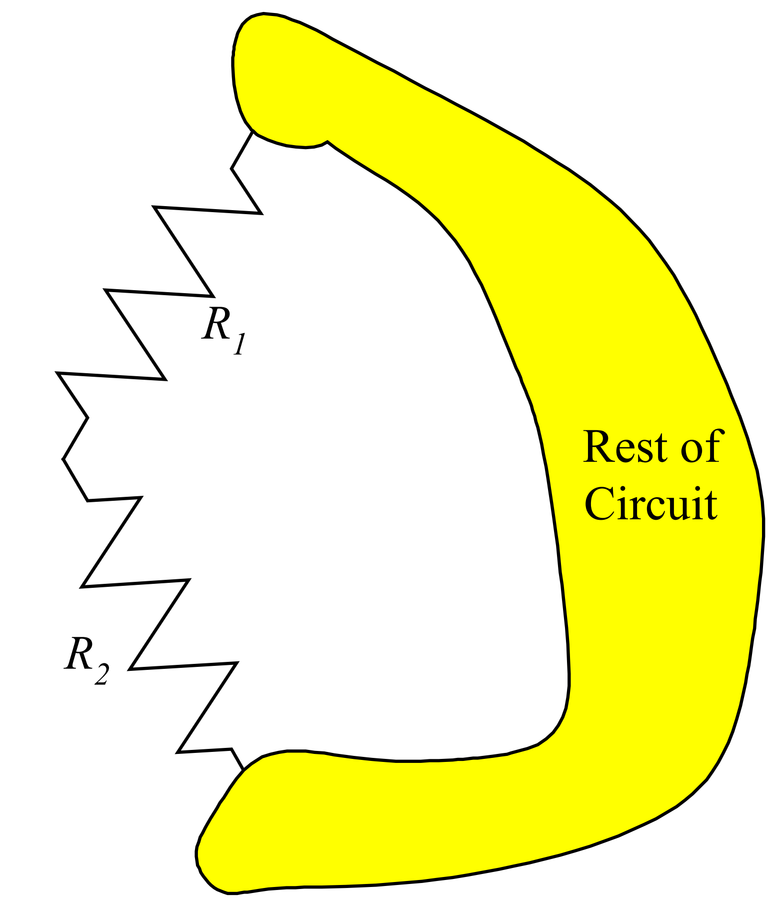


R_1 and R_2 are ***not*** in parallel

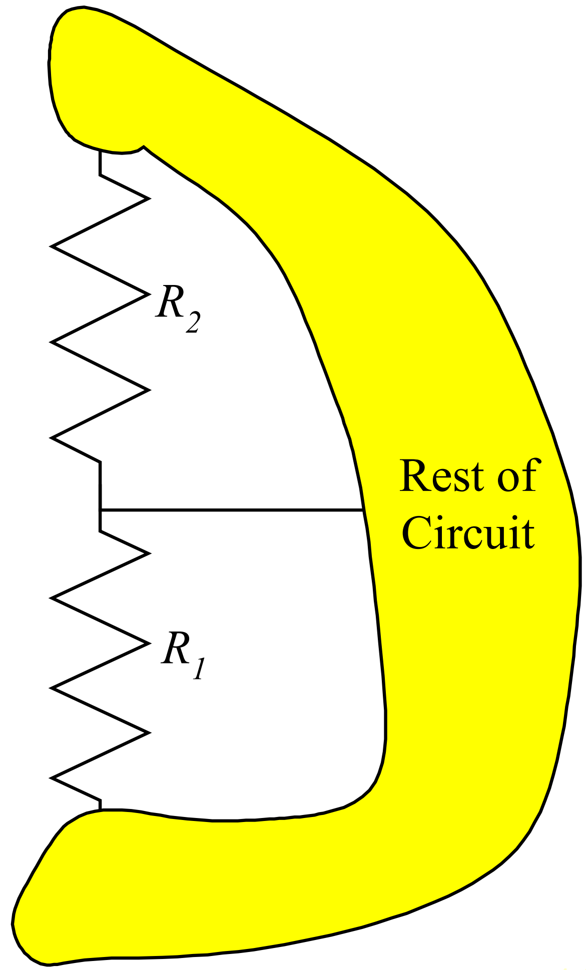
Go back to Overview slide.

Examples (Series)

- Some more examples are given here.



R_1 and R_2 are in series



R_1 and R_2 are ***not*** in series

How do we use equivalent circuits?

- This is yet another good question.
- We will often use these equivalents to simplify circuits, making them easier to solve. Sometimes, equivalent circuits are used in other ways. In some cases, one equivalent circuit is not simpler than another; rather one of them fits the needs of the particular circuit better. The delta-to-wye transformations that we cover next fit in this category. In yet other cases, we will have equivalent circuits for things that we would not otherwise be able to solve. For example, we will have equivalent circuits for devices such as diodes and transistors, that allow us to solve circuits that include these devices.
- The key point is this: Equivalent circuits are used throughout circuits and electronics. We need to use them correctly. ***Equivalent circuits are equivalent only with respect to the circuit outside them.***

Go back to Overview slide.

Week -7

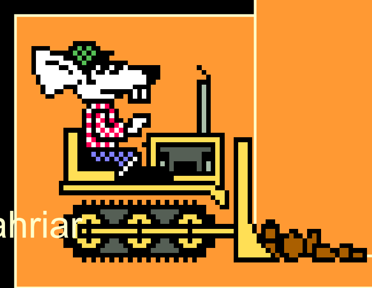
Page- (148-170)

Voltage Divider and Current Divider Rules

Voltage Divider Rule – Our First Circuit Analysis Tool

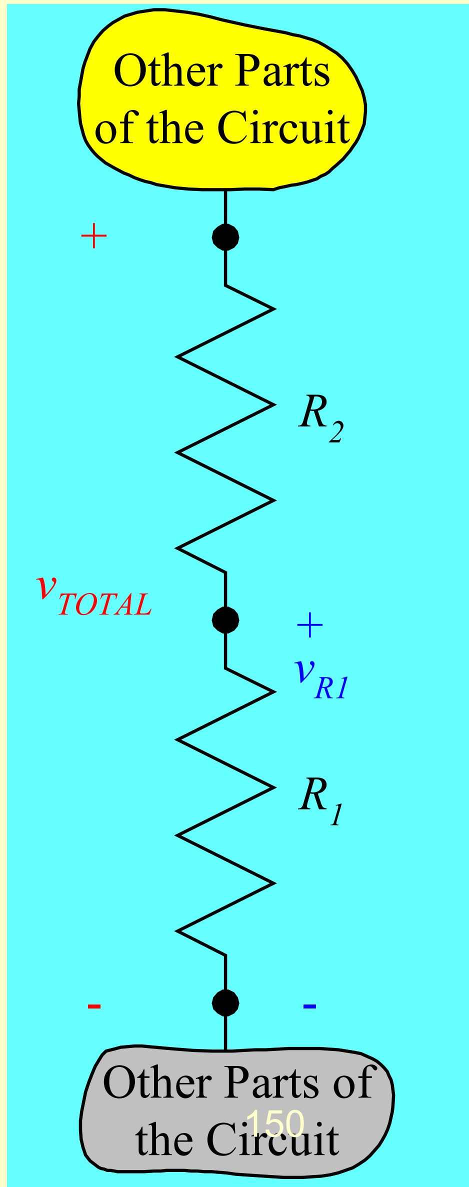
The Voltage Divider Rule (VDR) is the first of a long list of tools that we are going to develop to make circuit analysis quicker and easier. The idea is this: if the same situation occurs often, we can derive the solution once, and use it whenever it applies. As with any tools, the keys are:

1. Recognizing when the tool works and when it doesn't work.
2. Using the tool properly.



Voltage Divider Rule – Setting up the Derivation

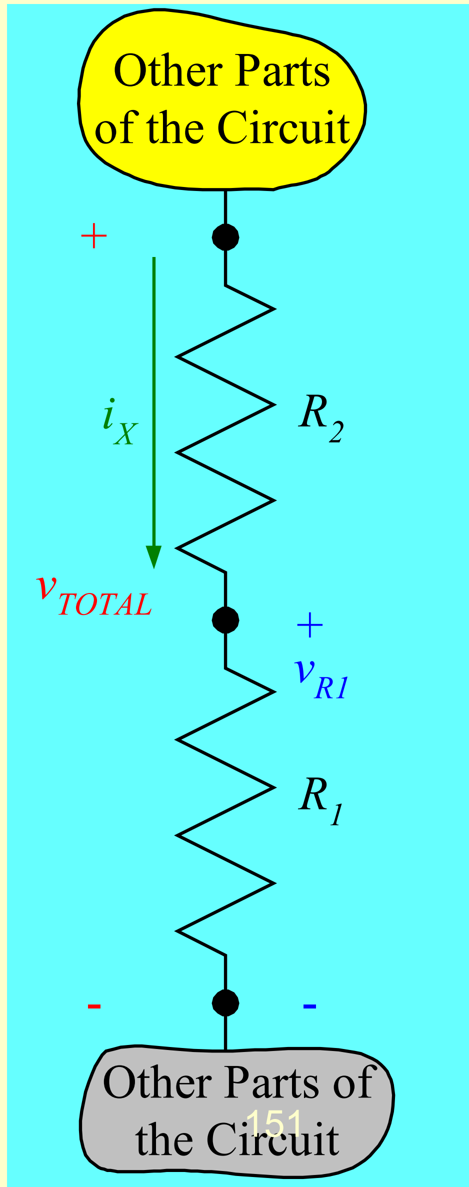
The Voltage Divider Rule involves the voltages across series resistors. Let's take the case where we have two resistors in series. Assume for the moment that the voltage across these two resistors, v_{TOTAL}, is known. Assume that we want the voltage across one of the resistors, shown here as v_{R1}. Let's find it.

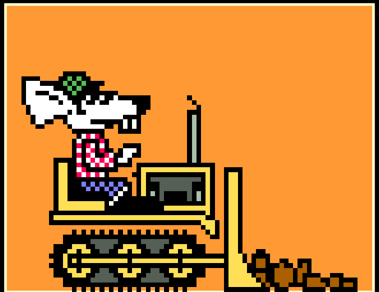


Voltage Divider Rule – Derivation Step 1

The current through both of these resistors is the same, since the resistors are in series. The current, i_X, is

$$i_X = \frac{v_{TOTAL}}{R_1 + R_2}.$$

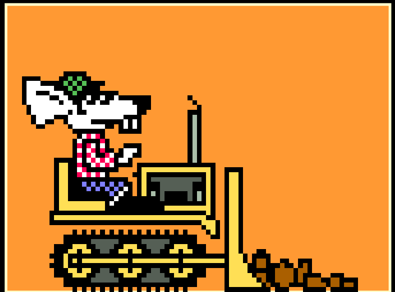


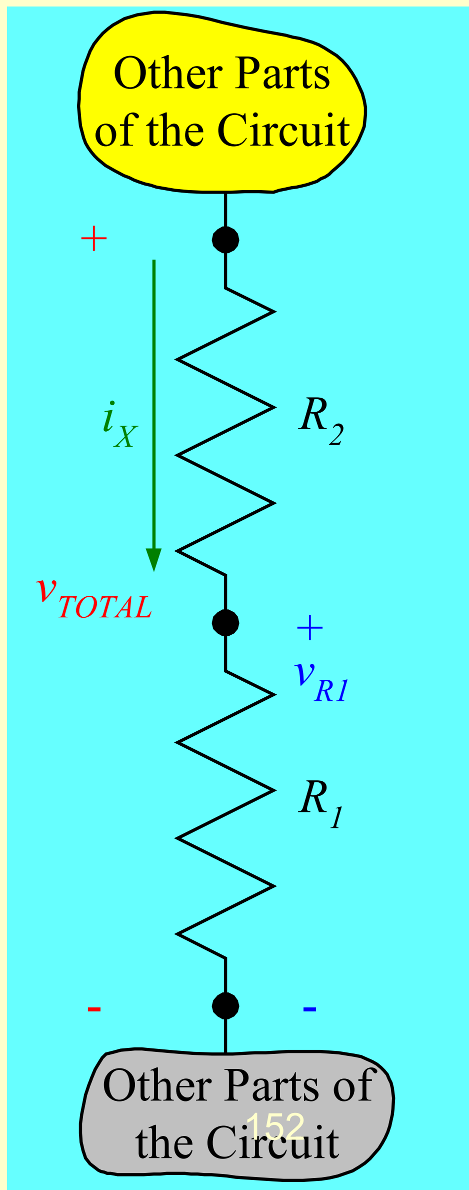


Voltage Divider Rule – Derivation Step 2

The current through resistor R_1 is the same current. The current, i_X, is

$$i_X = \frac{v_{R1}}{R_1}.$$



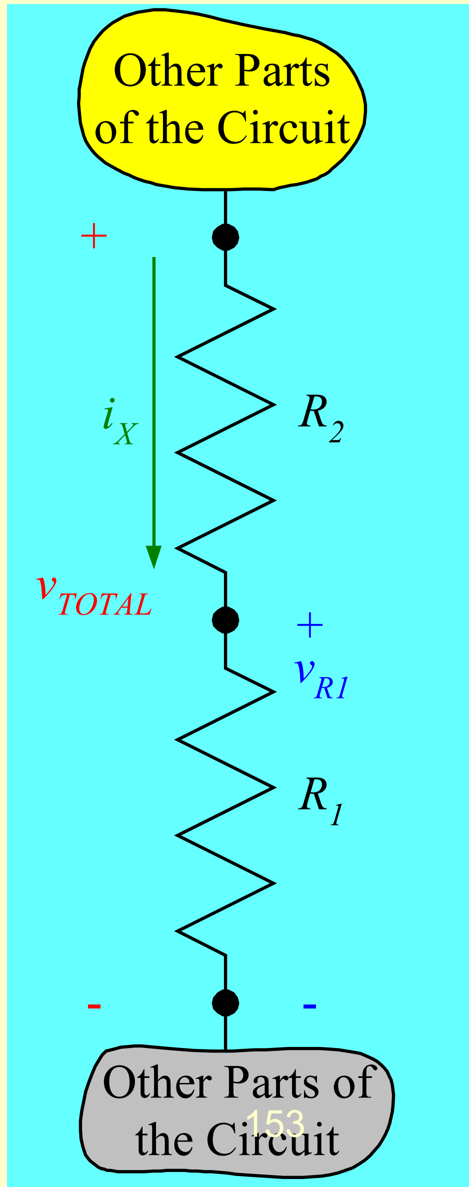


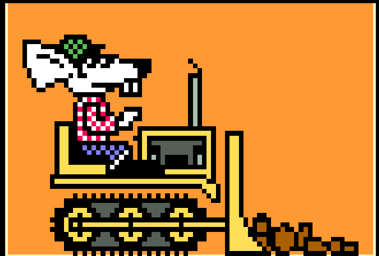
Voltage Divider Rule – Derivation Step 3

These are two expressions for the same current, so they must be equal to each other. Therefore, we can write

$$\frac{v_{R1}}{R_1} = \frac{v_{TOTAL}}{R_1 + R_2}.$$ Solving for v_{R1}, we get

$$v_{R1} = v_{TOTAL}\frac{R_1}{R_1 + R_2}.$$

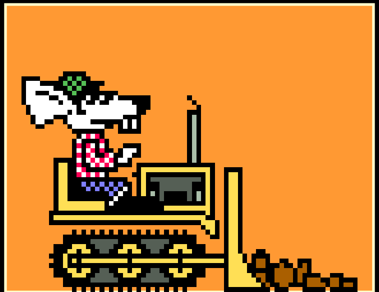


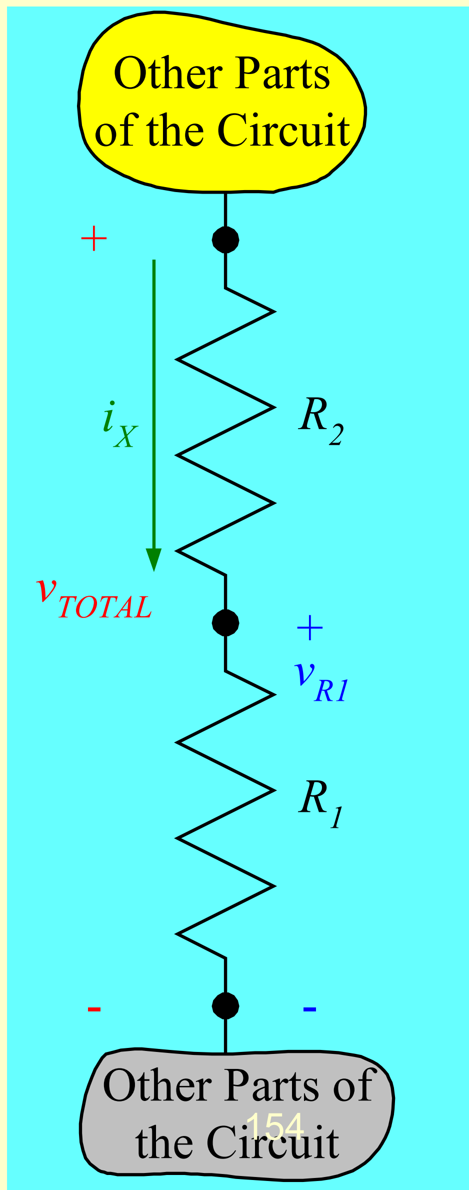


The Voltage Divider Rule

This is the expression we wanted. We call this the Voltage Divider Rule (VDR).

$$v_{R1} = v_{TOTAL} \frac{R_1}{R_1 + R_2}.$$





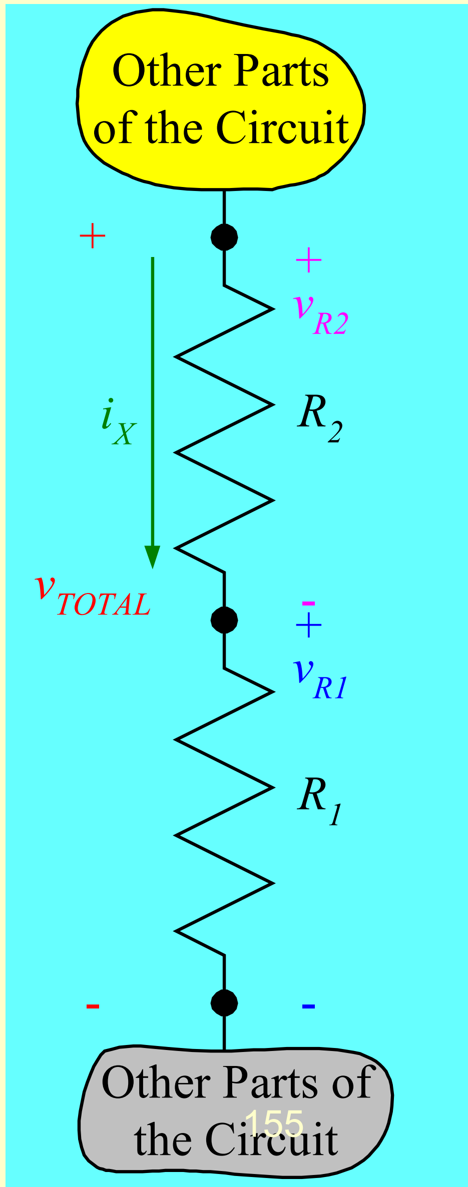
Go back to Overview slide.

Voltage Divider Rule – For Each Resistor

This is easy enough to remember that most people just memorize it. Remember that it only works for resistors that are in series. Of course, there is a similar rule for the other resistor. For the voltage across one resistor, we put that resistor value in the numerator.

$$v_{R1} = v_{TOTAL} \frac{R_1}{R_1 + R_2}.$$

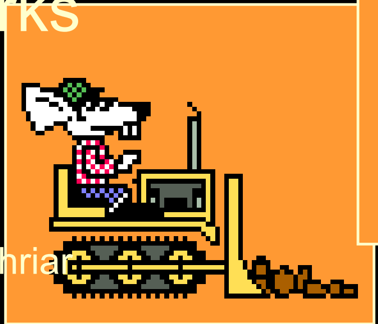
$$v_{R2} = v_{TOTAL} \frac{R_2}{R_1 + R_2}.$$

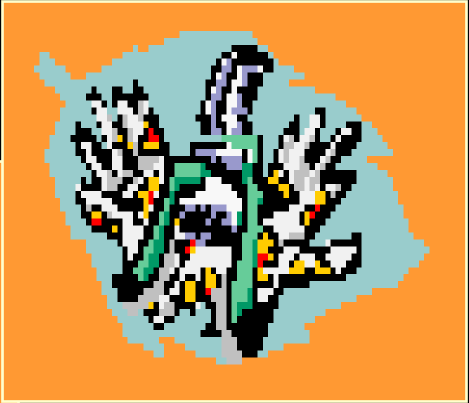


Current Divider Rule – Our Second Circuit Analysis Tool

The Current Divider Rule (CDR) is the second of a long list of tools that we are going to develop to make circuit analysis quicker and easier. Again, if the same situation occurs often, we can derive the solution once, and use it whenever it applies. As with any tools, the keys are:

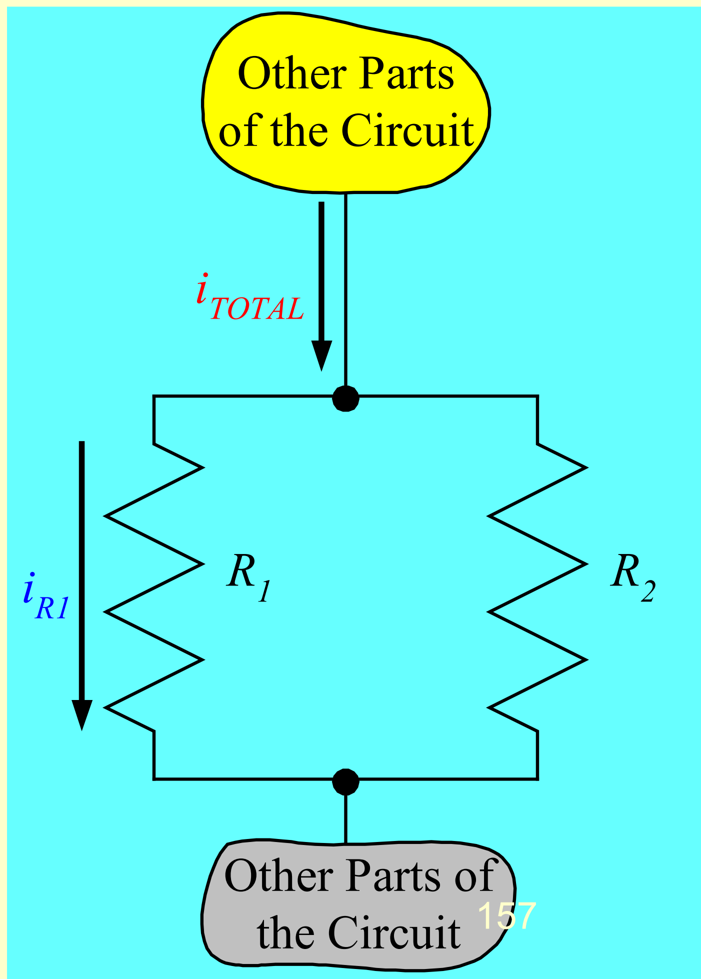
1. Recognizing when the tool works and when it doesn't work.
2. Using the tool properly.





Current Divider Rule – Setting up the Derivation

The Current Divider Rule involves the currents through parallel resistors. Let's take the case where we have two resistors in parallel. Assume for the moment that the current feeding these two resistors, i_{TOTAL}, is known. Assume that we want the current through one of the resistors, shown here as i_{R1}. Let's find it.

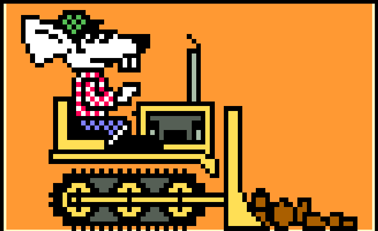


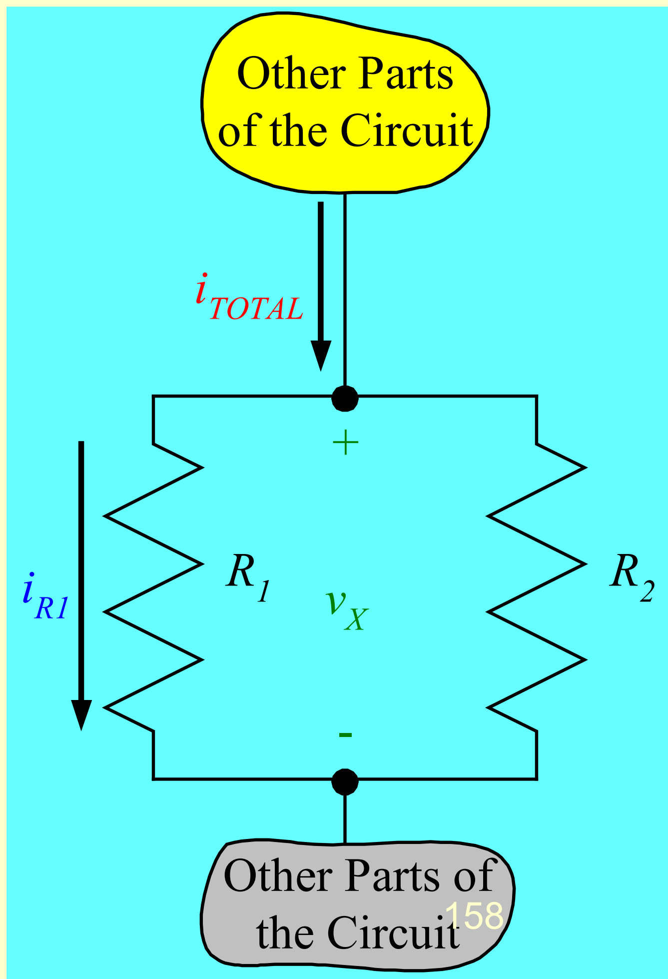
Current Divider Rule – Derivation Step 1

The voltage across both of these resistors is the same, since the resistors are in parallel. The voltage, v_X, is the current multiplied by the equivalent parallel resistance,

$$v_X = i_{TOTAL}\left(R_1 \| R_2\right), \text{ or}$$

$$v_X = i_{TOTAL}\left(\frac{R_1 R_2}{R_1 + R_2}\right).$$

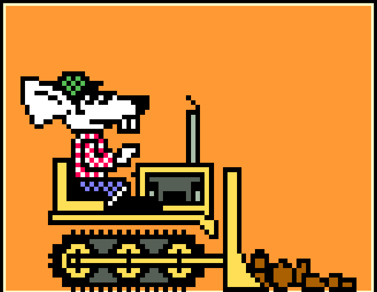


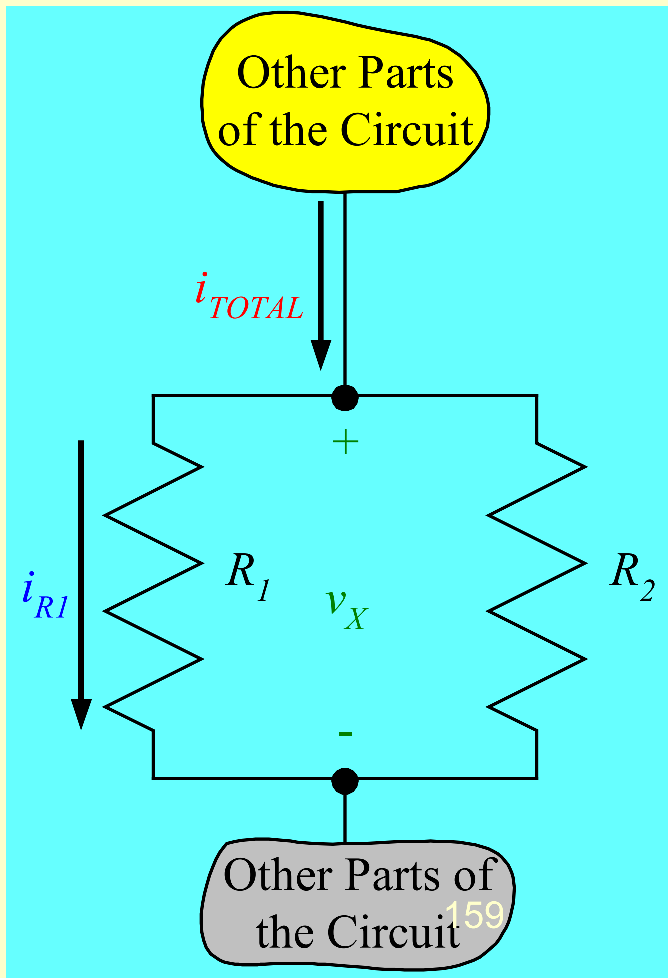


Current Divider Rule – Derivation Step 2

The voltage across resistor R_1 is the same voltage, v_X. The voltage, v_X, is

$$v_X = i_{R1} R_1.$$



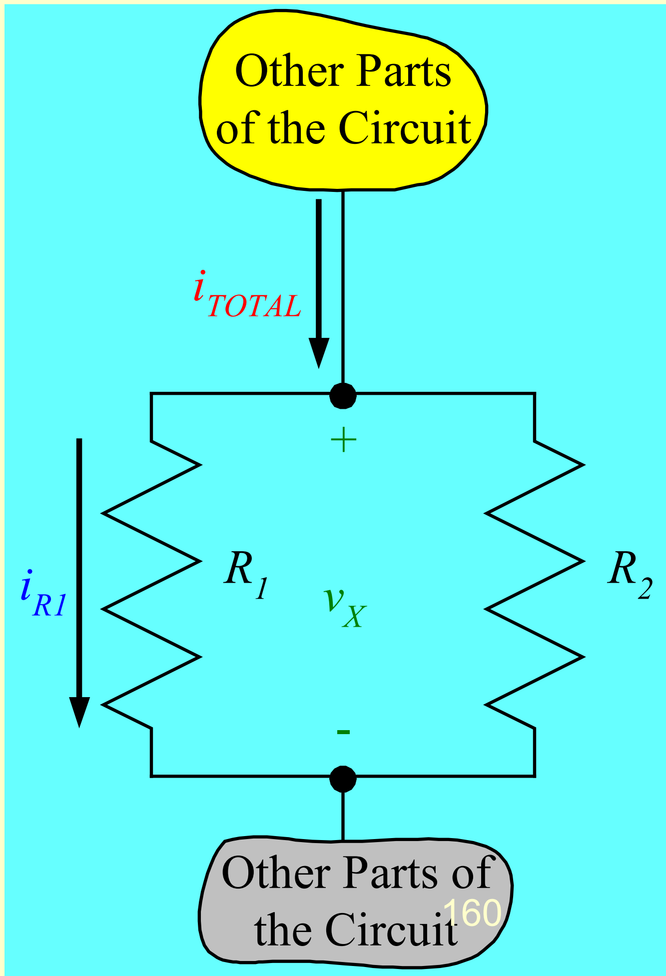


Current Divider Rule – Derivation Step 3

These are two expressions for the same voltage, so they must be equal to each other. Therefore, we can write

$$i_{R1}R_1 = i_{TOTAL}\frac{R_1R_2}{R_1+R_2}. \quad \text{Solve for } i_{R1};$$

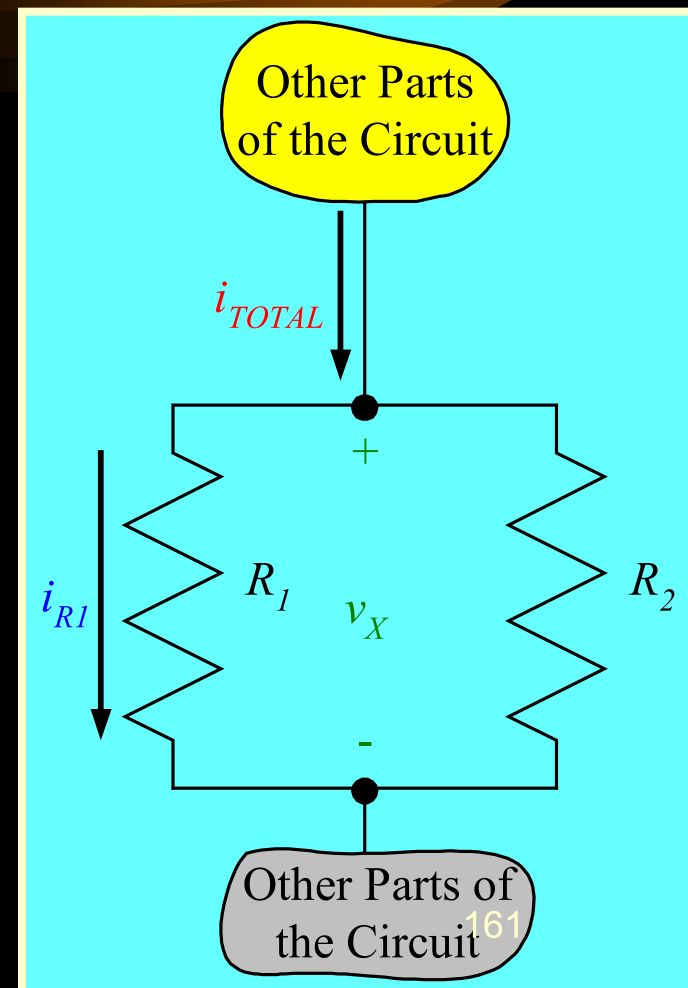
$$i_{R1} = i_{TOTAL}\frac{R_2}{R_1+R_2}.$$



The Current Divider Rule

This is the expression we wanted. We call this the Current Divider Rule (CDR).

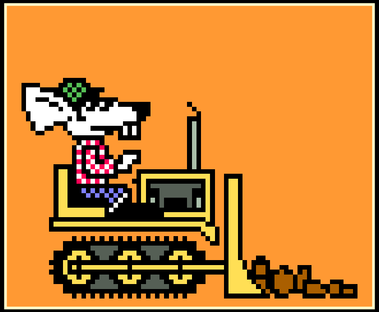
$$i_{R1} = i_{TOTAL} \frac{R_2}{R_1 + R_2}.$$



Go back to Overview slide.

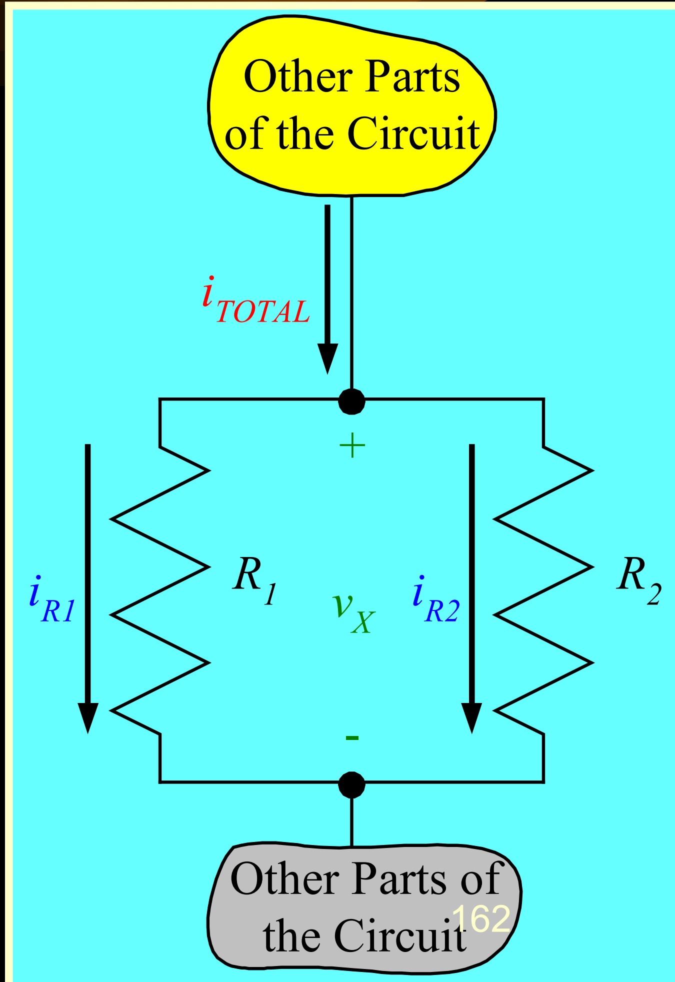
Current Divider Rule – For Each Resistor

Most people just memorize this. Remember that it only works for resistors that are in parallel. Of course, there is a similar rule for the other resistor. For the current through one resistor, we put the opposite resistor value in the numerator.



$$i_{R1} = i_{TOTAL} \frac{R_2}{R_1 + R_2}.$$

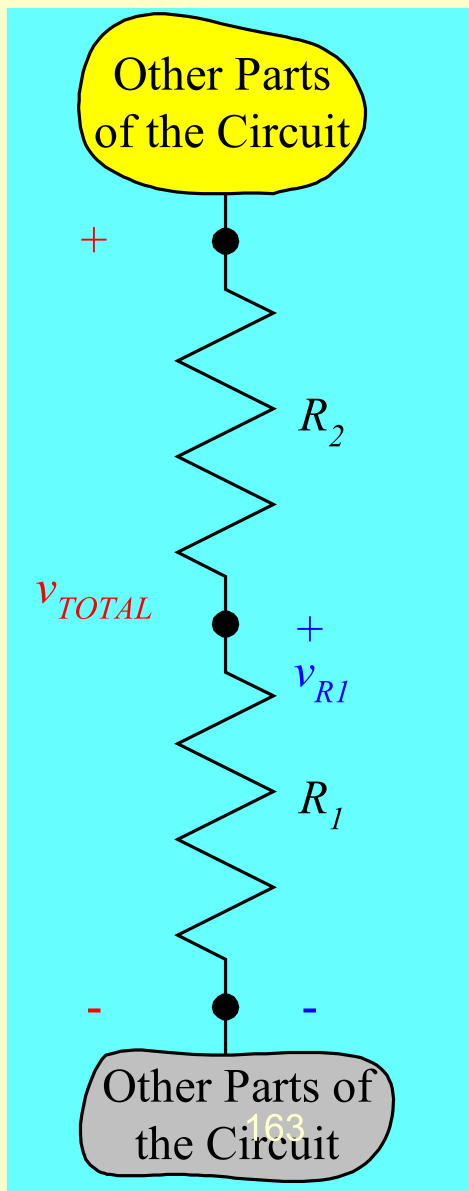
$$i_{R2} = i_{TOTAL} \frac{R_1}{R_1 + R_2}.$$



Signs in the Voltage Divider Rule

As in most every equation we write, we need to be careful about the sign in the Voltage Divider Rule (VDR). Notice that when we wrote this expression, there is a positive sign. This is because the voltage v_{TOTAL} is in the same relative polarity as v_{R1}.

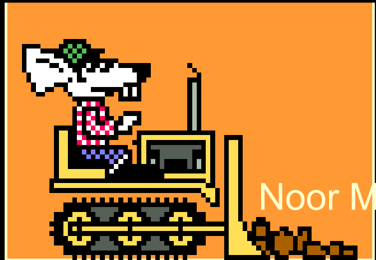
$$v_{R1} = +v_{TOTAL}\frac{R_1}{R_1 + R_2}.$$

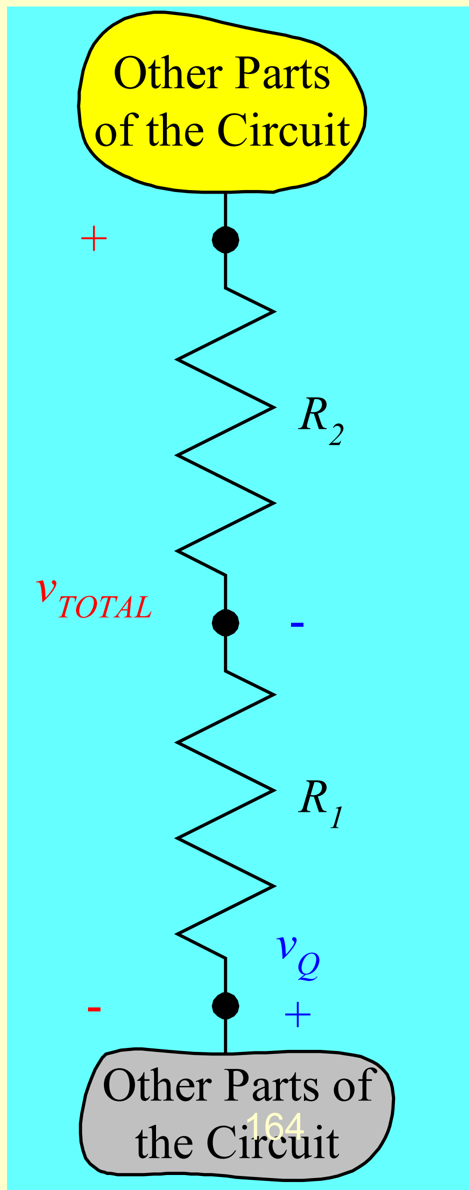


Negative Signs in the Voltage Divider Rule

If, instead, we had solved for v_Q, we would need to change the sign in the equation. This is because the voltage v_{TOTAL} is in the opposite relative polarity from v_Q.

$$v_Q = -v_{TOTAL}\frac{R_1}{R_1 + R_2}.$$



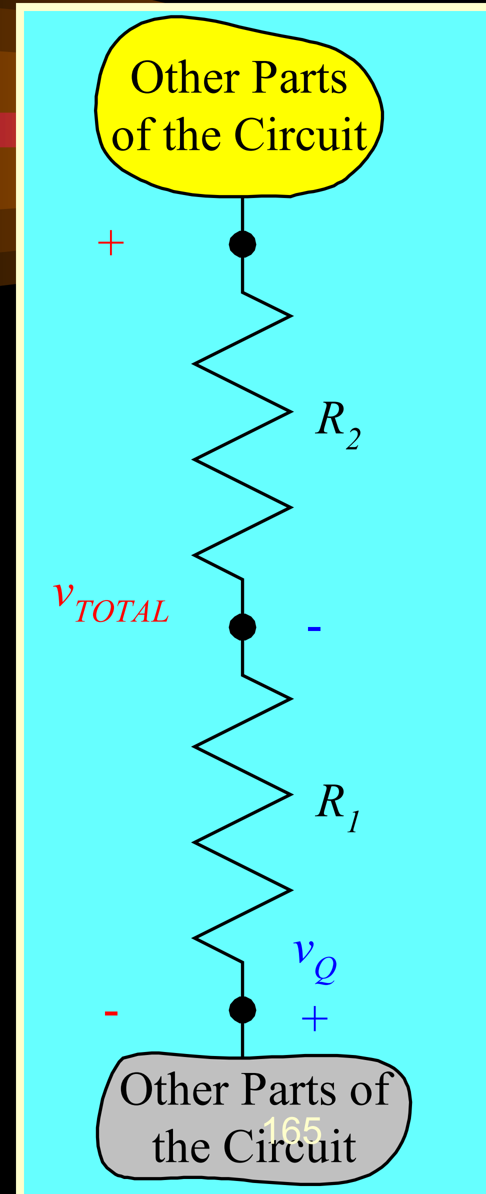


Go back to Overview slide.

Check for Signs in the Voltage Divider Rule

The rule for proper use of this tool, then, is to check the relative polarity of the voltage across the series resistors, and the voltage across one of the resistors.

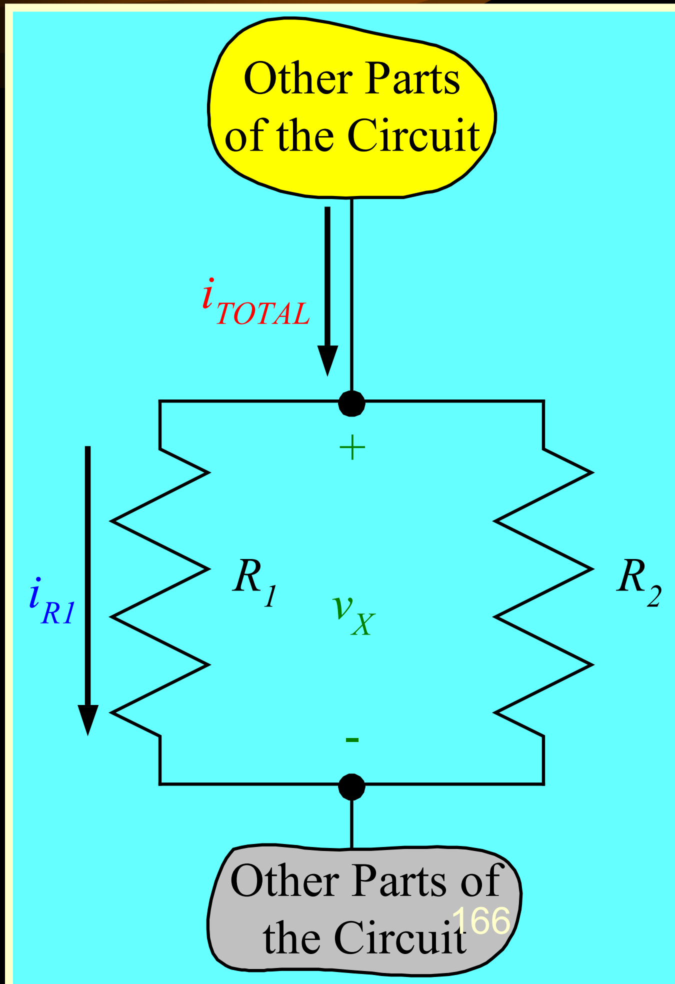
$$v_Q = -v_{TOTAL} \frac{R_1}{R_1 + R_2}.$$



Signs in the Current Divider Rule

As in most every equation we write, we need to be careful about the sign in the Current Divider Rule (CDR). Notice that when we wrote this expression, there is a positive sign. This is because the current i_{TOTAL} is in the same relative polarity as i_{R1}.

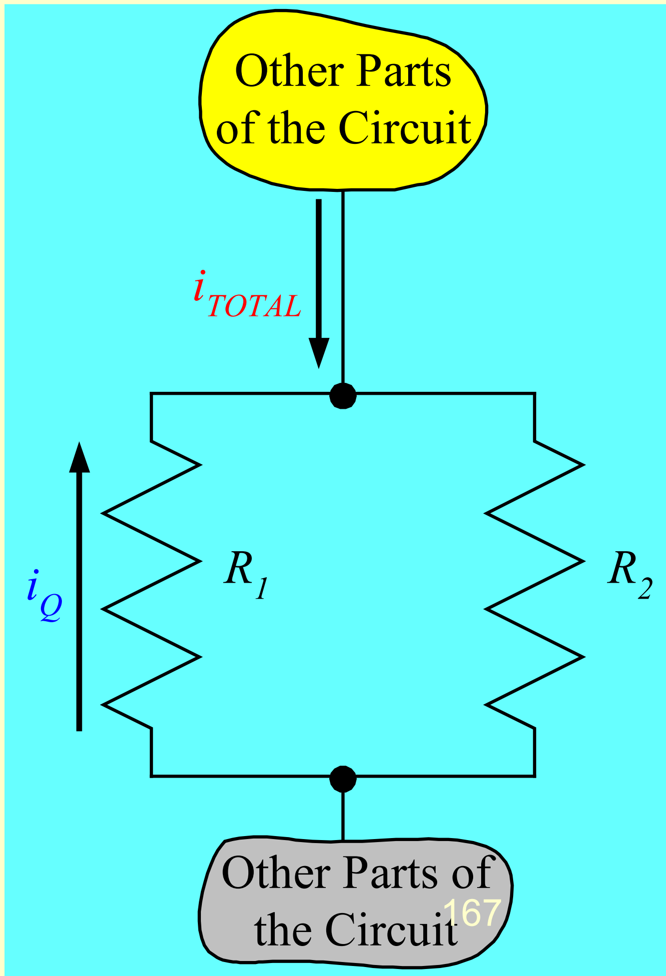
$$i_{R1} = +i_{TOTAL} \frac{R_2}{R_1 + R_2}.$$



Negative Signs in the Current Divider Rule

If, instead, we had solved for i_Q, we would need to change the sign in the equation. This is because the current i_{TOTAL} is in the opposite relative polarity from i_Q.

$$i_Q = -i_{TOTAL} \frac{R_2}{R_1 + R_2}.$$

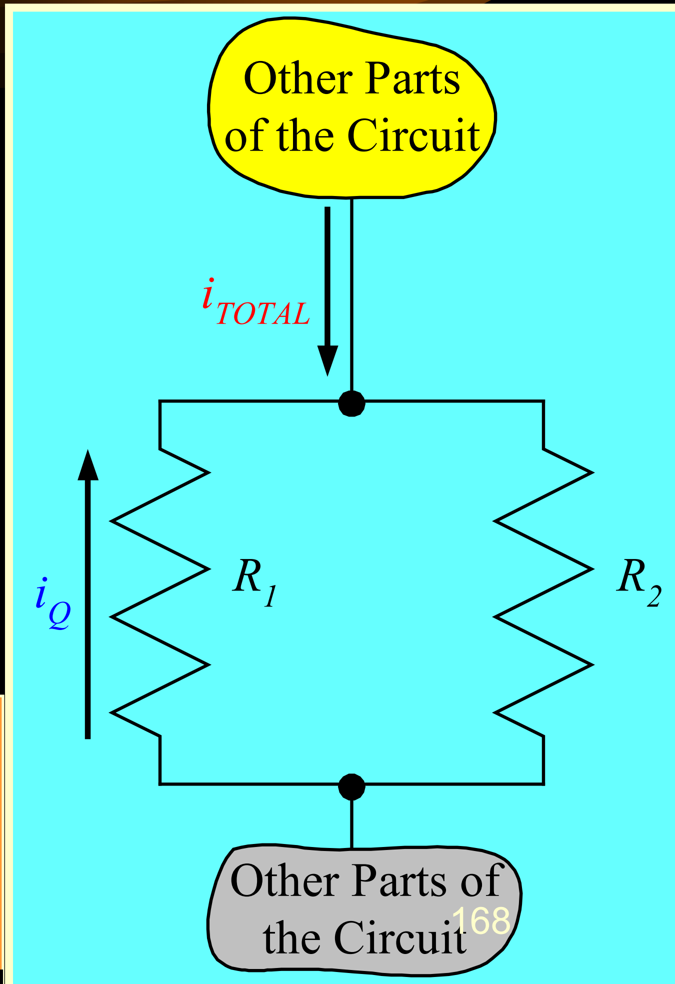


Go back to Overview slide.

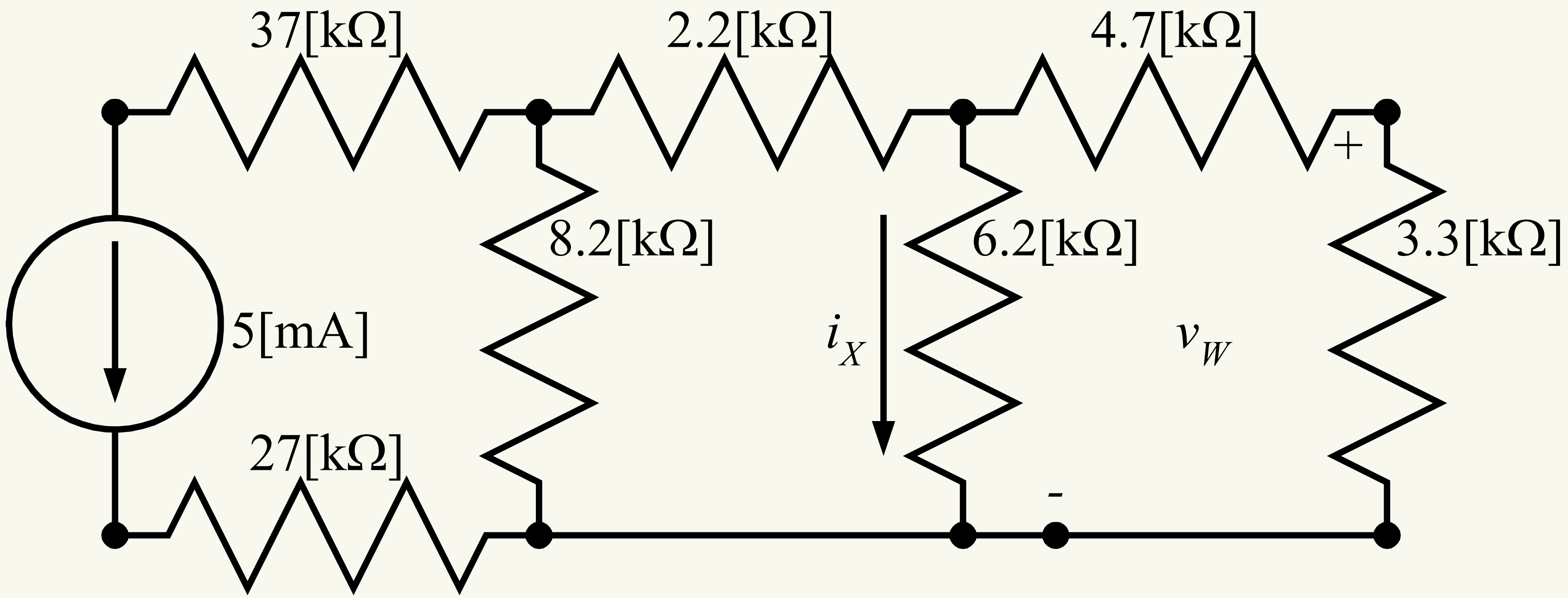
Check for Signs in the Current Divider Rule

The rule for proper use of this tool, then, is to check the relative polarity of the current through the parallel resistors, and the current through one of the resistors.

$$i_Q = -i_{TOTAL} \frac{R_2}{R_1 + R_2}.$$



Example Problem #6



Find i_X and v_W.

Week -8

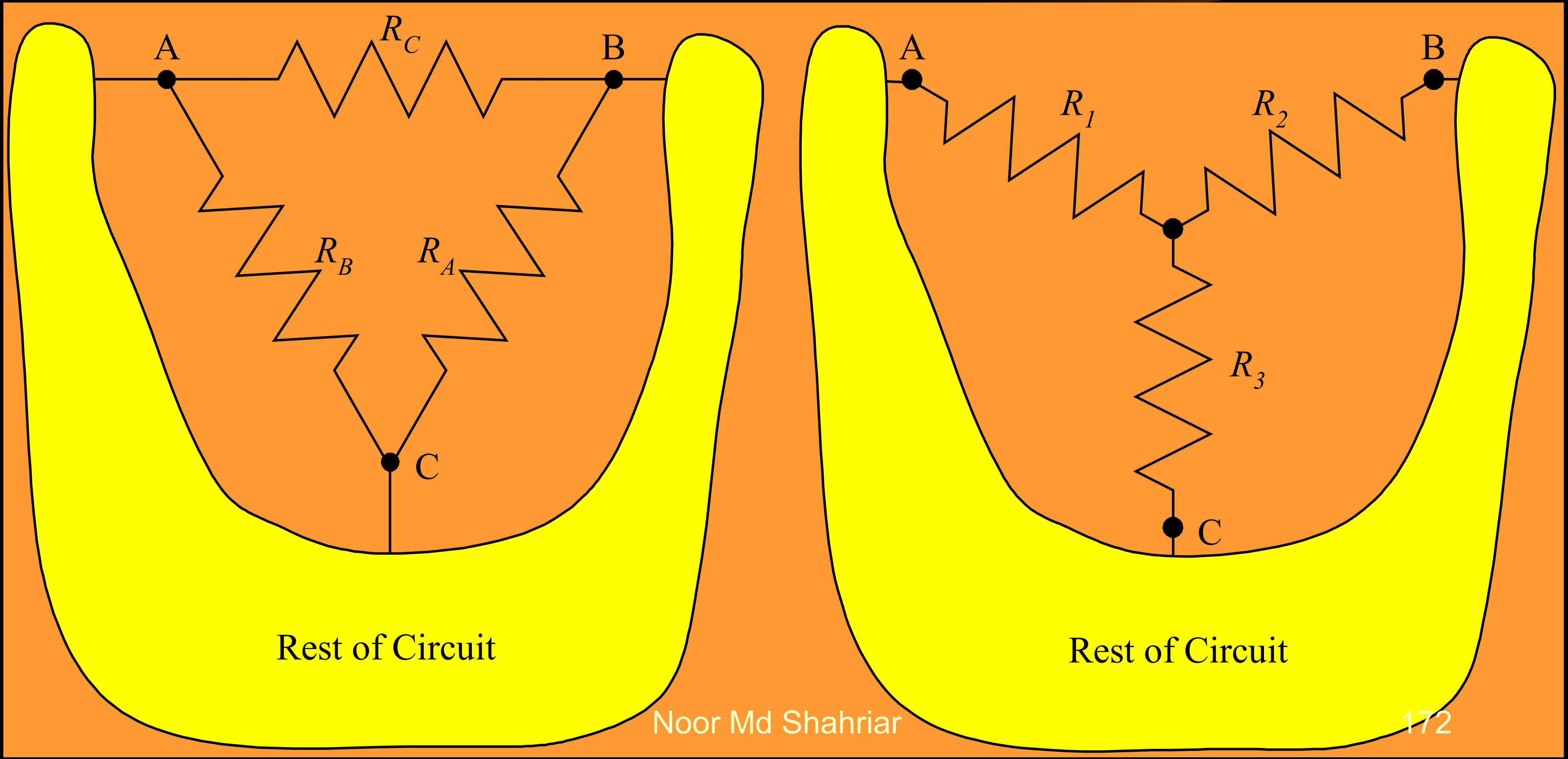
Page- (171-186)

Delta-to-Wye Transformations

- The transformations, or equivalent circuits, that we cover next are called delta-to-wye, or wye-to-delta transformations. They are also sometimes called pi-to-tee or tee-to-pi transformations. For these lecture sets, we will call them the delta-to-wye transformations.
- These are equivalent circuit pairs. They apply for parts of circuits that have three terminals. Each version of the equivalent circuit has three resistors.
- Many courses do not cover these particular equivalent circuits at this point, delaying coverage until they are specifically needed during the discussion of three phase circuits. However, they are an excellent example of equivalent circuits, and can be used in some cases to solve circuits more easily.

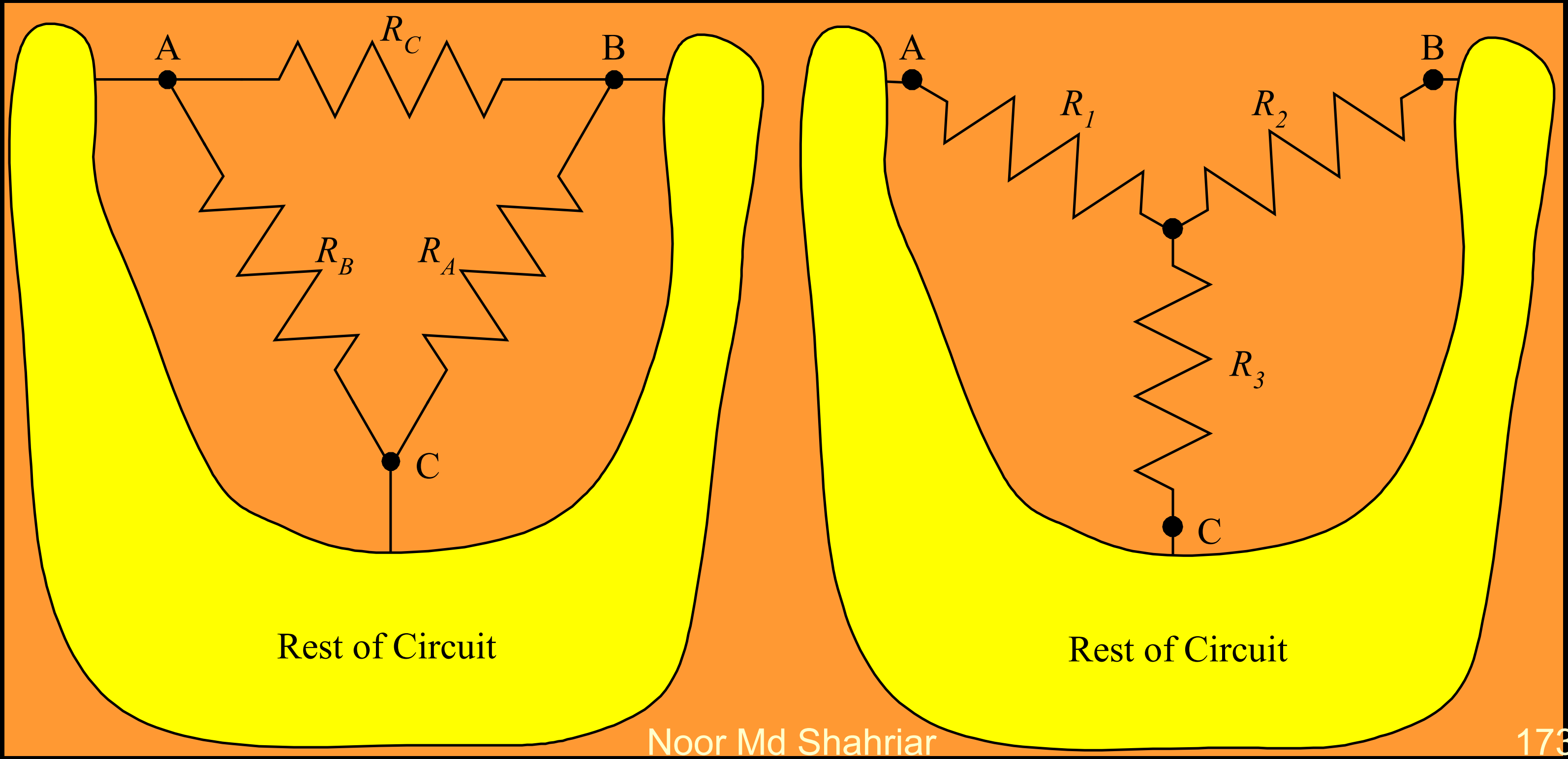
Delta-to-Wye Transformations

Three resistors in a part of a circuit with three terminals can be replaced with another version, also with three resistors. The two versions are shown here. Note that none of these resistors is in series with any other resistor, nor in parallel with any other resistor. The three terminals in this example are labeled A, B, and C.



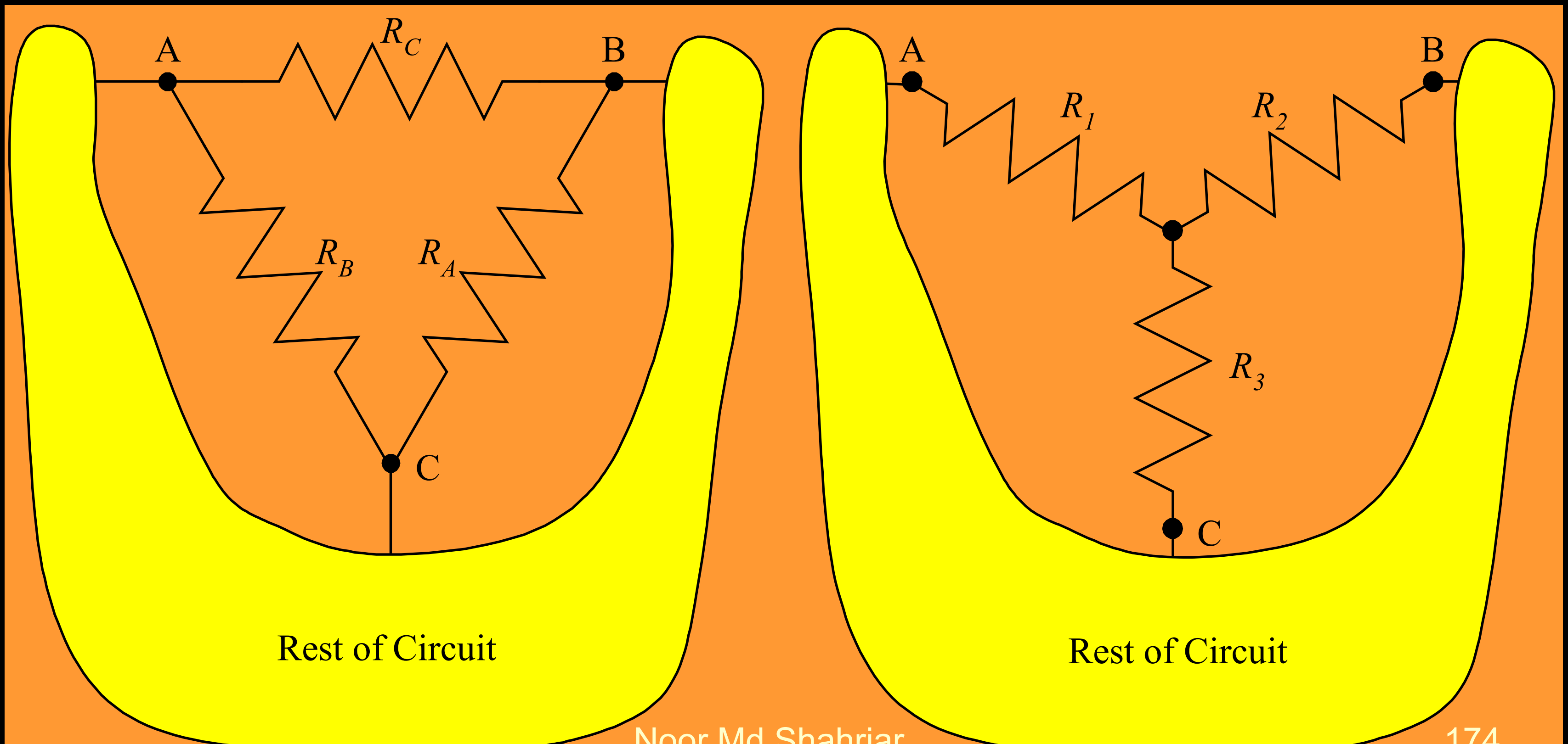
Delta-to-Wye Transformations (Notes on Names)

The version on the left hand side is called the delta connection, for the Greek letter Δ. The version on the right hand side is called the wye connection, for the letter Y. The delta connection is also called the pi (π) connection, and the wye interconnection is also called the tee (T) connection. All these names come from the shapes of the drawings.



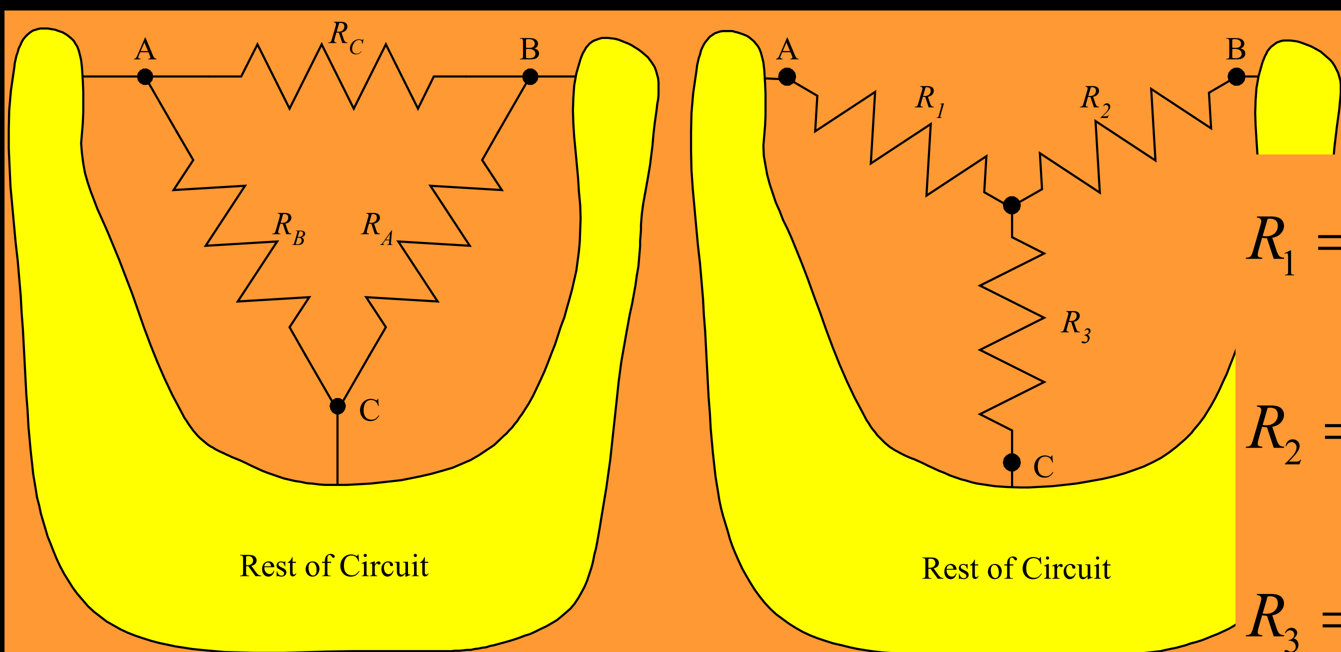
Delta-to-Wye Transformations (More Notes)

When we go from the delta connection (on the left) to the wye connection (on the right), we call this the delta-to-wye transformation. Going in the other direction is called the wye-to-delta transformation. One can go in either direction, as needed. These are equivalent circuits.



Delta-to-Wye Transformation Equations

When we perform the delta-to-wye transformation (going from left to right) we use the equations given below.



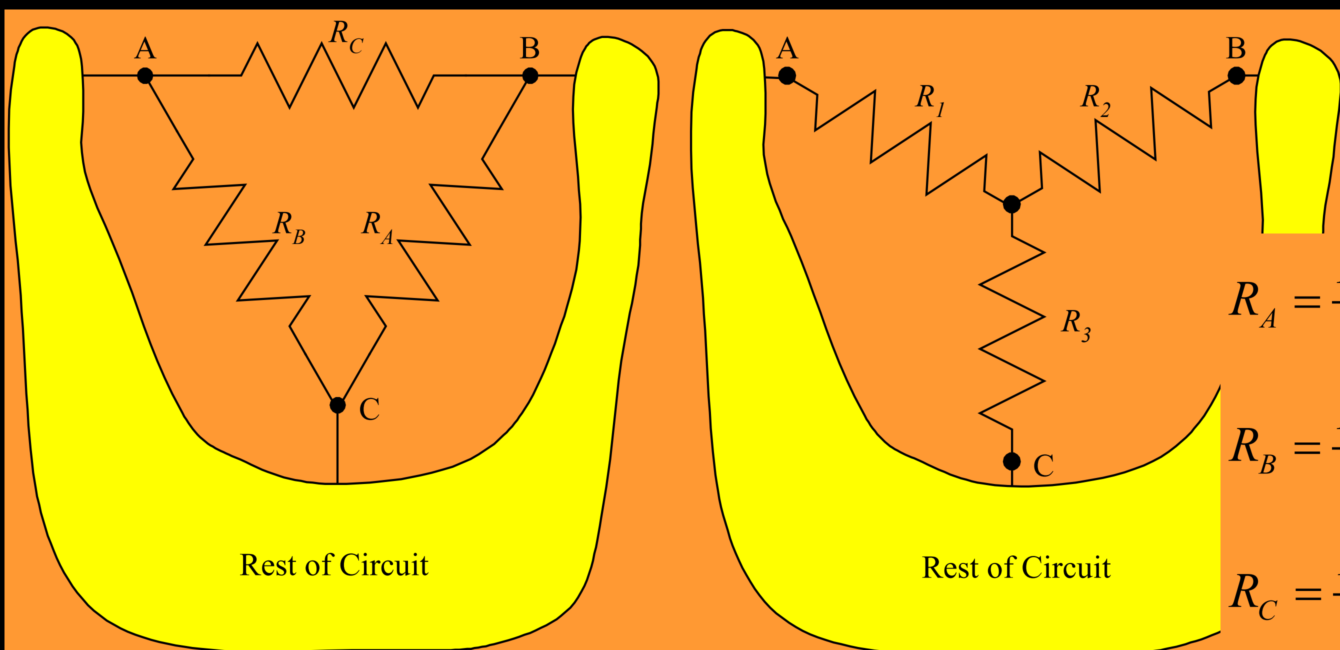
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C},$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}, \text{ and}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}.$$

Wye-to-Delta Transformation Equations

When we perform the wye-to-delta transformation (going from right to left) we use the equations given below.



$$R_A = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1},$$

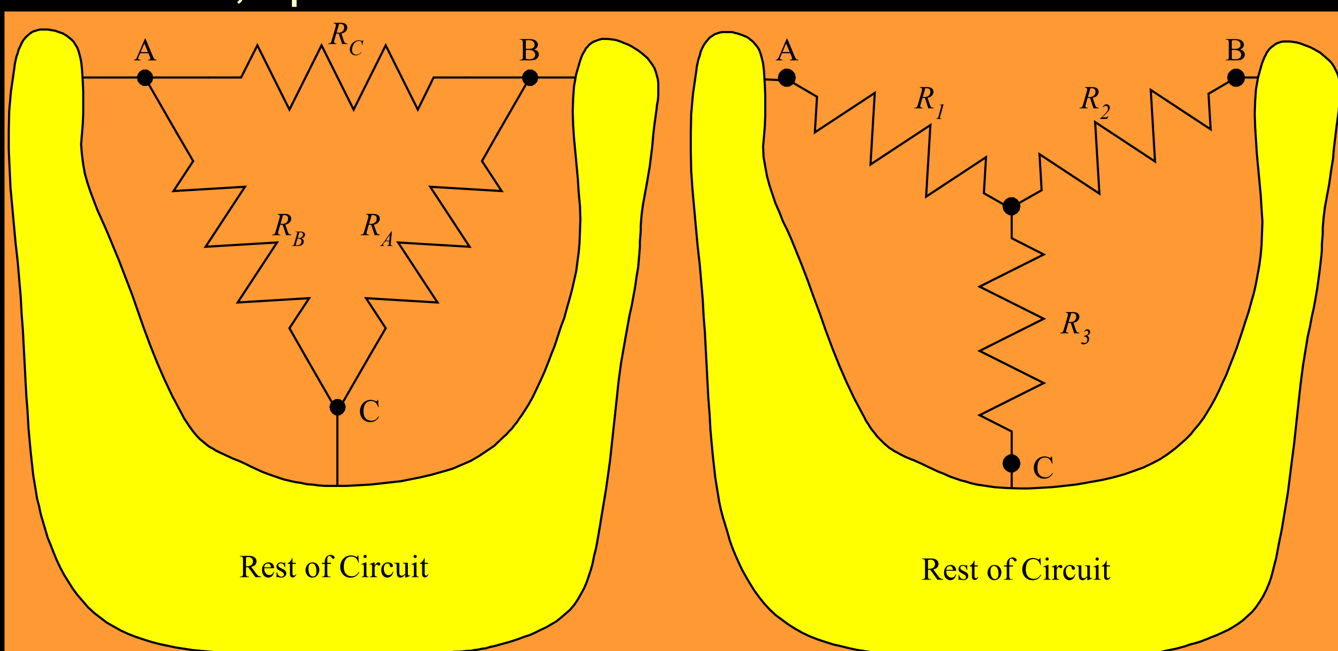
$$R_B = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}, \text{ and}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}.$$

Deriving the Equations

While these equivalent circuits are useful, perhaps the most important insight is gained from asking where these useful equations come from. How were these equations derived?

The answer is that they were derived using the fundamental rule for equivalent circuits. These two equivalent circuits have to behave the same way no matter what circuit is connected to them. So, we can choose specific circuits to connect to the equivalents. We make the derivation by solving for equivalent resistances, using our series and parallel rules, under different, specific conditions.



$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

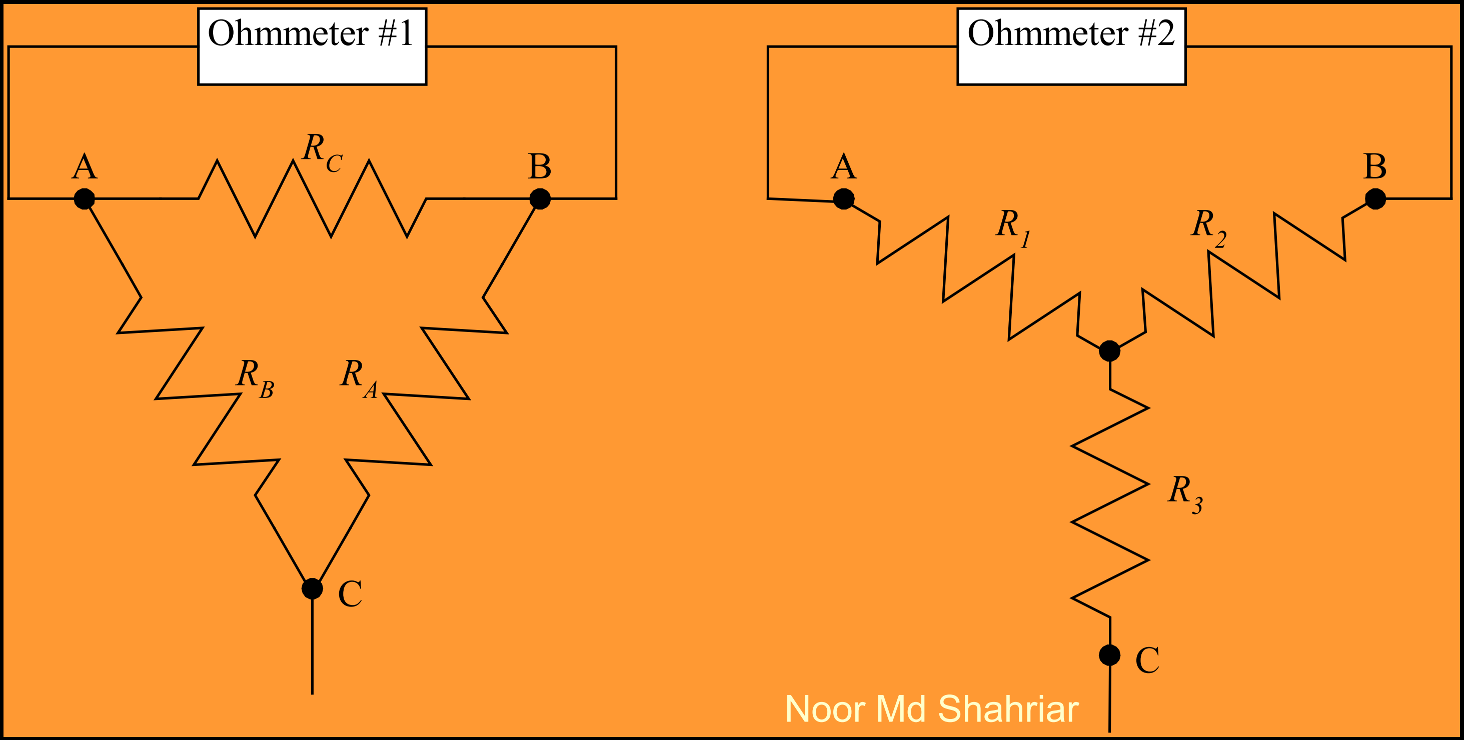
$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

Equation 1

We can calculate the equivalent resistance between terminals A and B, when C is not connected anywhere. The two cases are shown below. This is the same as connecting an ohmmeter, which measures resistance, between terminals A and B, while terminal C is left disconnected.

Ohmmeter #1 reads $R_{EQ1} = R_C \,||\, (R_A + R_B)$. Ohmmeter #2 reads $R_{EQ2} = R_1 + R_2$.

These must read the same value, so $R_C \,||\, (R_A + R_B) = R_1 + R_2$.

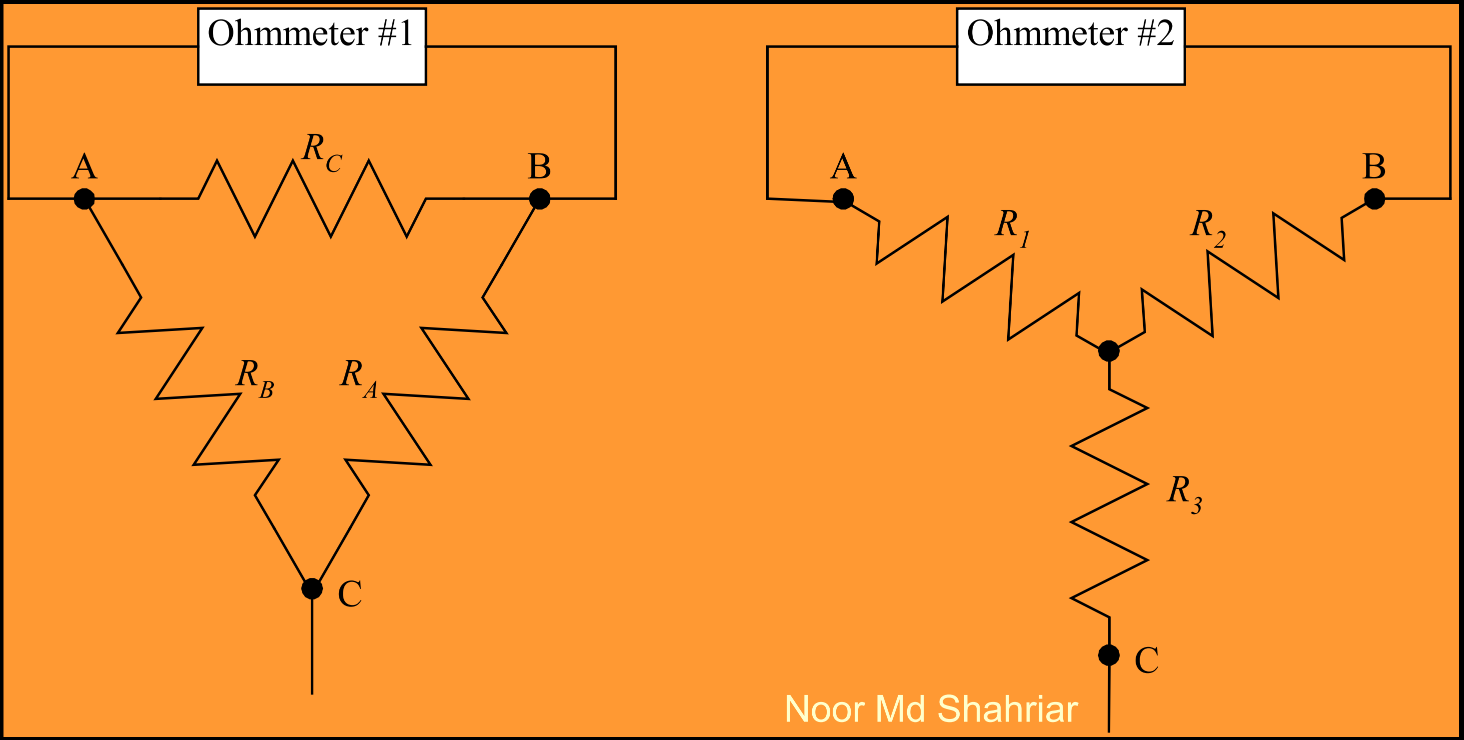


Equations 2 and 3

So, the equation that results from the first situation is

$$R_C \parallel (R_A + R_B) = R_1 + R_2.$$

We can make this measurement two other ways, and get two more equations. Specifically, we can measure the resistance between A and C, with B left open, and we can measure the resistance between B and C, with A left open.



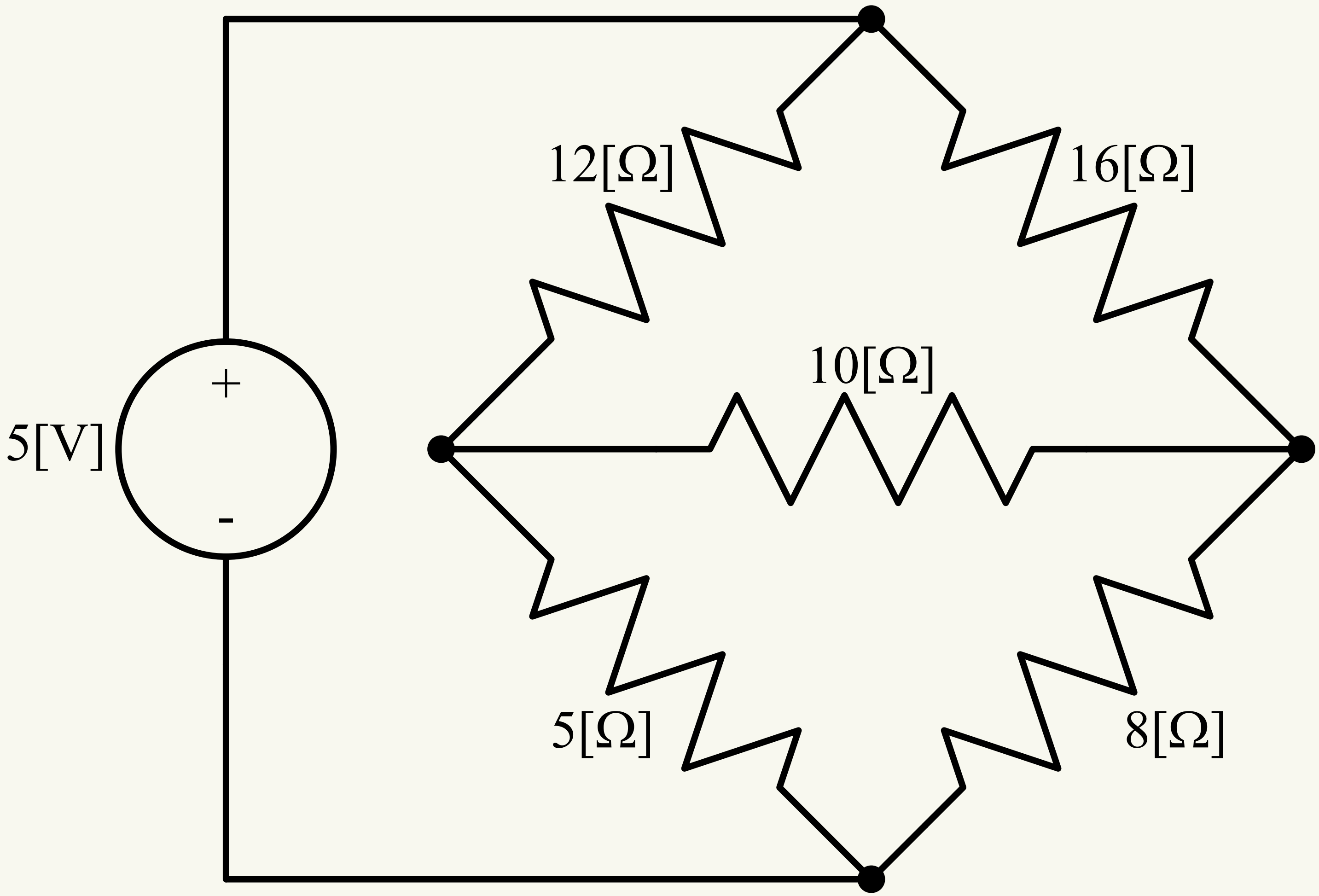
All Three Equations

The three equations we can obtain are

$$R_C \| (R_A + R_B) = R_1 + R_2,$$
$$R_B \| (R_A + R_C) = R_1 + R_3, \text{ and}$$
$$R_A \| (R_B + R_C) = R_2 + R_3.$$

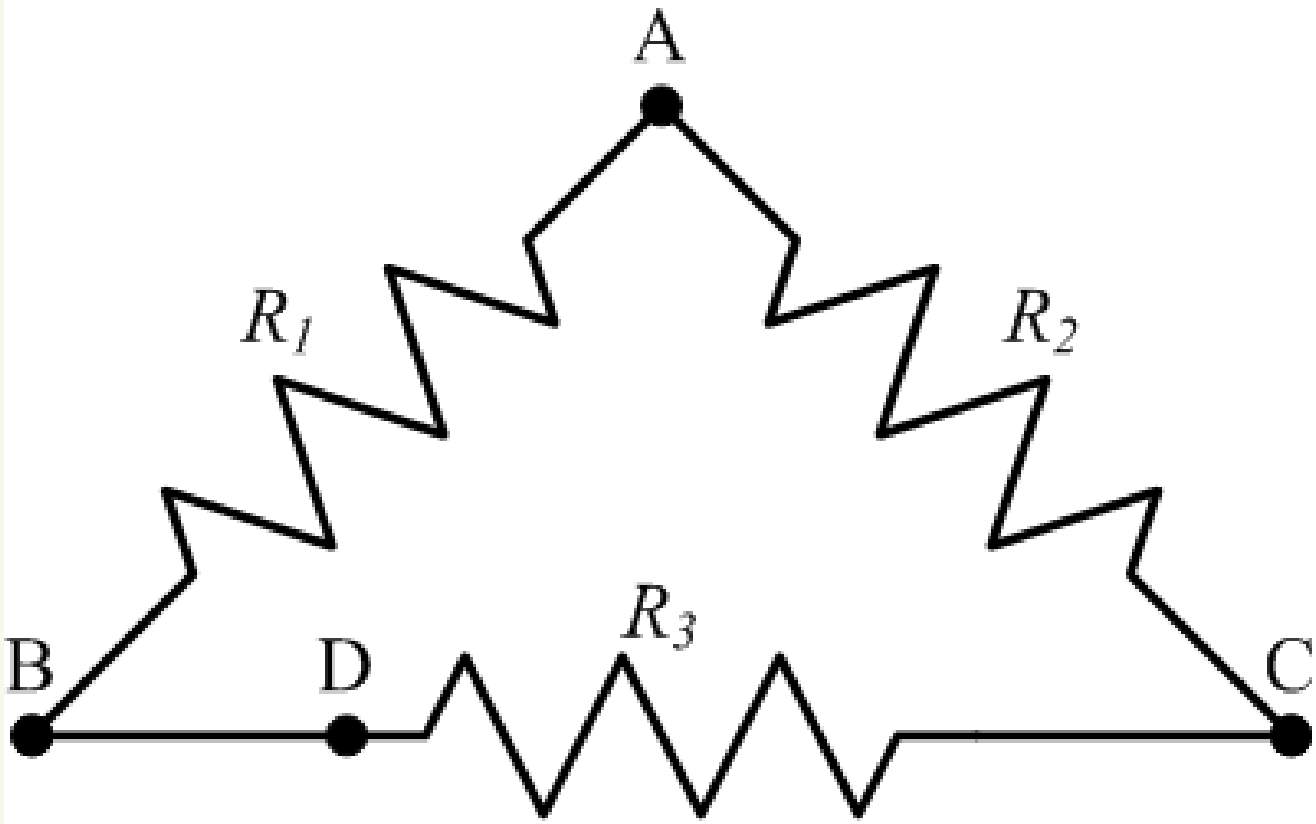
This is all that we need. These three equations can be manipulated algebraically to obtain either the set of equations for the delta-to-wye transformation (by solving for R_1, R_2, and R_3), or the set of equations for the wye-to-delta transformation (by solving for R_A, R_B, and R_C).

Example Problem #1



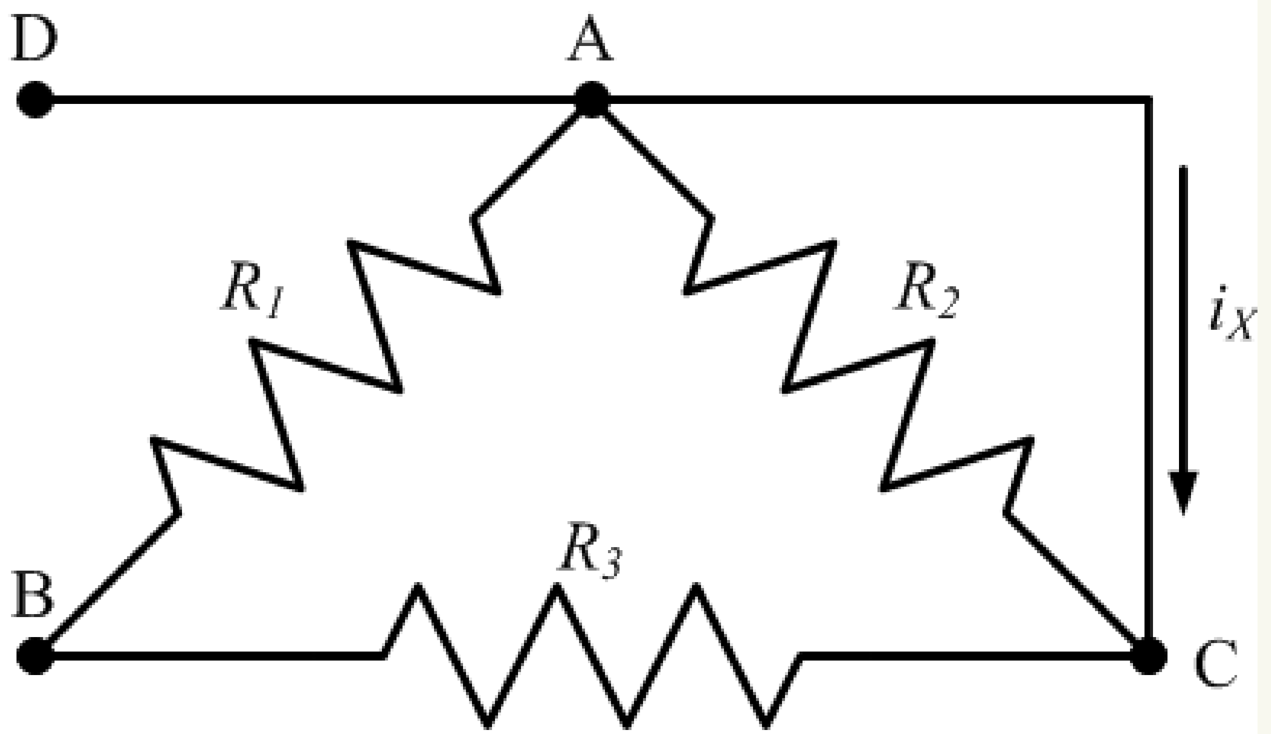
Find the power delivered by the source in this circuit.

Example Problem #2



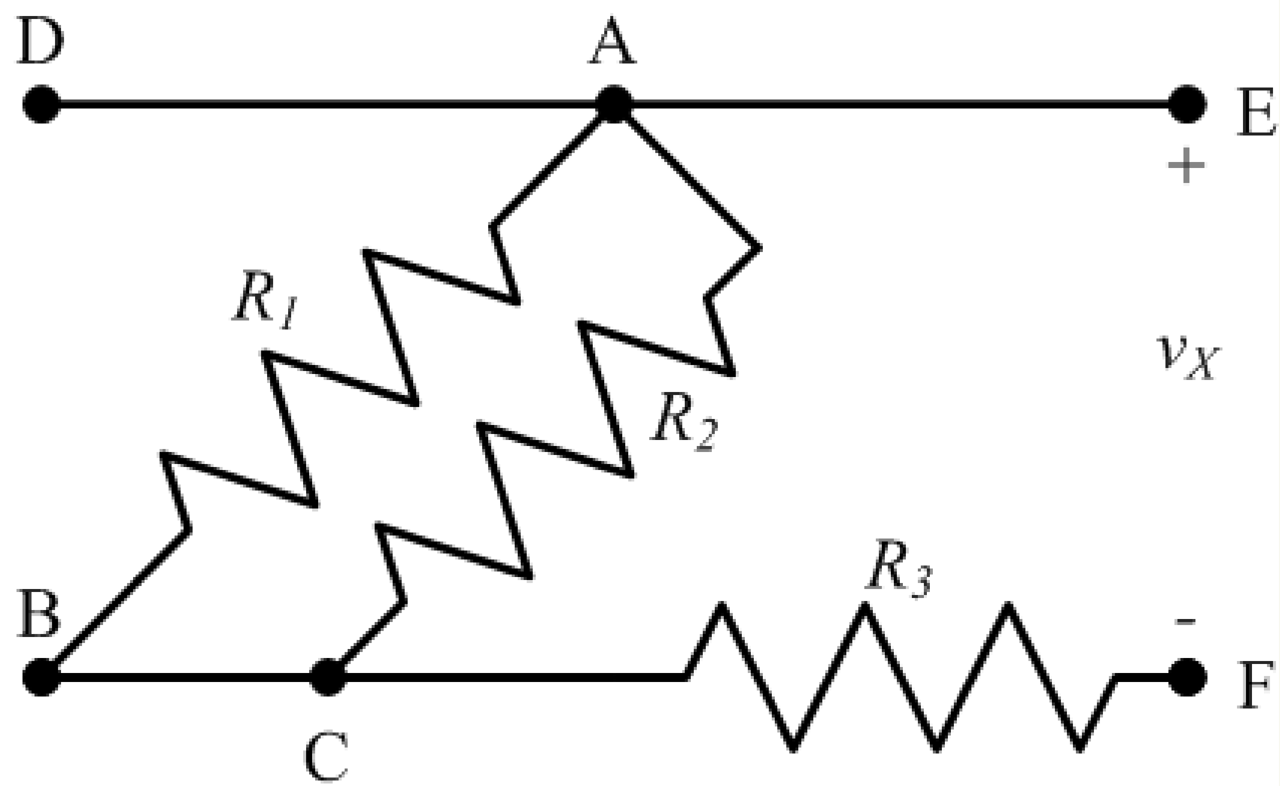
If we are finding the equivalent resistance, are R_1 and R_2 in series?

Example Problem #3



We are finding the equivalent resistance as seen from terminals B and D.
Can R_2 be removed?
If so, should it be replaced by anything?

Example Problem #4



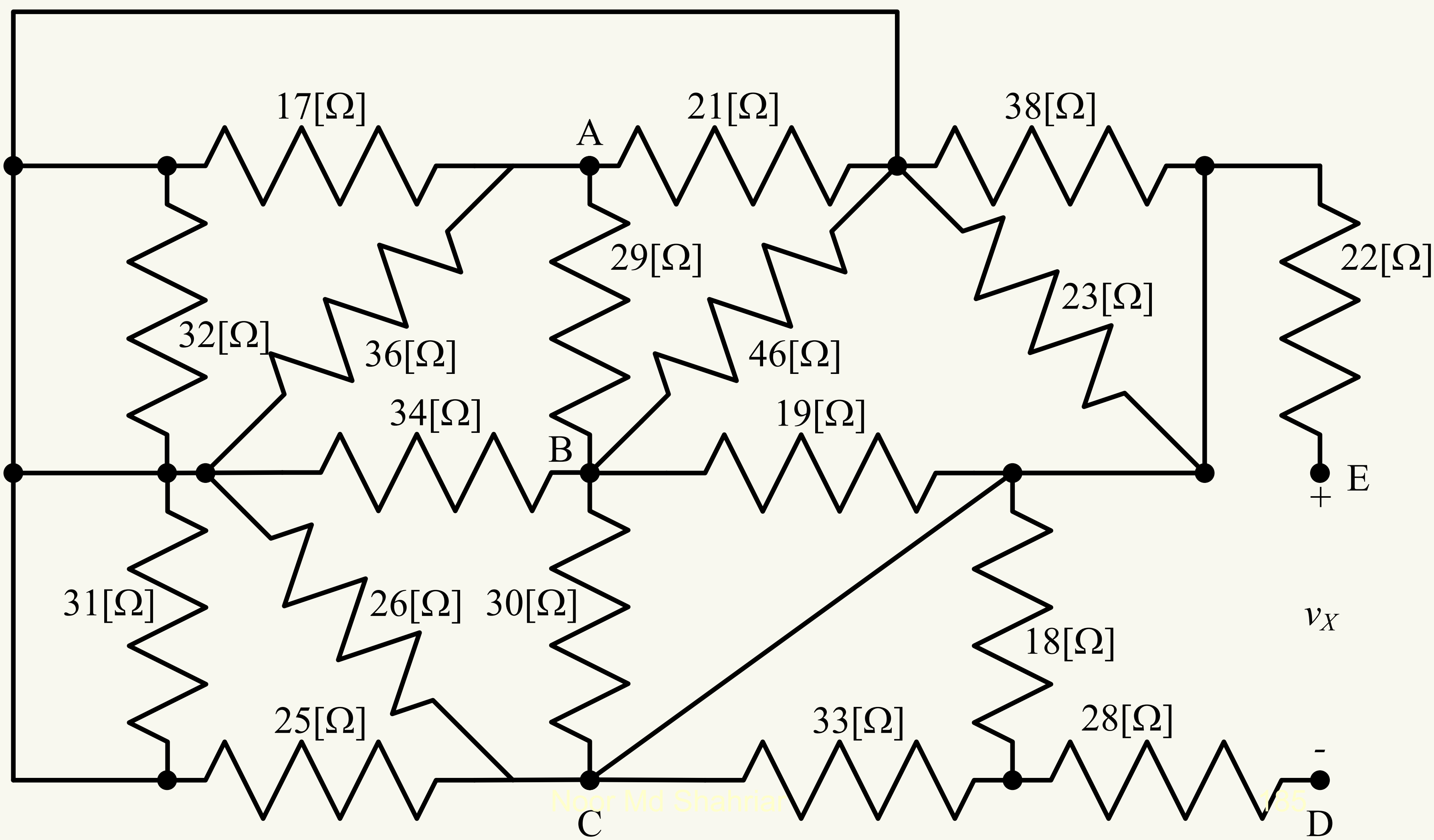
We are finding the equivalent resistance as seen from terminals B and D.

Can R_3 be removed?

If so, should it be replaced by anything?

Example Problem #5

Find the equivalent resistance as seen from terminals A and B.



Week -9

Page- (187-257)

The Node Voltage Method

Some Basic Definitions

- **Node** – a place where two or more components meet
- **Essential Node** – a place where three or more components meet
- **Reference Node** – a special essential node that we choose as a reference point for voltages

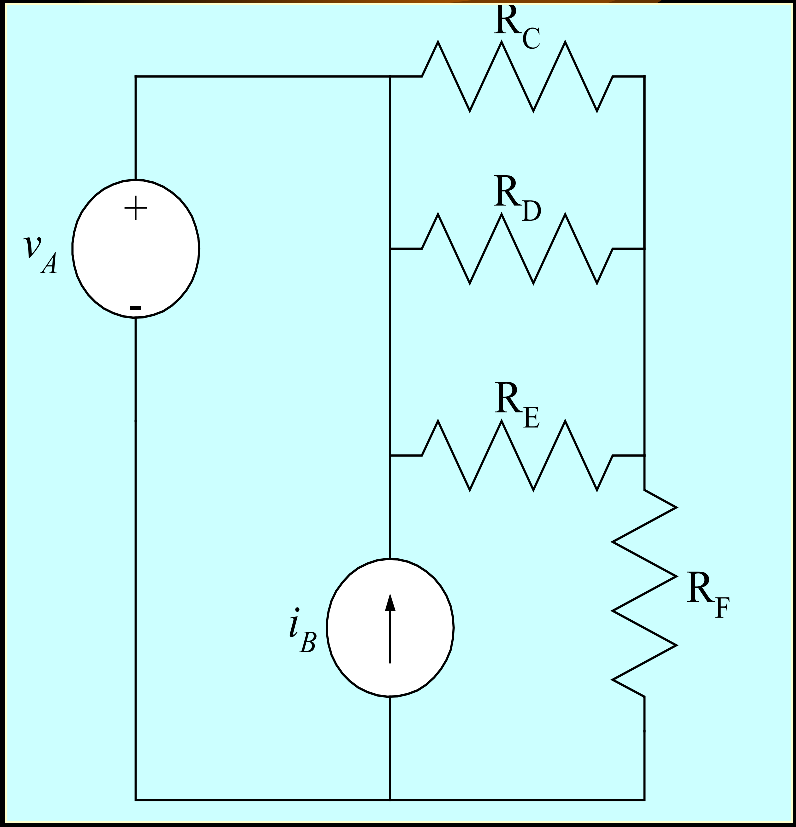
You may be familiar with the word node from its use as a location in computer networks. It has a similar meaning there, a place where computers are connected.

Review Nodes

Skip Review of Nodes

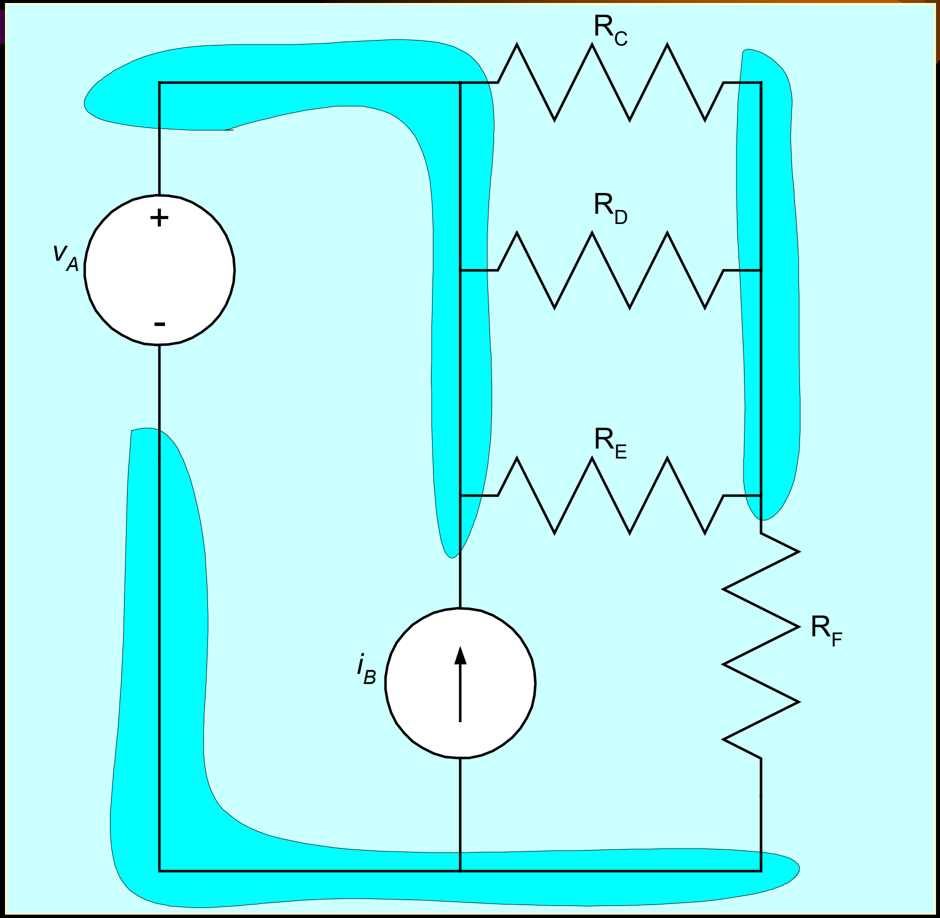
Some Review – Nodes

- A node is defined as a place where two or more components are connected.
- The key thing to remember is that we connect components with wires. It doesn't matter how many wires are being used; it only matters how many components are connected together.
- How many nodes are there in this circuit here?



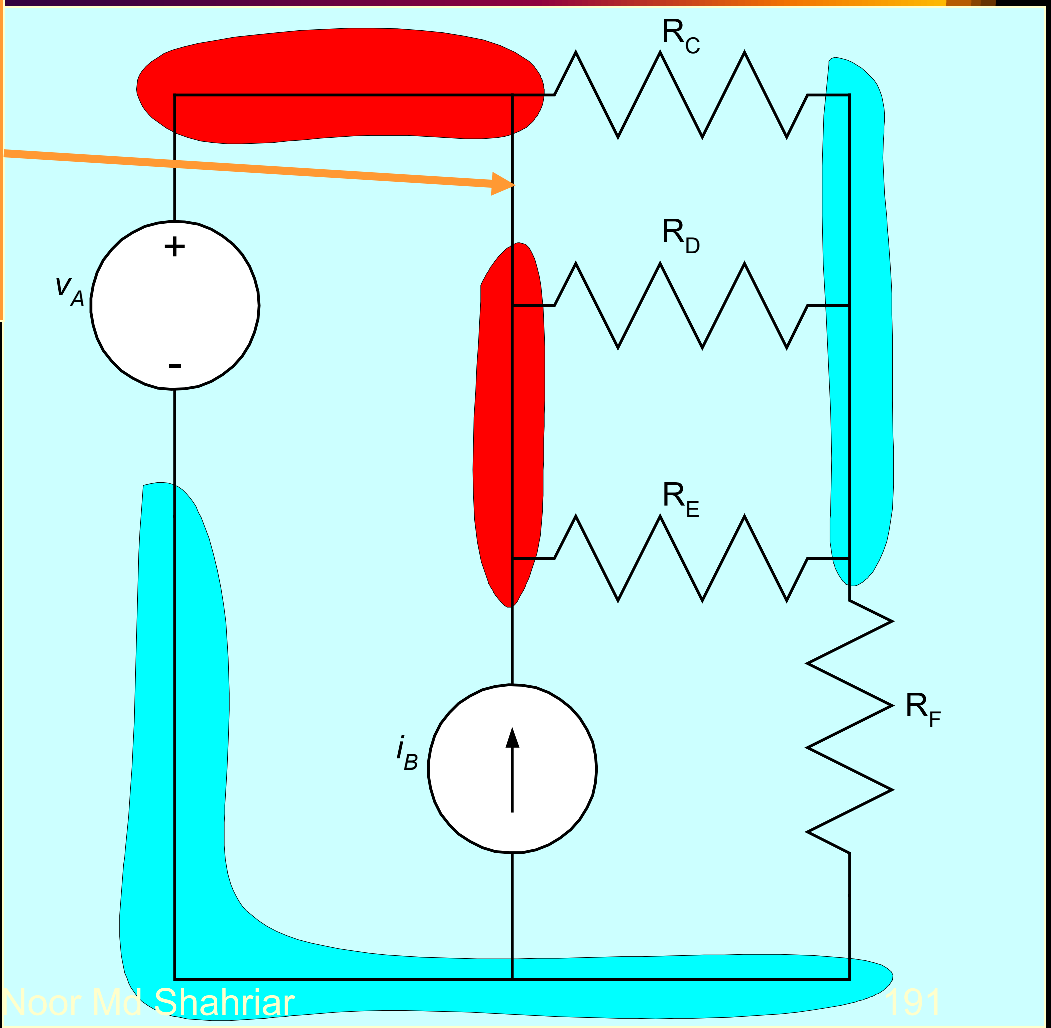
How Many Nodes – Correct Answer

- In the example circuit schematic given here, there are three nodes. These nodes are shown in dark blue here.
- Some students count more than three nodes in a circuit like this. When they do, it is usually because they have considered two points connected by a wire to be two nodes.
- There are also three essential nodes. Each of these three nodes has at least 3 components connected to it.



How Many Nodes – Wrong Answer

Wire connecting two nodes means that these are really a single node.



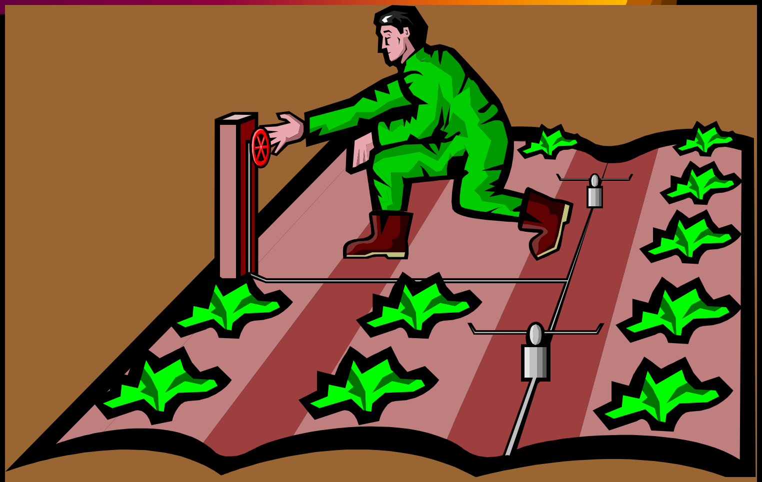
- In the example circuit schematic given here, the two red nodes are really the same node. There are not four nodes.
- Remember, two nodes connected by a wire were really only one node in the first place.

The Node-Voltage Method (NVM)

The Node-Voltage Method (NVM) is a systematic way to write all the equations needed to solve a circuit, and to write just the number of equations needed. The idea is that any other current or voltage can be found from these node voltages.

This method is not that important in very simple circuits, but in complicated circuits it gives us an approach that will get us all the equations that we need, and no extras.

It is also good practice for the writing of KCL and KVL equations. Many students believe that they know how to do this, but make errors in more complicated situations. Our work on the NVM will help correct some of those errors.



The Node-Voltage Method is a system. And like the sprinkler system here, the goal is be sure that nothing gets missed, and everything is done correctly. We want to write all the equations, the minimum number of equations, and nothing but **correct** equations.

The Node-Voltage Method (NVM)

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We will explain these steps by going through several examples.

Review KCL Equations Skip KCL Review

Kirchhoff's Current Law (KCL) – a Review

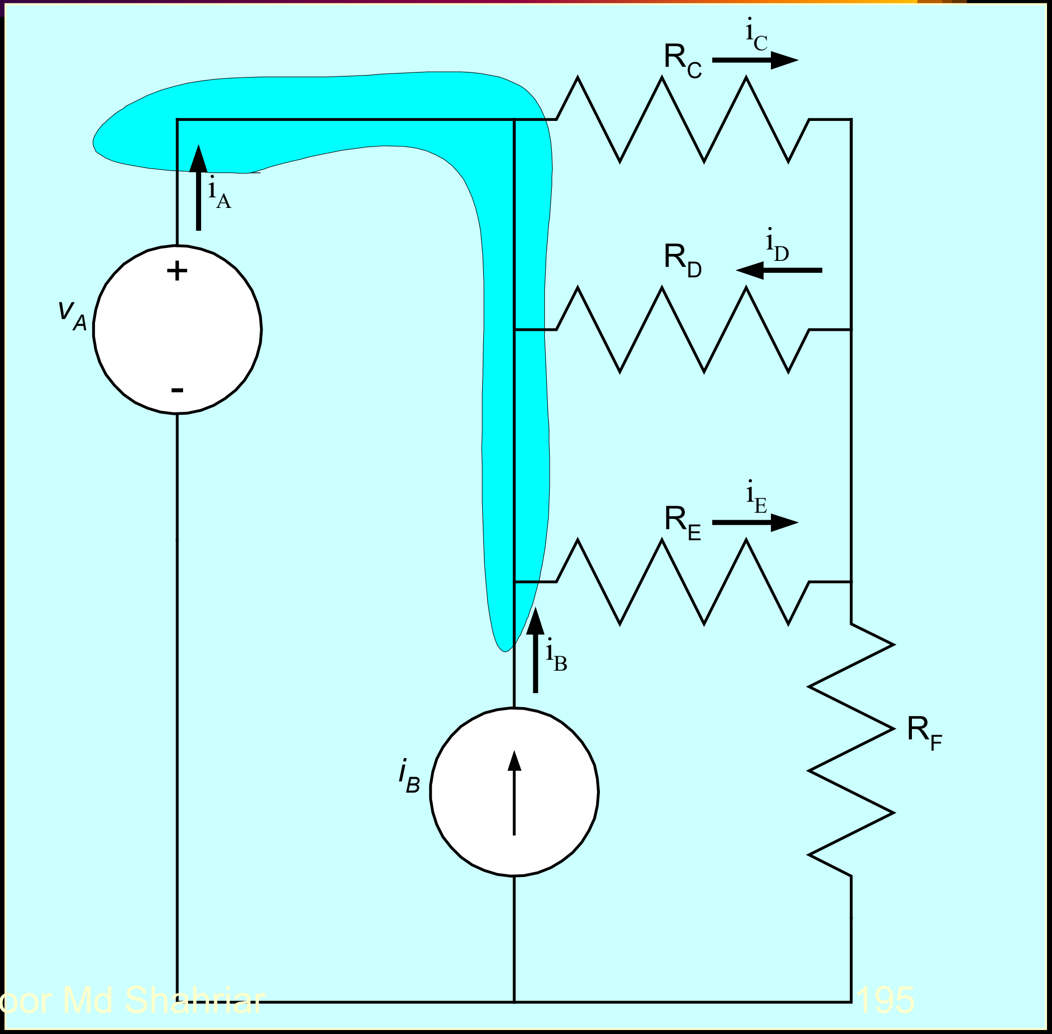
The algebraic (or signed) summation of currents through any closed surface must equal zero.

For this set of material, we will always assign a positive sign to a term that refers to a reference current that leaves a closed surface, and a negative sign to a term that refers to a reference current that enters a closed surface.

Kirchhoff's Current Law (KCL) – a Review Example

- For this set of material, we will always assign a positive sign to a term that refers to a current that leaves a node, and a negative sign to a term that refers to a current that enters a node.
- In this example, we have already assigned reference polarities for all of the currents for the nodes indicated in darker blue.
- For this circuit, and using my rule, we have the following equation:

$$-i_A + i_C - i_D + i_E - i_B = 0$$

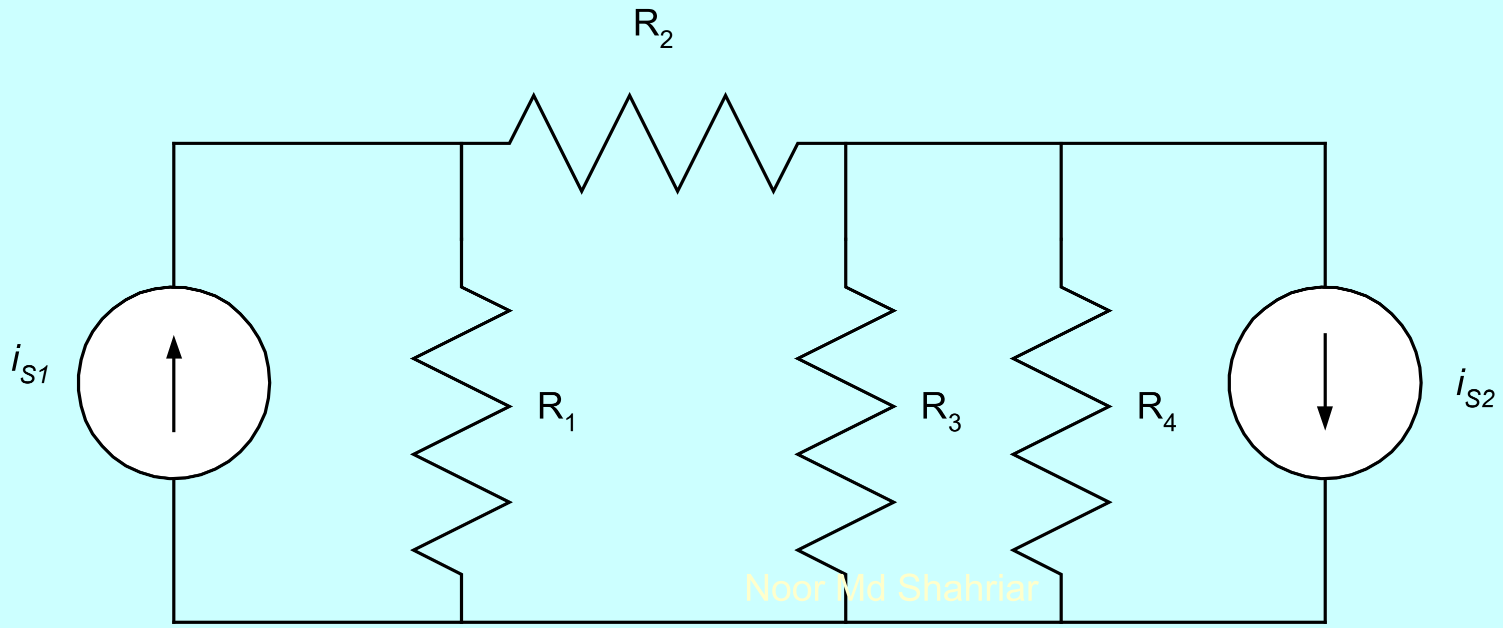


NVM – 1st Example

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

For most students, it seems to be best to introduce the NVM by doing examples. We will start with simple examples, and work our way up to complicated examples. Our first example circuit is given here.

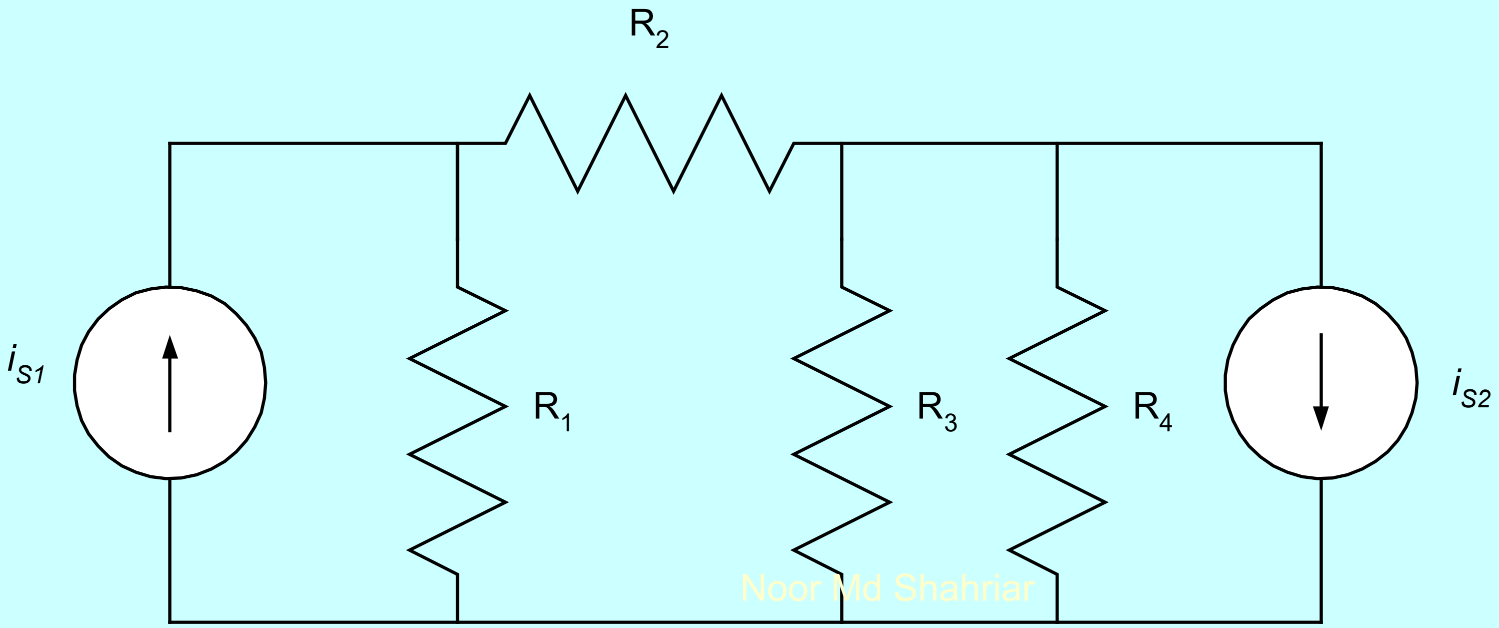


NVM – 1st Example – Step 1

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We need to find all the essential nodes, and only the essential nodes. How many are there?

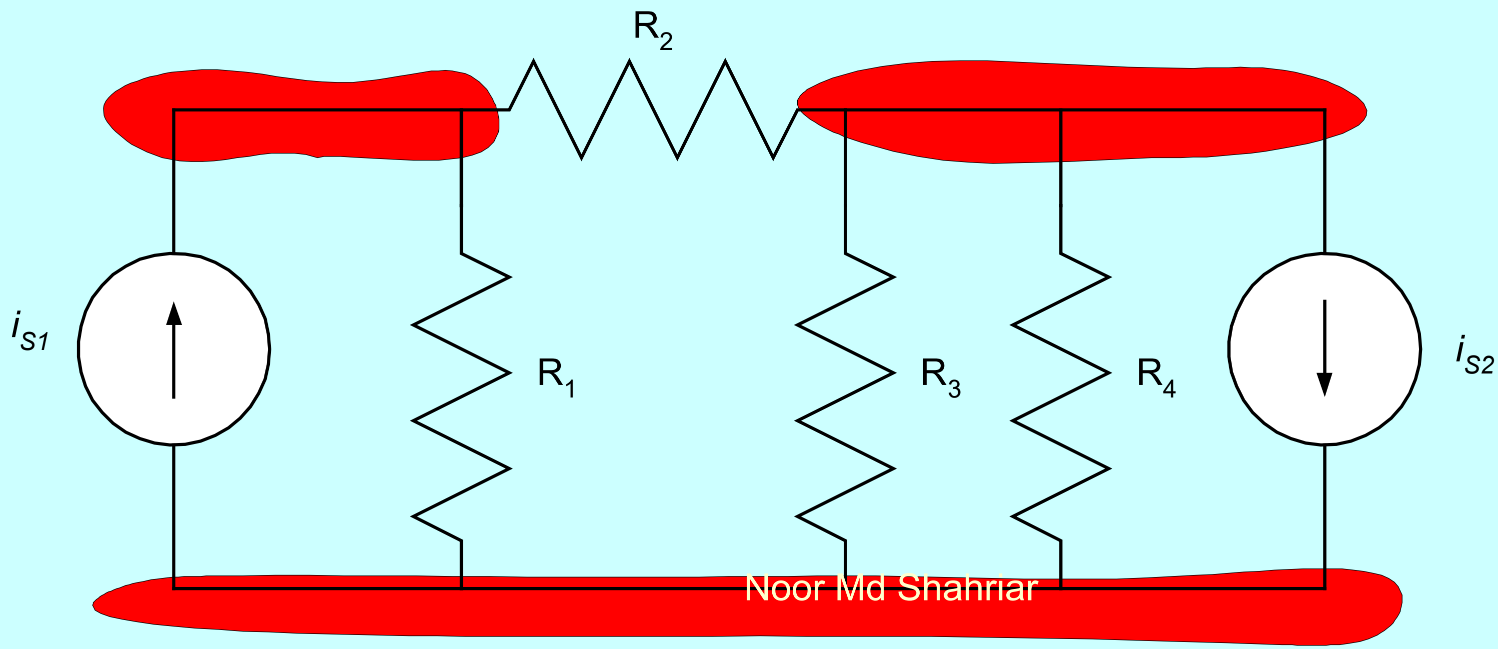


NVM – 1st Example – Step 1(Done)

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

There are three essential nodes, each of which is shown in red on the diagram below.

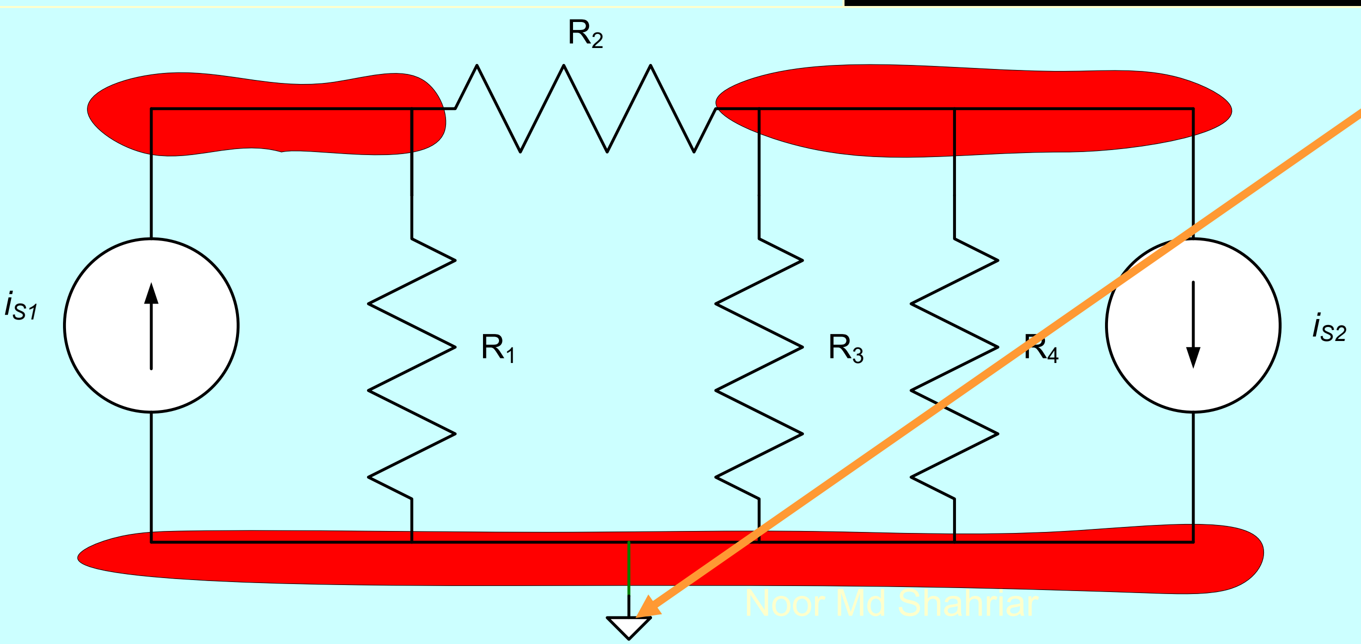


NVM – 1st Example – Step 2

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We could choose any of the three essential nodes as the reference node. However, there are better choices. Remember that we need to write a KCL equation for each essential node, except for the reference node. The best idea, then, is to pick the node with the most connections, to eliminate the most difficult equation. Here this is the bottom node. It is labeled to show that it is the reference node.



This symbol is used to designate the reference node. There are different symbols used for this designation. This choice of symbols is not important. Making a designation **is** important.

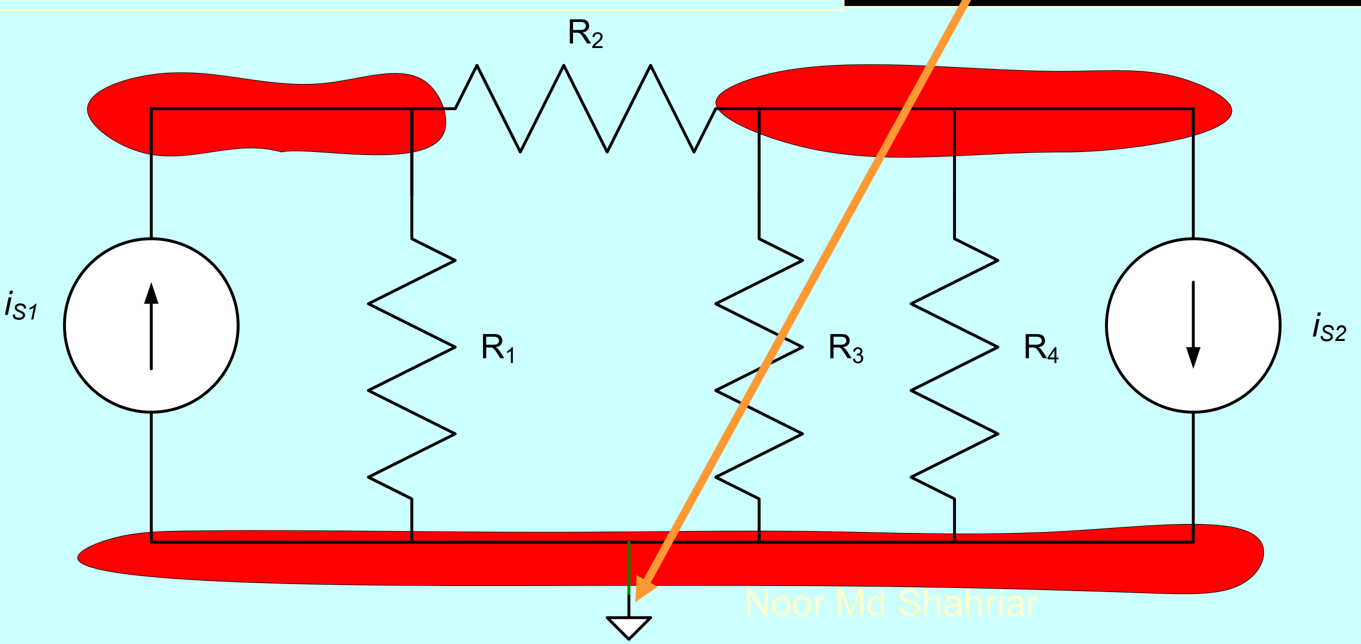
NVM – 1st Example – Step 2 Note

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Among the symbols that you might see to designate the reference node are the ones shown below. The choice we use is the one used in most textbooks.

Reference Node Symbols



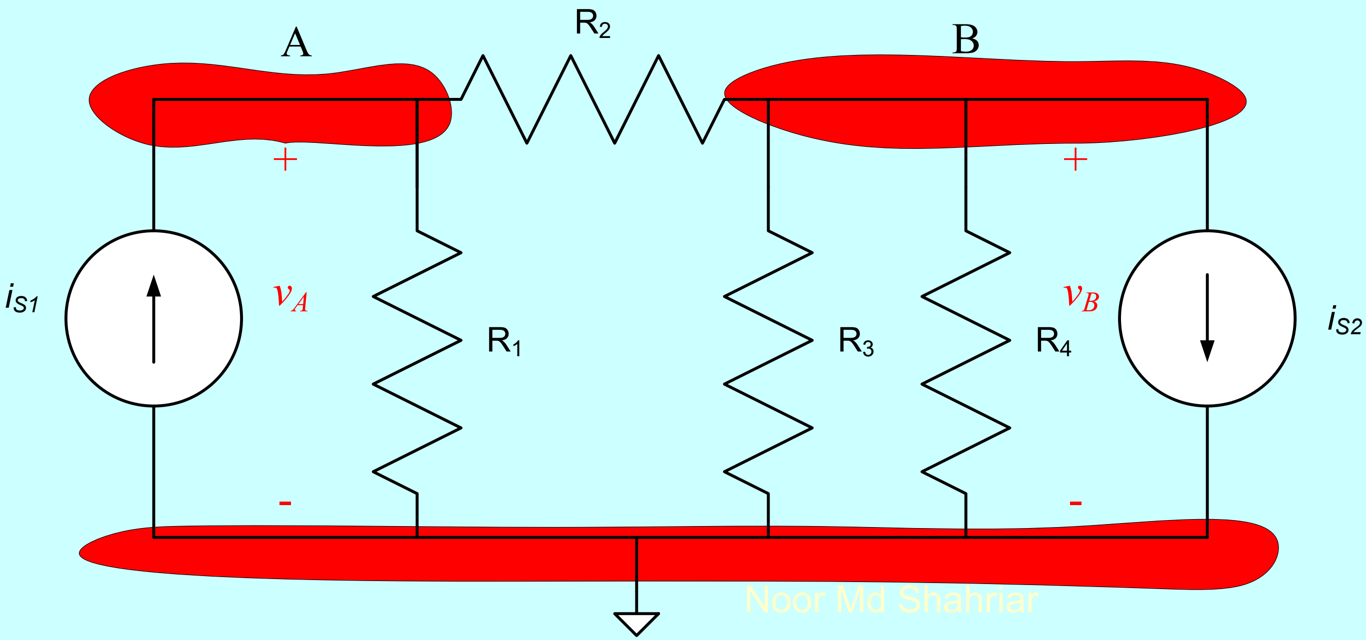
Actually, each of these symbols has a specific meaning in a formal circuit schematic. However, for our purposes here, the distinction is not important.

NVM – 1st Example – Step 3

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We have defined the node voltages, v_A and v_B. They are shown in red. For clarity, we have also named the nodes themselves, A and B.



Note: As with any voltage, the polarity must be defined. We have defined the voltages by showing the voltages with a "+" and "-" sign for each. Strictly speaking, this should not be necessary. The words in step 3 make the polarity clear. Some texts do not label the voltages on the schematic. For clarity, we will label the voltages in these notes.

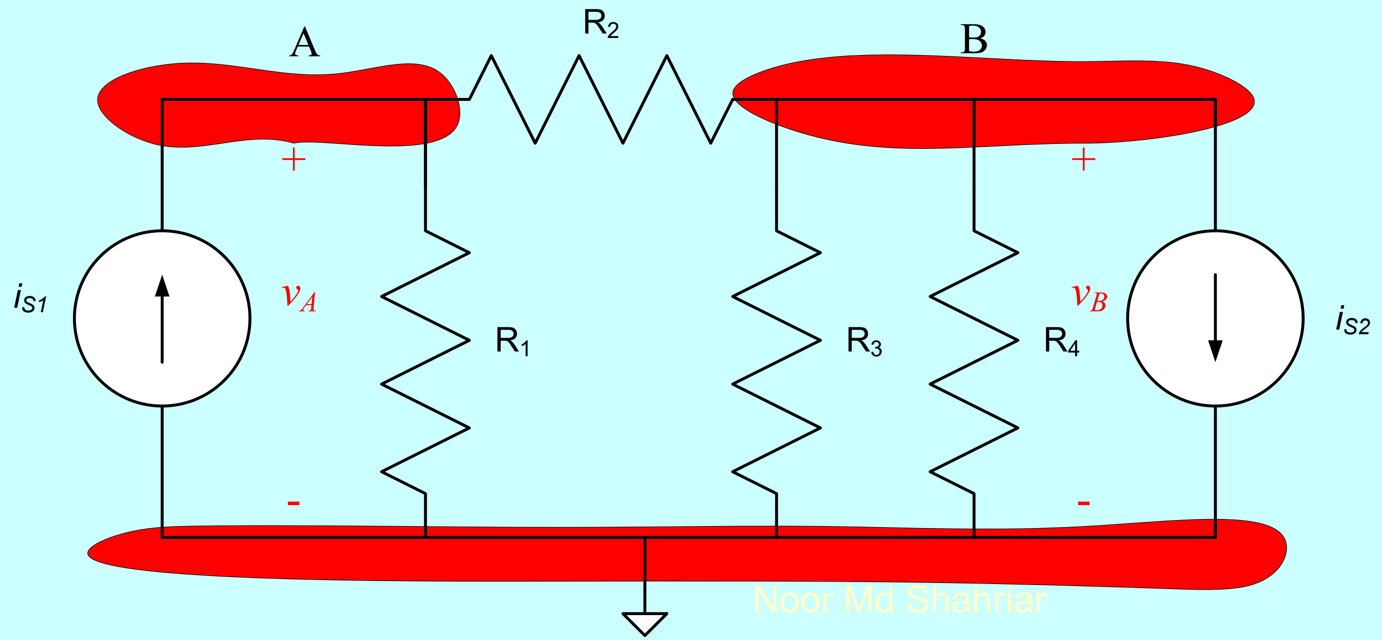
NVM – 1st Example – Step 4, Part 1

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Now, we need to write a KCL equation for each non-reference essential node. That means an equation for A and one for B. Let's start with A. The equation is:

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0.$$



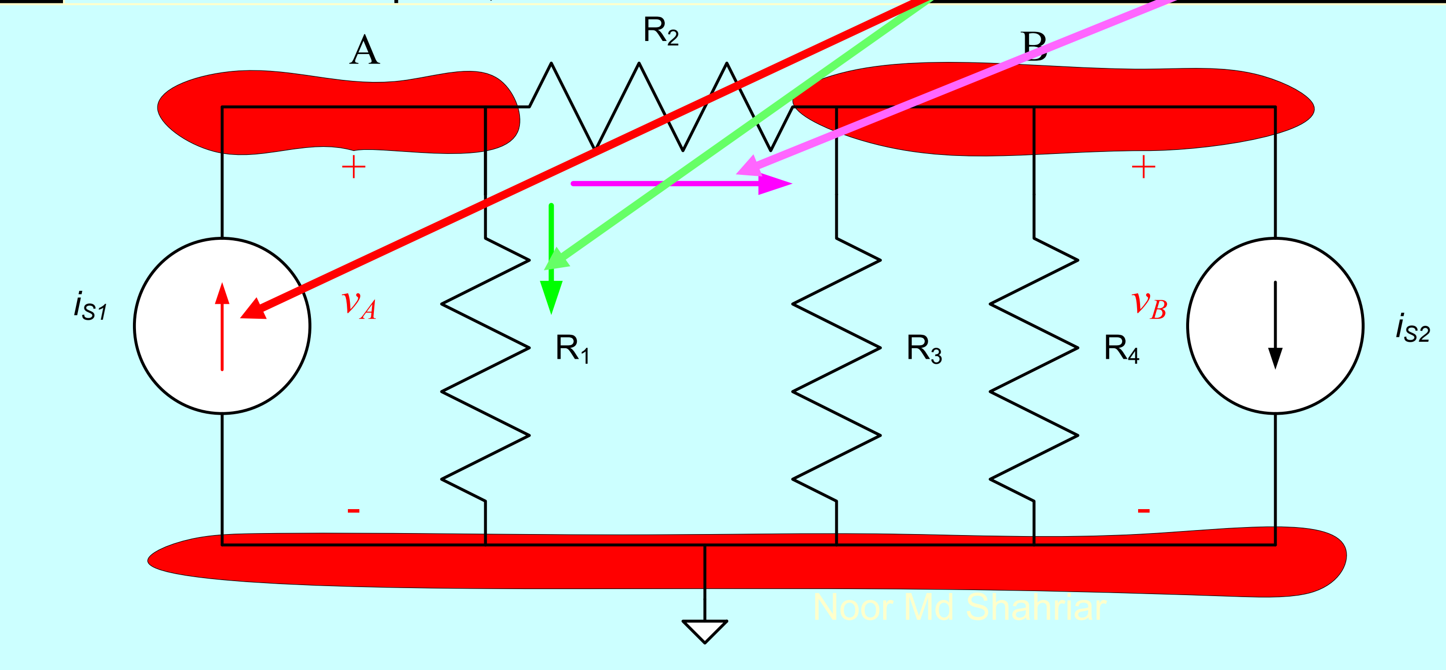
NVM – 1st Example – Step 4, Part 2

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Now, we need to write a KCL equation for each non-reference essential node. That means an equation for A and one for B. Let's start with A. The equation is:

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

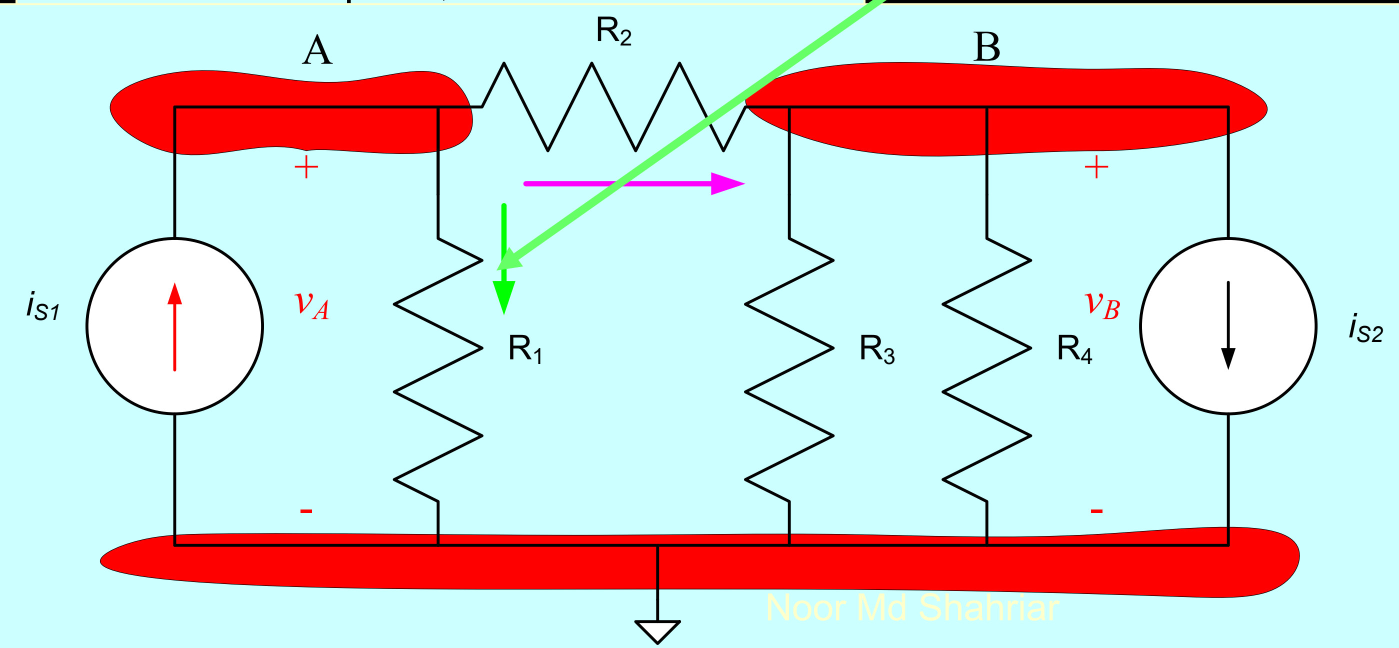


The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

The first term comes from Ohm's Law. The voltage v_A is the voltage across R_1. Thus, the current shown in green is v_A/R_1, out of node A, and thus has a + sign in this equation.

$$+\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



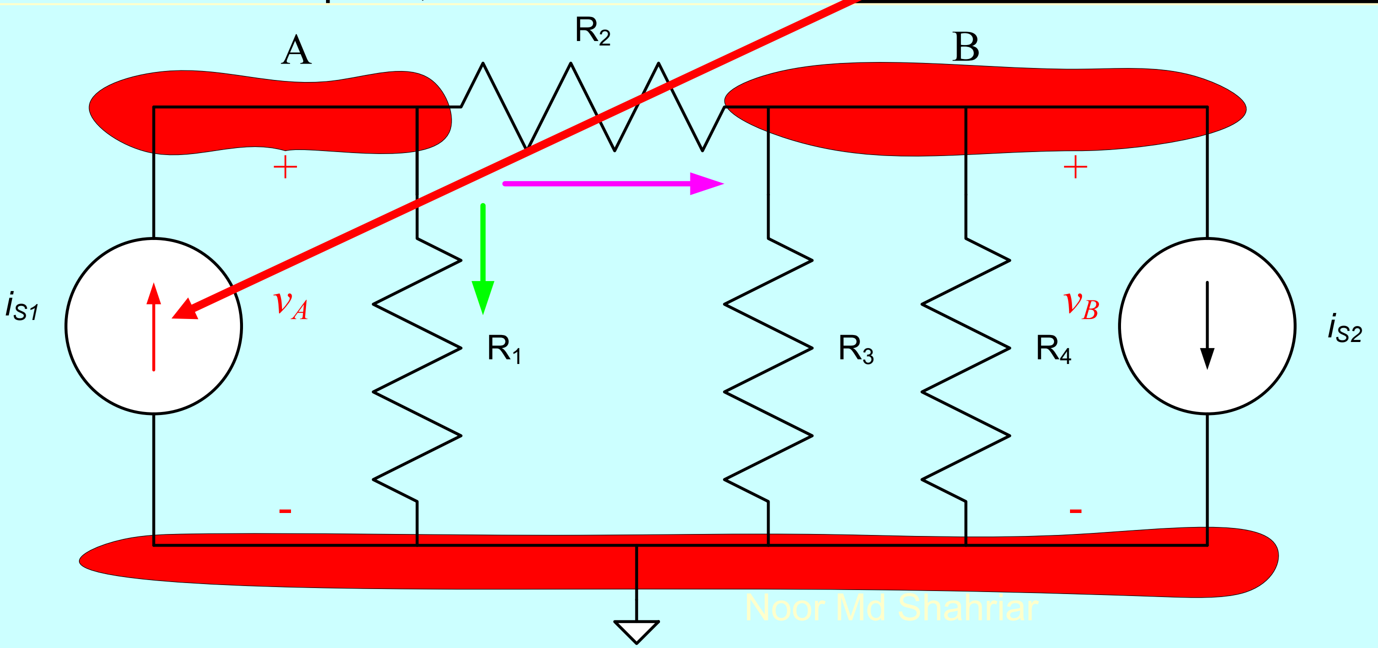
NVM – Currents Explained 2

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

The current through the current source is, by definition, given by the value of that current source. Since the reference polarity of the current is entering node A, it has a "-" sign.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



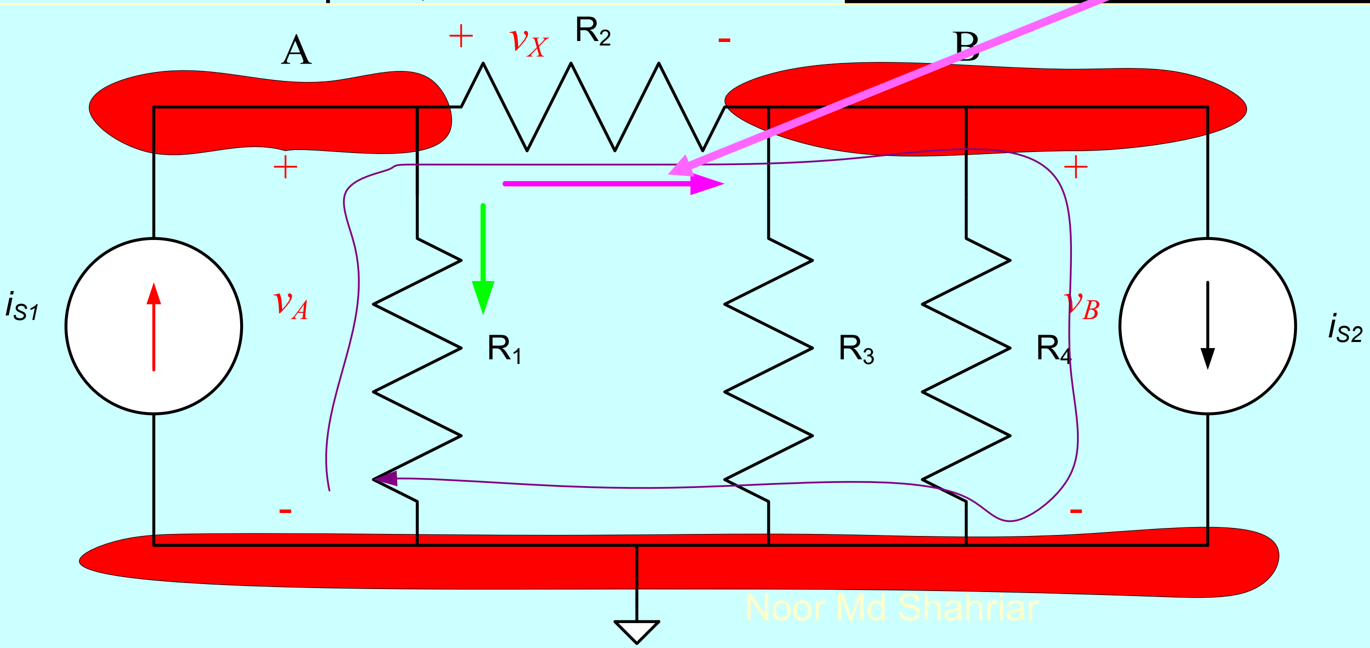
NVM – Currents Explained 3

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

This current expression also comes from Ohm's Law. The voltage v_X is the voltage across the resistor R_2, and results in a current in the polarity shown.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



To prove to yourself that $v_X = v_A - v_B$, take KVL around the loop shown. The voltage at A with respect to B, is $v_A - v_B$, where v_A and v_B are both node voltages.

NVM – 1st Example – Step 4, Part 3

The Node-Voltage Method steps are:

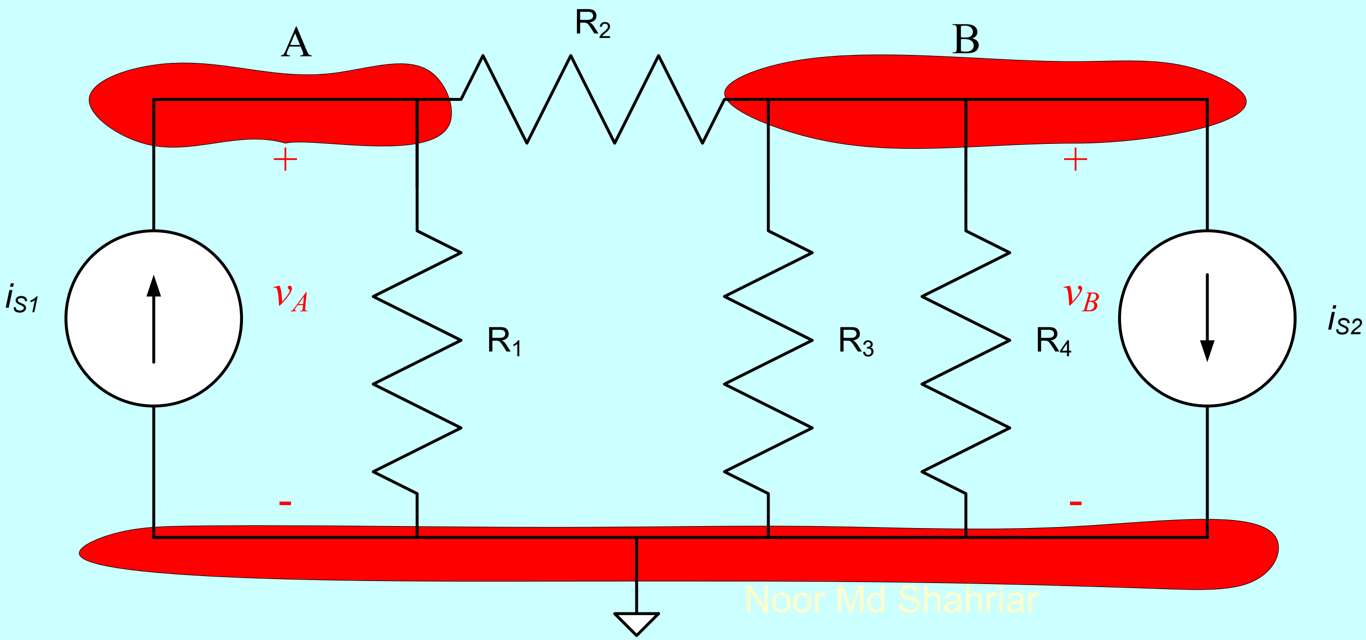
1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

The KCL equation for the A node was:

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0.$$

The KCL equation for the B node is:

$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0.$$



Be very careful that you understand the signs of all these terms. One of the big keys in these problems is to get the signs correct. If you have questions, review this material.

NVM – 1st Example – Step 4 – Notes

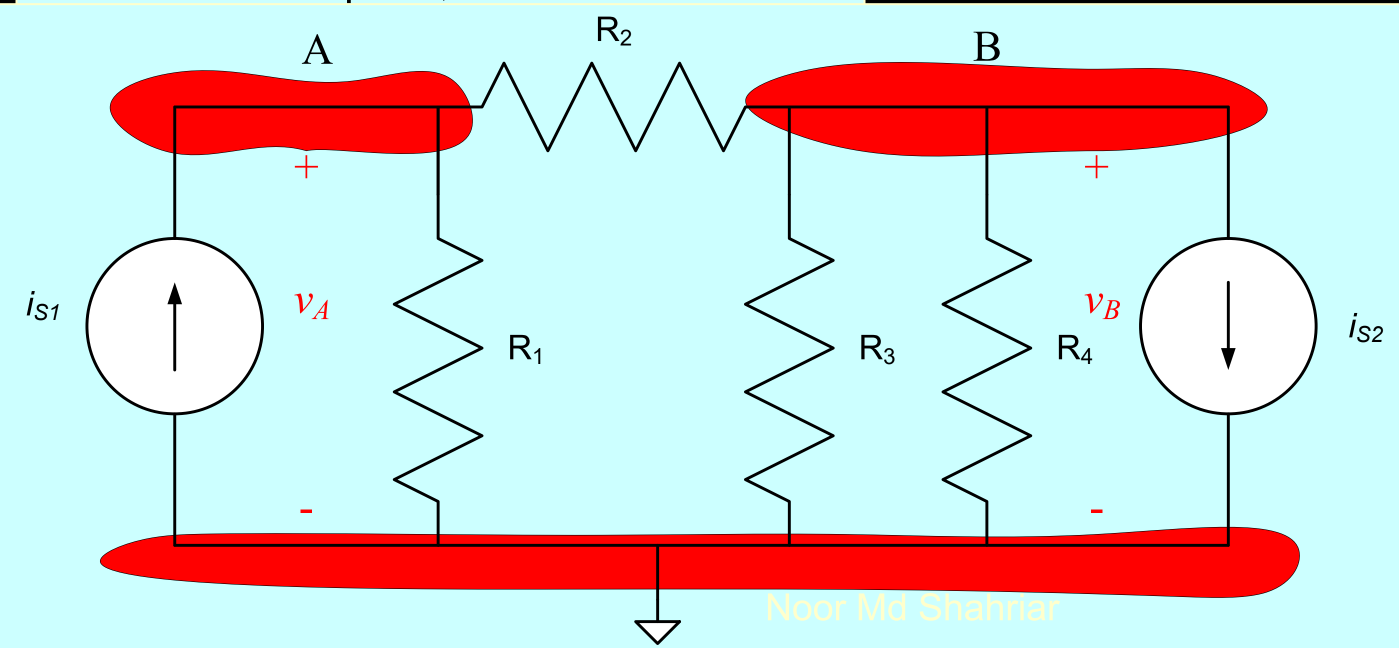
The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Some notes that may be helpful:

a) We are actually writing KCL for the closed surfaces shown. You might want to actually sketch in your diagrams a closed surface like this, so that you don't miss any currents.

b) When we write these equations using the conventions we picked, the A node equation has a positive sign associated with all the terms with v_A, and a negative sign with all other node-voltage terms. This is a good way to check your equations.



$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$

NVM – 1st Example – Step 5

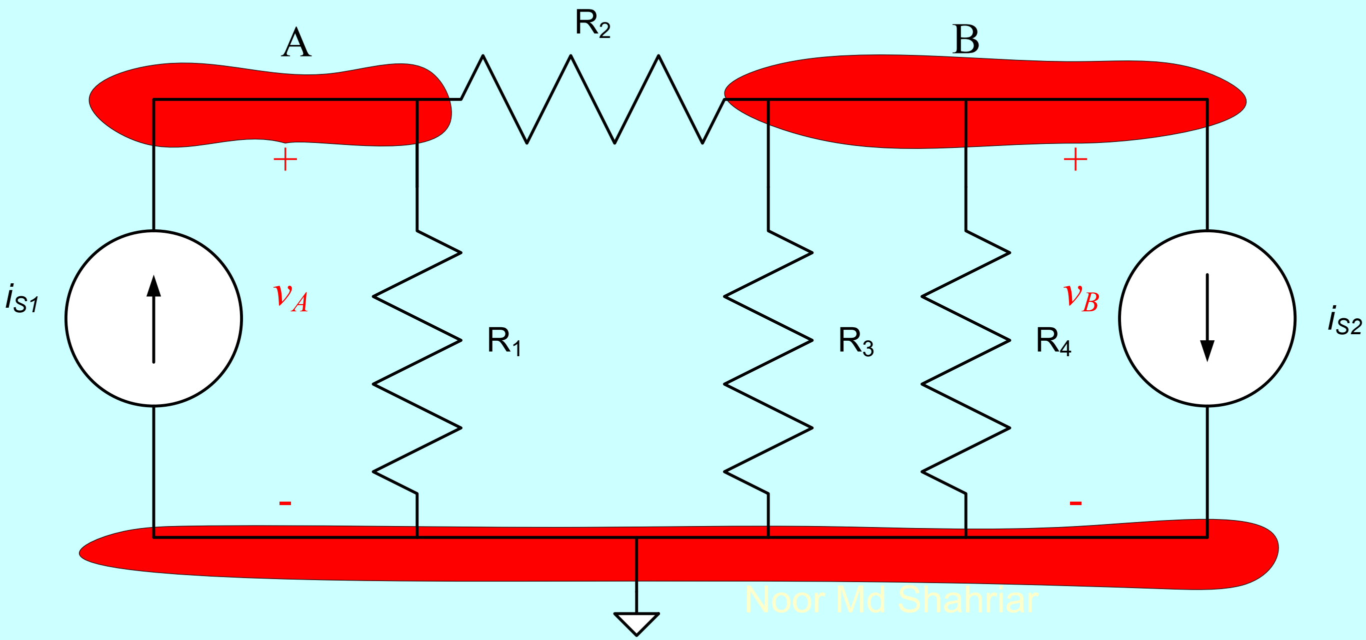
The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, at the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

There are no dependent sources in this circuit, so we can skip step 5. We should now have the same number of equations (2) as unknowns (2), and we can solve.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

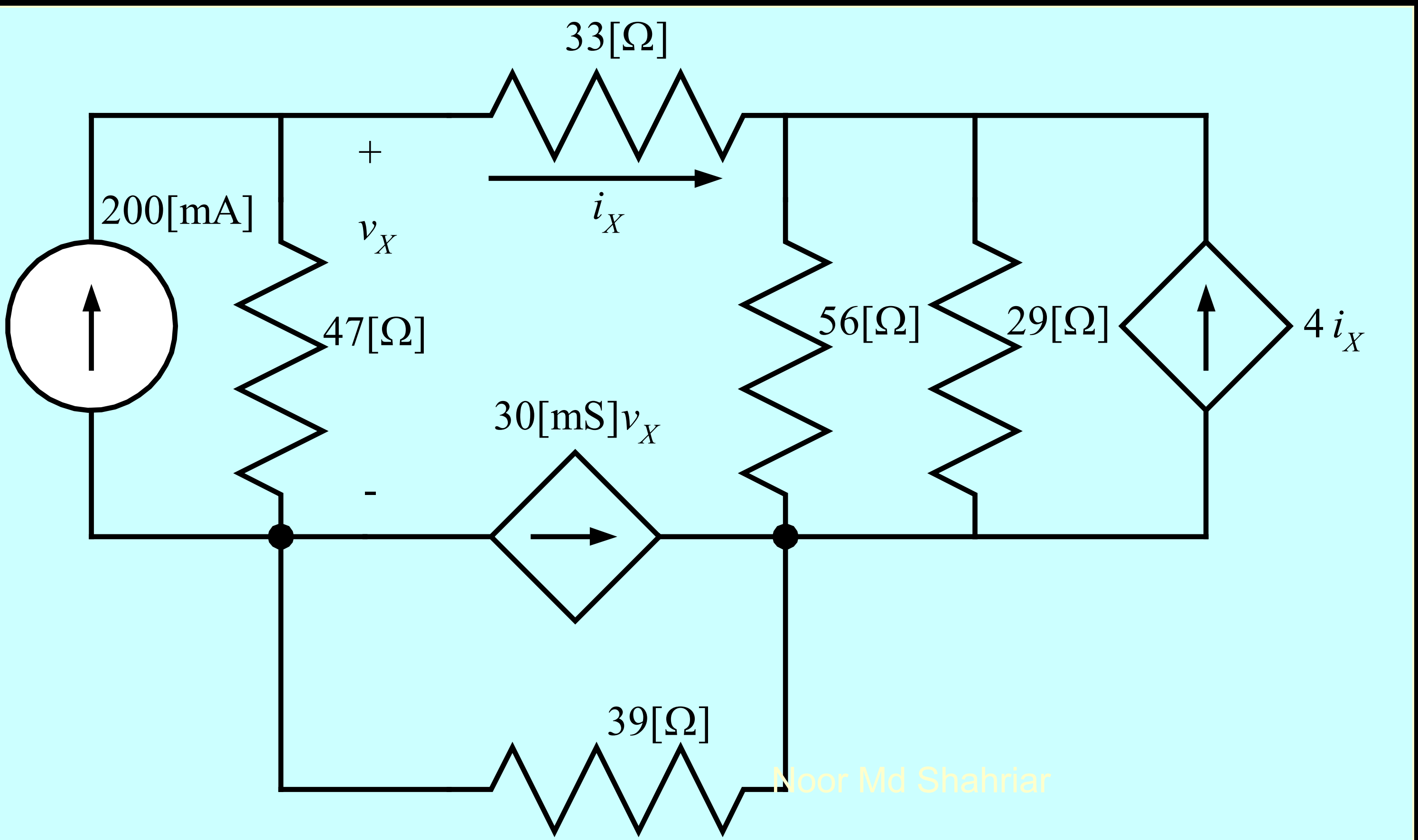
$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$



Note that we have assumed that all the values of the resistors and sources have been given. If not, we will need to get more information before we can solve.

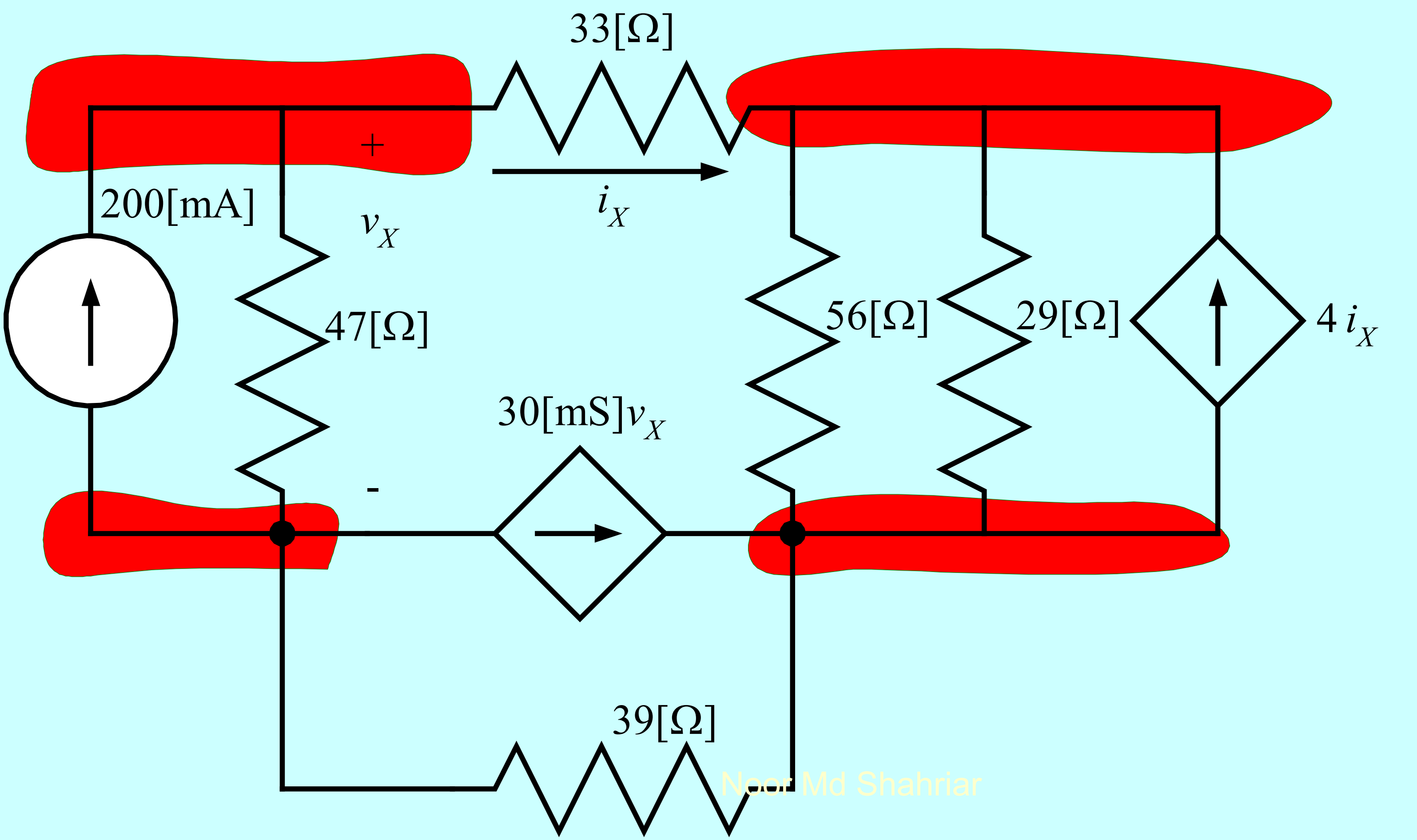
NVM – 2nd Example

Our second example circuit is given here. Numerical values are given in this example. Let's find the current i_X shown, using the Node-Voltage Method.



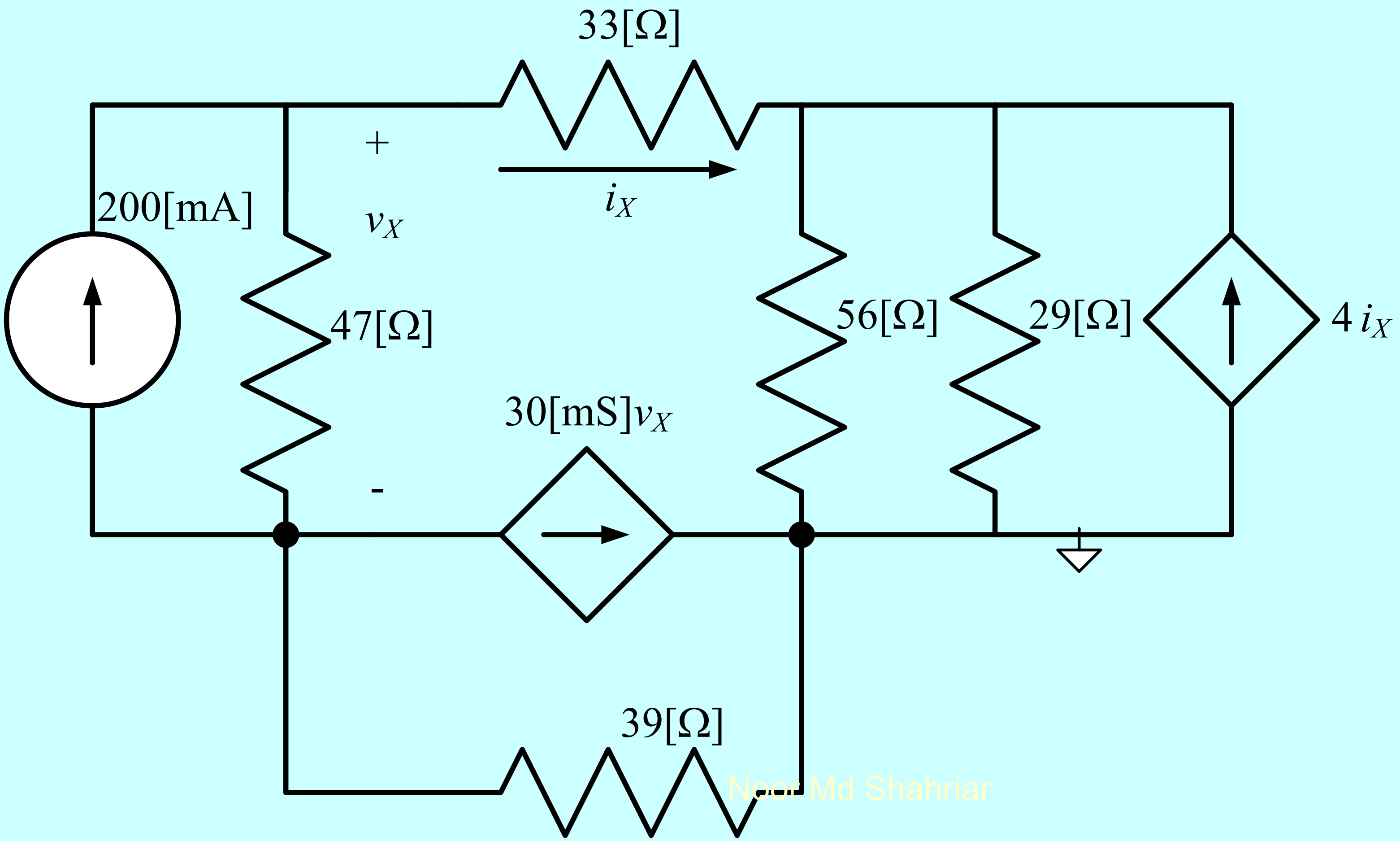
NVM – 2nd Example – Step 1

We have 4 essential nodes. We marked them in red in this slide, but will not mark them in the slides that follow. On your diagrams, you can always draw them. Remember that two nodes connected by a wire were really only one node.



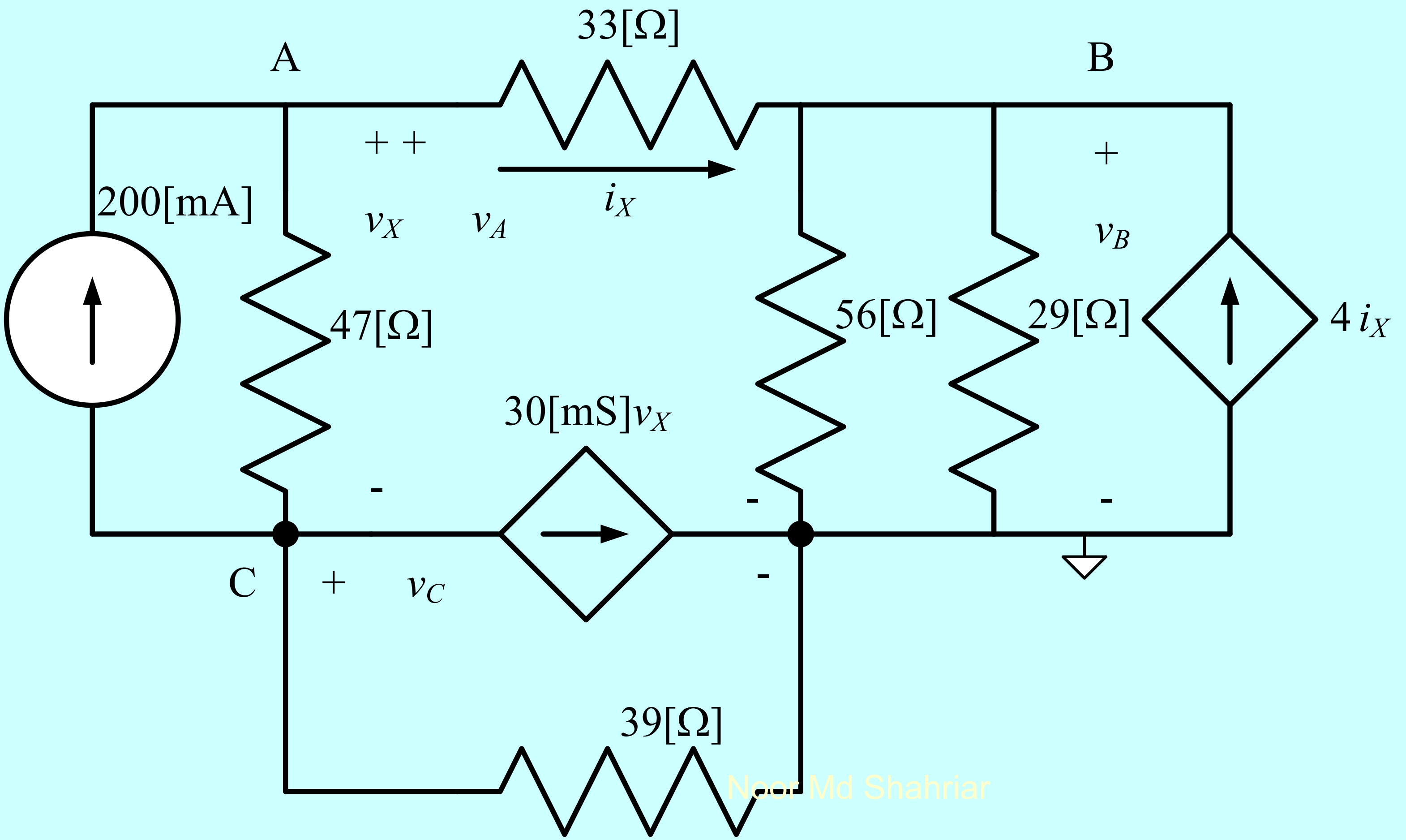
NVM – 2nd Example – Step 2

We have chosen the bottom right node as the reference node. This choice is a reasonable one, since it has 5 components connected to it, more than any other essential node.



NVM – 2nd Example – Step 3

We have defined the three node voltages. Note that each node voltage is the voltage at the essential node with respect to the reference node.

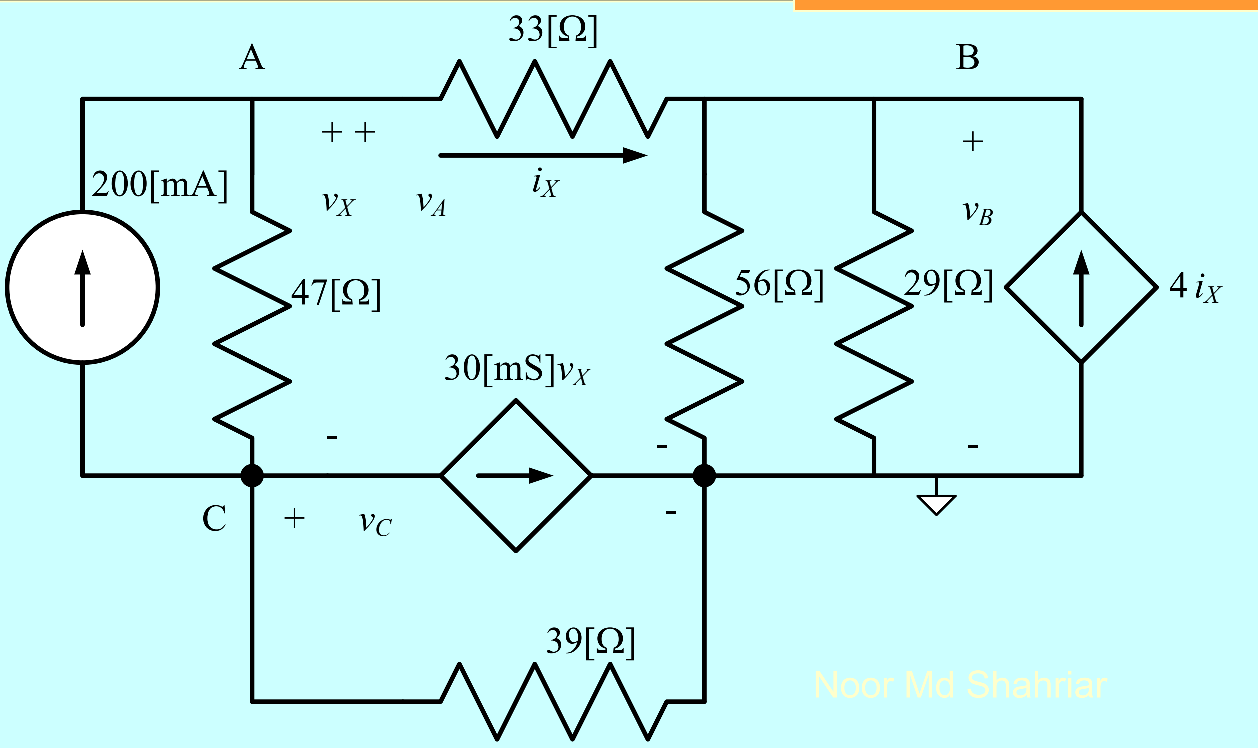


NVM – 2nd Example – Step 4

$$\text{A:}\quad -200[\text{mA}] + \frac{v_A - v_B}{33[\Omega]} + \frac{v_A - v_C}{47[\Omega]} = 0,$$

$$\text{B:}\quad -4i_X + \frac{v_B}{29[\Omega]} + \frac{v_B}{56[\Omega]} + \frac{v_B - v_A}{33[\Omega]} = 0, \text{ and}$$

$$\text{C:}\quad 200[\text{mA}] + \frac{v_C - v_A}{47[\Omega]} + 30[\text{mS}]v_X + \frac{v_C}{39[\Omega]} = 0.$$



Now, we write KCL equations for nodes A, B, and C. These are given here. We have labeled each equation with the name of the node for which it was written.

NVM – 2nd Example – Step 5

Hopefully, it is now clear why we needed step 5. Until this point, we have 3 equations and 5 unknowns. We need two more equations.

$$\text{A:} \quad -200[\text{mA}] + \frac{v_A - v_B}{33[\Omega]} + \frac{v_A - v_C}{47[\Omega]} = 0$$

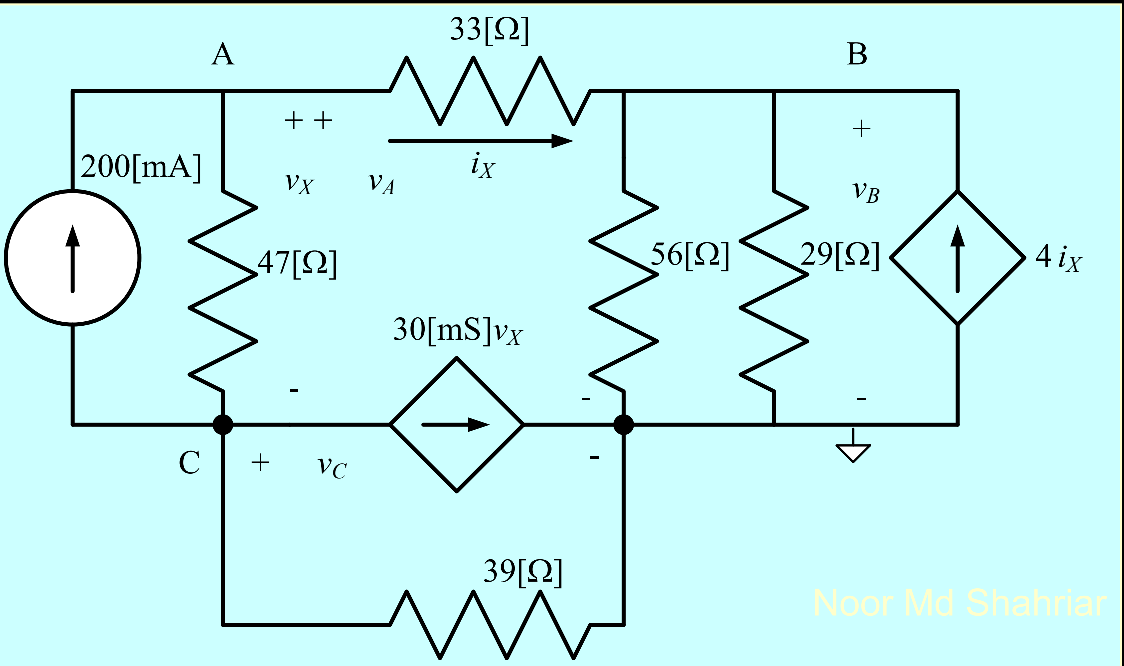
$$\text{B:} \quad -4i_X + \frac{v_B}{29[\Omega]} + \frac{v_B}{56[\Omega]} + \frac{v_B - v_A}{33[\Omega]} = 0$$

$$\text{C:} \quad 200[\text{mA}] + \frac{v_C - v_A}{47[\Omega]} + 30[\text{mS}]v_X + \frac{v_C}{39[\Omega]} = 0$$

We get these equations by writing equations for i_X and v_X, using KCL, KVL and Ohm's Law, and using the node-voltages already defined. If we have to define new variables, it will mean we need more equations. Let's write the two equations we need:

$$i_X = \frac{v_A - v_B}{33[\Omega]}, \text{ and}$$

$$v_X = v_A - v_C.$$



Now, we have 5 equations and 5 unknowns.

NVM – 2nd Example – Solution

We have the following equations.

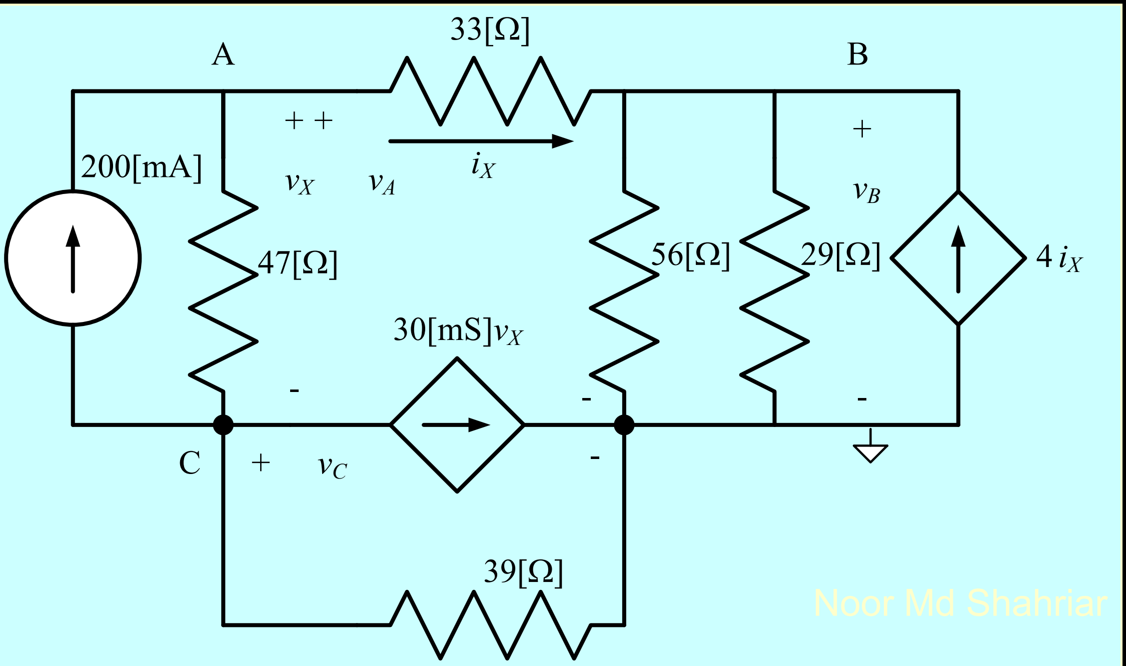
$$\text{A:}\quad -200[\text{mA}] + \frac{v_A - v_B}{33[\Omega]} + \frac{v_A - v_C}{47[\Omega]} = 0$$

$$\text{B:}\quad -4i_X + \frac{v_B}{29[\Omega]} + \frac{v_B}{56[\Omega]} + \frac{v_B - v_A}{33[\Omega]} = 0$$

$$\text{C:}\quad 200[\text{mA}] + \frac{v_C - v_A}{47[\Omega]} + 30[\text{mS}]v_X + \frac{v_C}{39[\Omega]} = 0$$

$$i_X = \frac{v_A - v_B}{33[\Omega]}$$

$$v_X = v_A - v_C$$



The solution of these five equations yields

$v_A = -1.29[\text{V}]$,

$v_B = -0.96[\text{V}]$,

$v_C = -11.2[\text{V}]$,

$v_X = 9.87[\text{V}]$, and

$i_X = -10.0[\text{mA}]$.

How many node-voltage equations do I need to write?

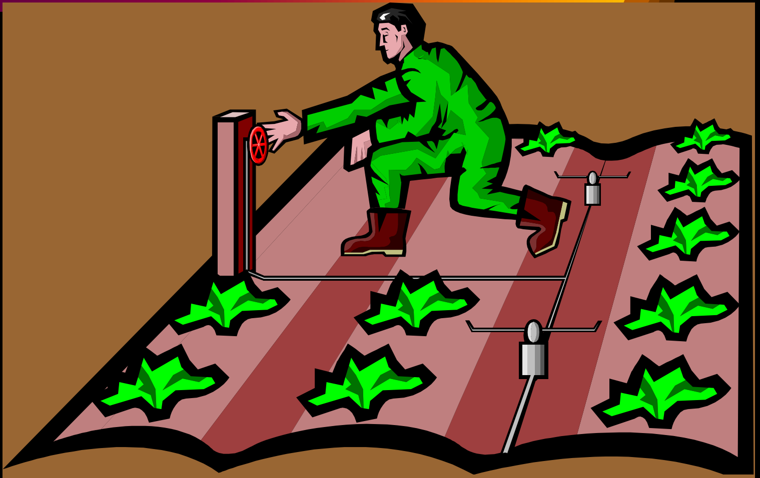
- This is a very important question. It is a good idea to figure this out before beginning a problem. Then, you will know how many equations to write before you are done.
- The fundamental rule is this: If there are n_e essential nodes, you need to write n_e-1 equations. Remember that one essential node is the reference node, and we do not write a KCL equation for the reference node.
- If there are dependent sources present, then the number of equations has to increase. In general, each dependent source introduces a variable which is unknown. If v is the number of variables that dependent sources depend on, then you need to write n_e -1+v equations.

Go back to Overview slide.

Node-Voltage Method with Voltage Sources

The Node-Voltage Method (NVM)

The Node-Voltage Method (NVM) is a systematic way to write all the equations needed to solve a circuit, and to write just the number of equations needed. The idea is that any other current or voltage can be found from these node voltages.



The Node-Voltage Method is a system. And like the sprinkler system here, the goal is be sure that nothing gets missed, and everything is done correctly. We want to write all the equations, the minimum number of equations, and nothing but **correct** equations.

The Steps in the Node-Voltage Method (NVM)

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

These steps were explained in detail in the last set of lecture notes.

Voltage Sources and the NVM

The NVM steps are:

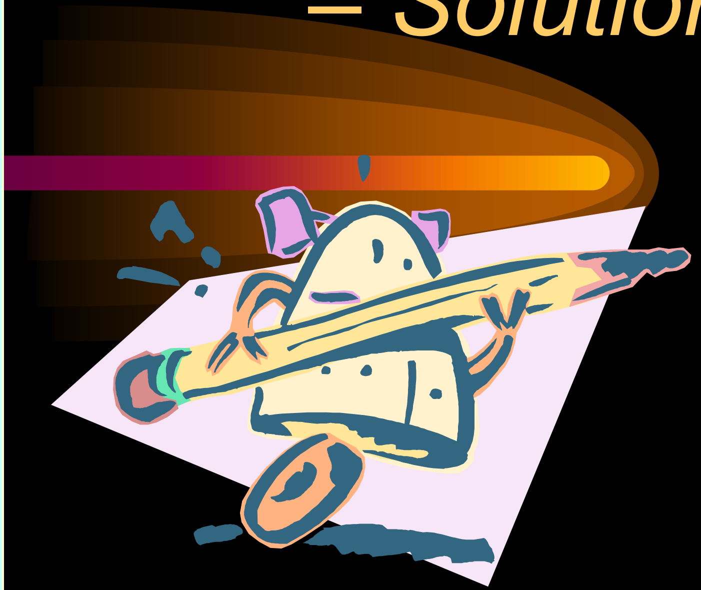
1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

A problem arises when using the NVM when there are voltage sources present. The problem is in Step 4. The current through a voltage source can be anything; the current depends on what the voltage source is connected to. Therefore, it is not clear what to write for the KCL expression. We could introduce a new current variable, but we would rather not introduce another variable. In addition, if all we do is directly write KCL equations, we cannot include the value of the voltage source.

Voltage Sources and the NVM – Solution

The NVM steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

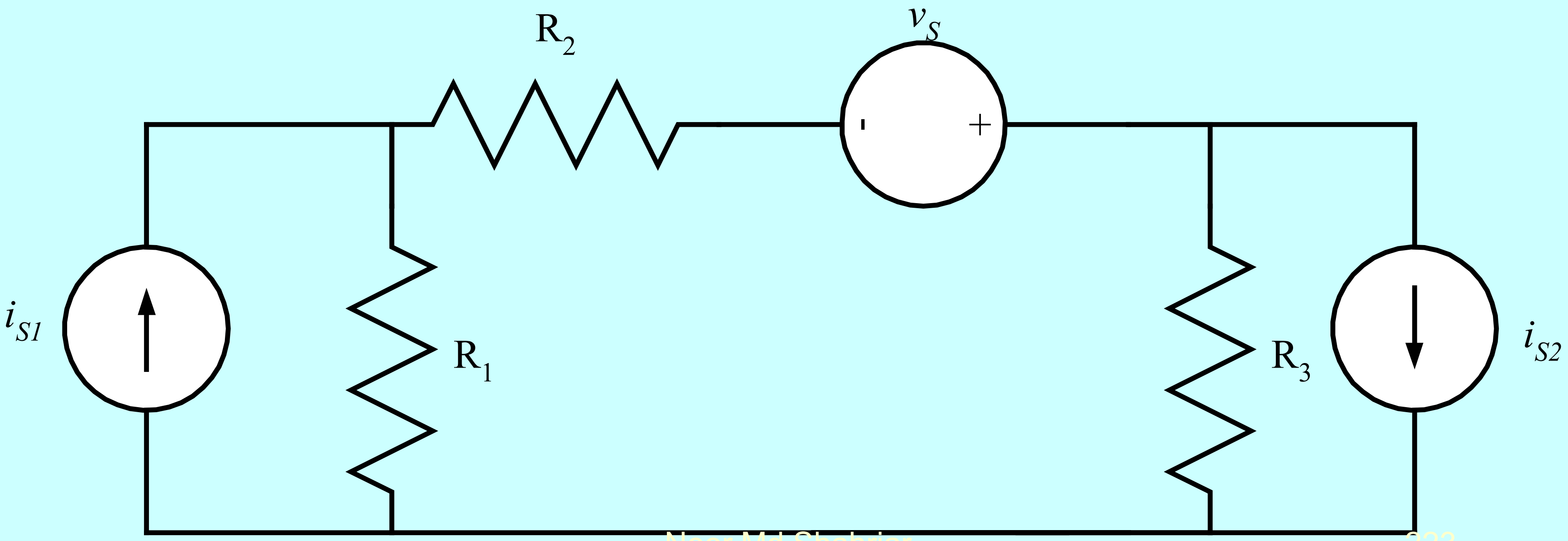


The solution for what to do when there is a voltage source present depends on how it appears. There are three possibilities. We will handle each of them in turn. The three possibilities are:

1. A voltage source in series with another element.
2. A voltage source between the reference node and another essential node.
3. A voltage source between two non-reference essential nodes.

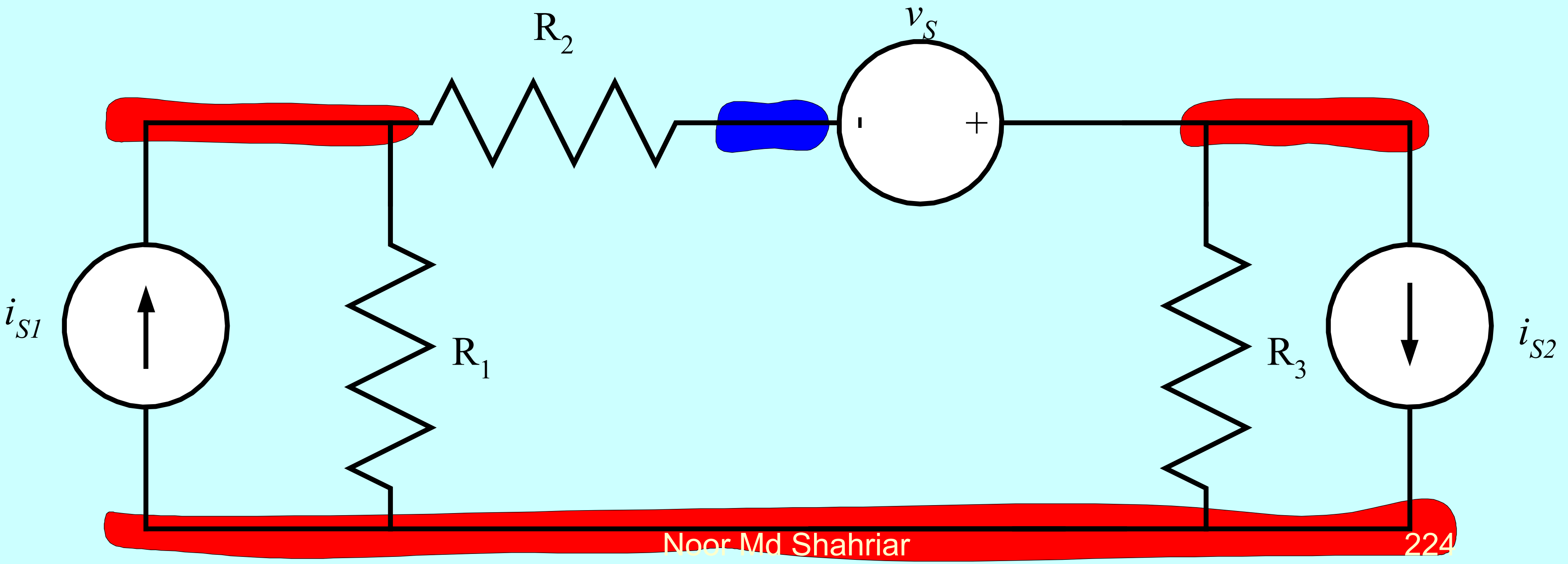
NVM – Voltage Source in Series with Another Element

As before, it seems to be best to introduce the NVM by doing examples. Our first example circuit is given here. We will go through the entire solution, but our emphasis will be on step 4. Note that here the voltage source v_S is in series with the resistor R_2.



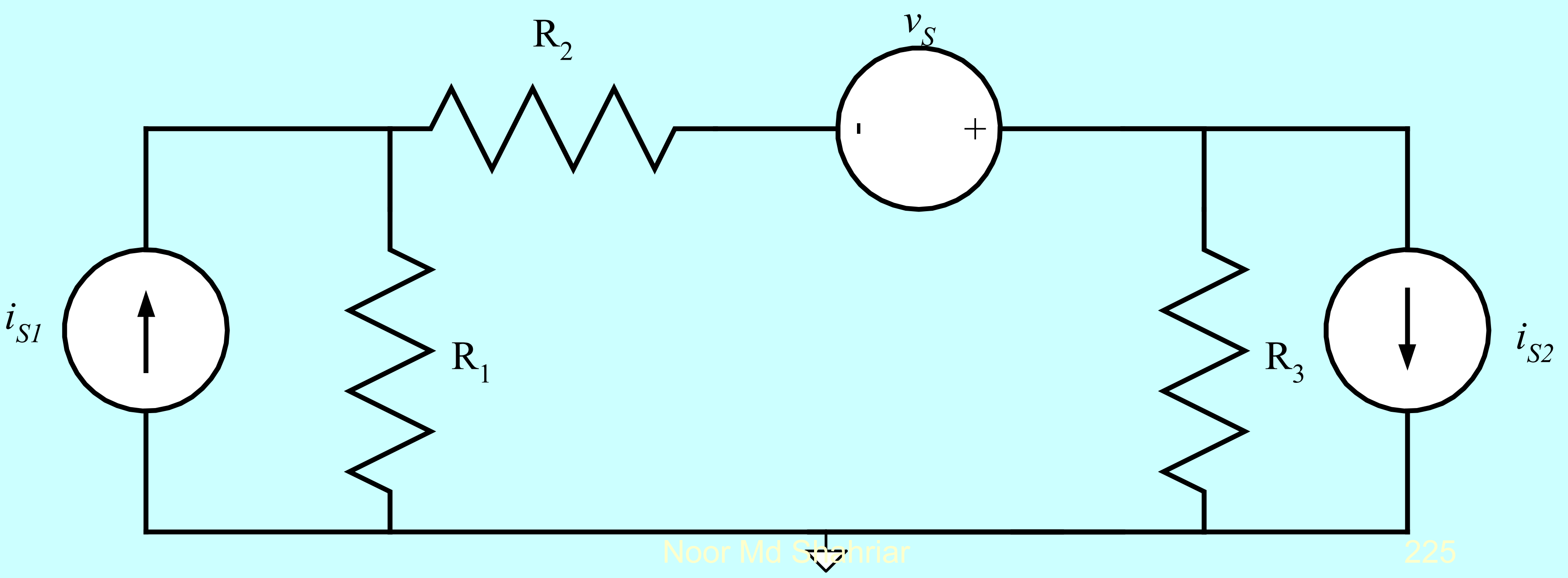
NVM – Voltage Source in Series Step 1

The first step is to identify the essential nodes. There are three, marked in red. The fourth node, marked in dark blue, is not an essential node. It only connects two components, not three.



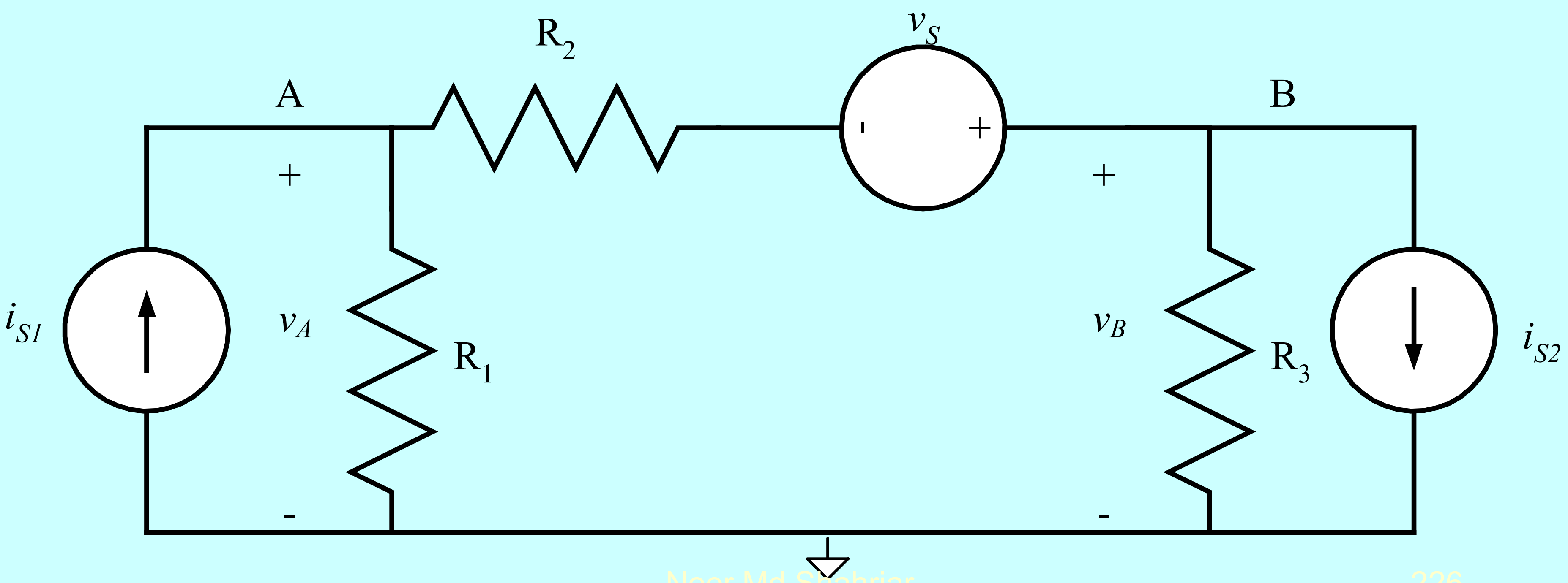
NVM – Voltage Source in Series Step 2

The second step is to define one essential node as the reference node. This is done here. The bottom node is picked since it has four connections.



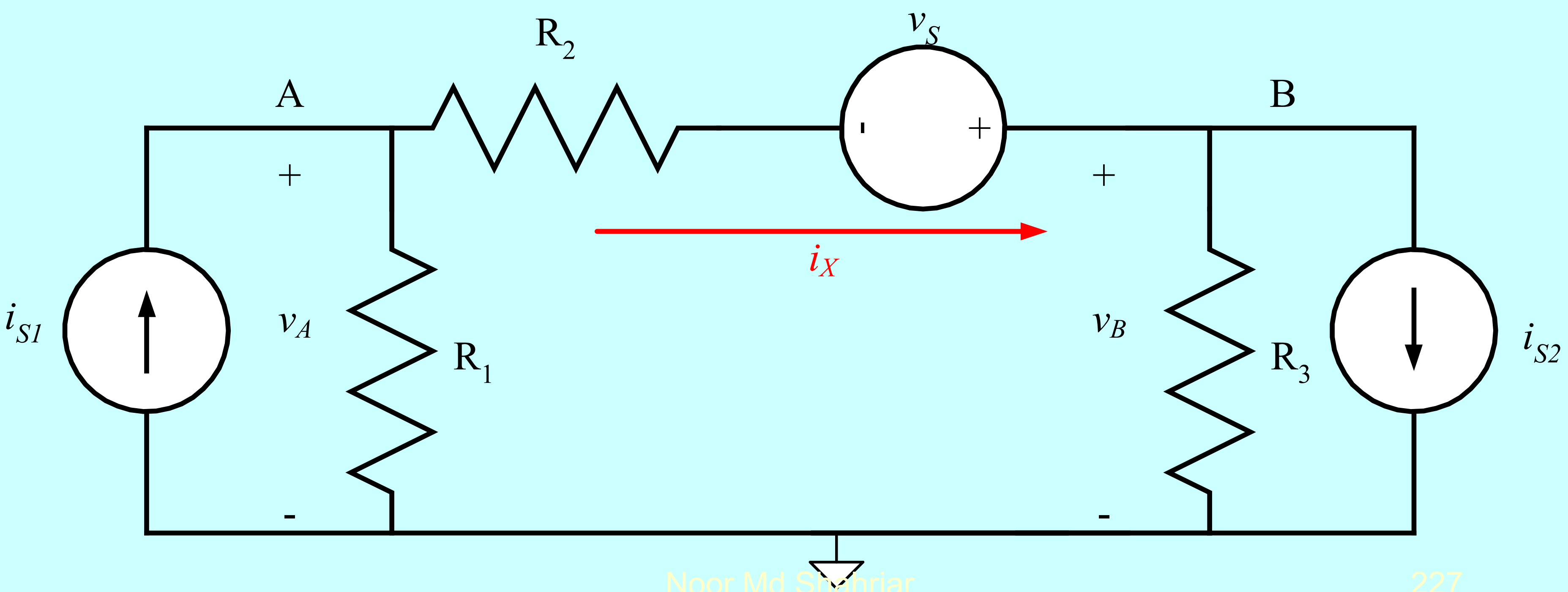
NVM – Voltage Source in Series Step 3

The third step is to define the node voltages. We have two to define.



NVM – Voltage Source in Series Step 4 – Part 1

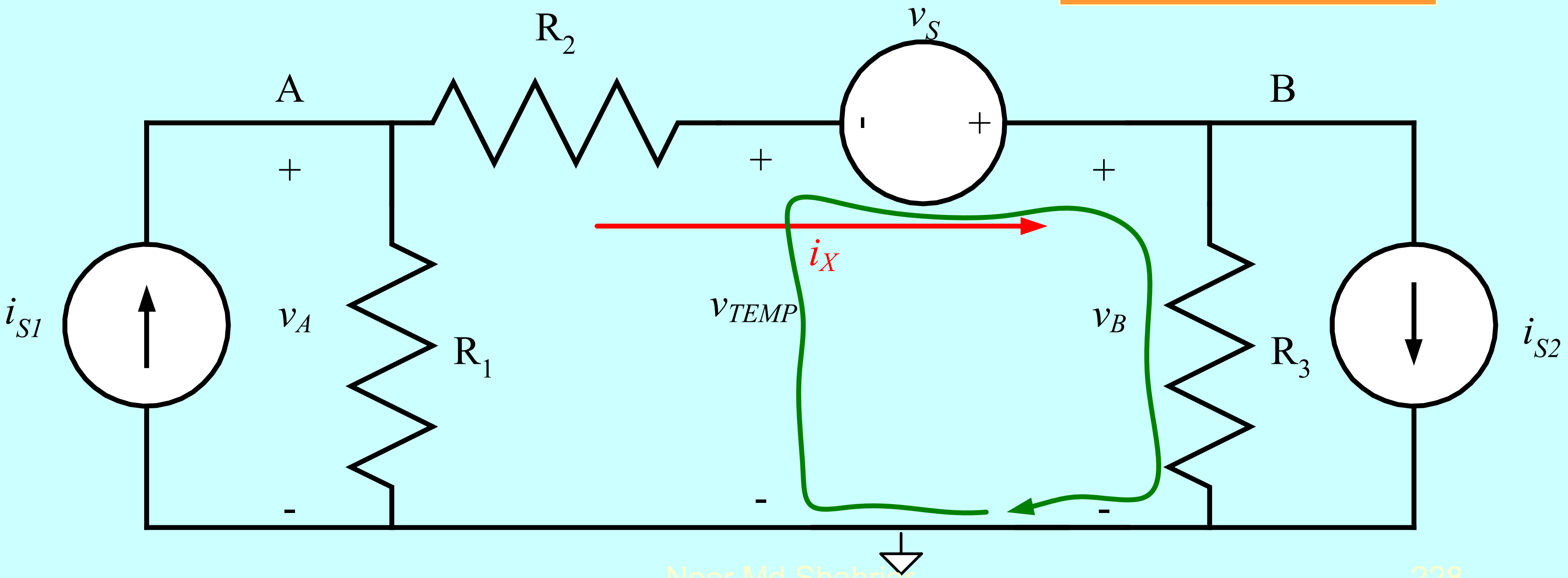
The fourth step is to write KCL equations for nodes A and B. The difficult term to write will be for the current going through the voltage source and through R_2. This current is shown with a red current arrow below.



NVM – Voltage Source in Series Step 4 – Part 2

This current shown with a red current arrow below can be expressed using the resistor R_2. The key is to be able to determine the voltage across the resistor in terms of the existing variables.
Note that the voltage v_{TEMP} shown is given by $v_{TEMP} = v_B - v_S$. We can show this by writing KVL around the loop shown.

$$v_B - v_{TEMP} - v_S = 0.$$

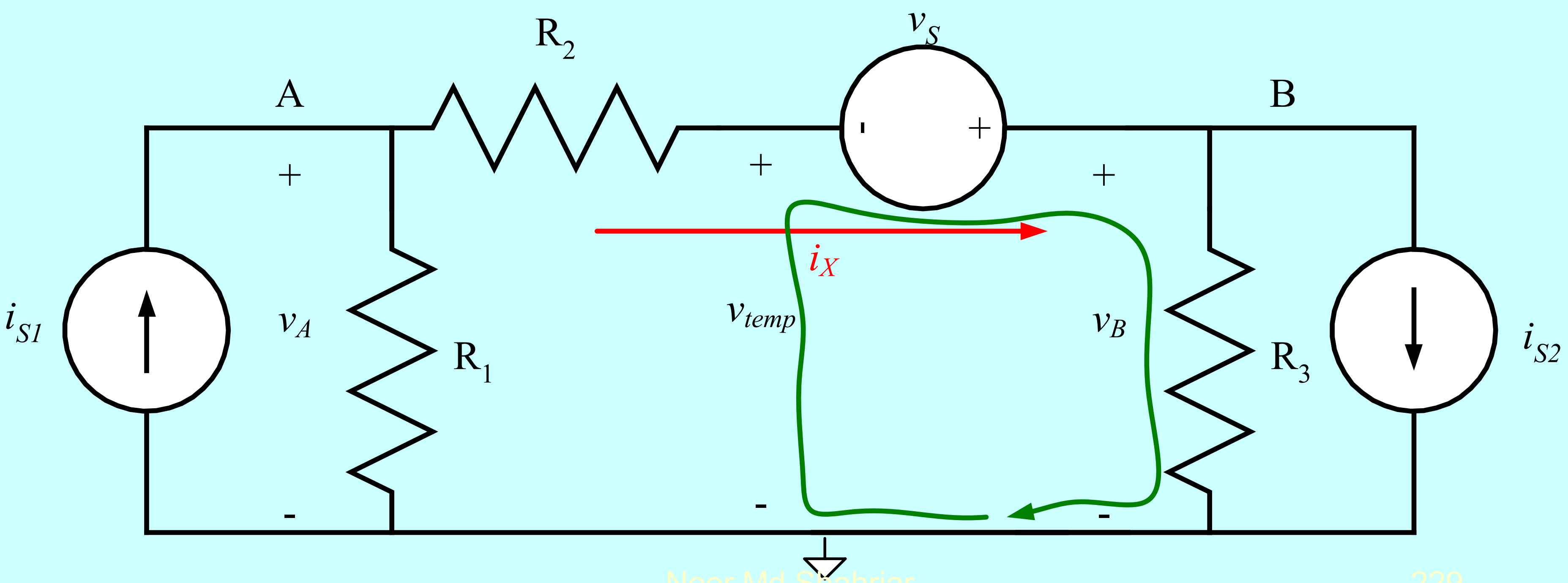


NVM – Voltage Source in Series Step 4 – Part 3

This current shown with a red current arrow below can be expressed using the voltage across the resistor R_2. The current is

$$i_X = \frac{v_A - v_{temp}}{R_2} =$$

$$i_X = \frac{v_A - (v_B - v_S)}{R_2}.$$

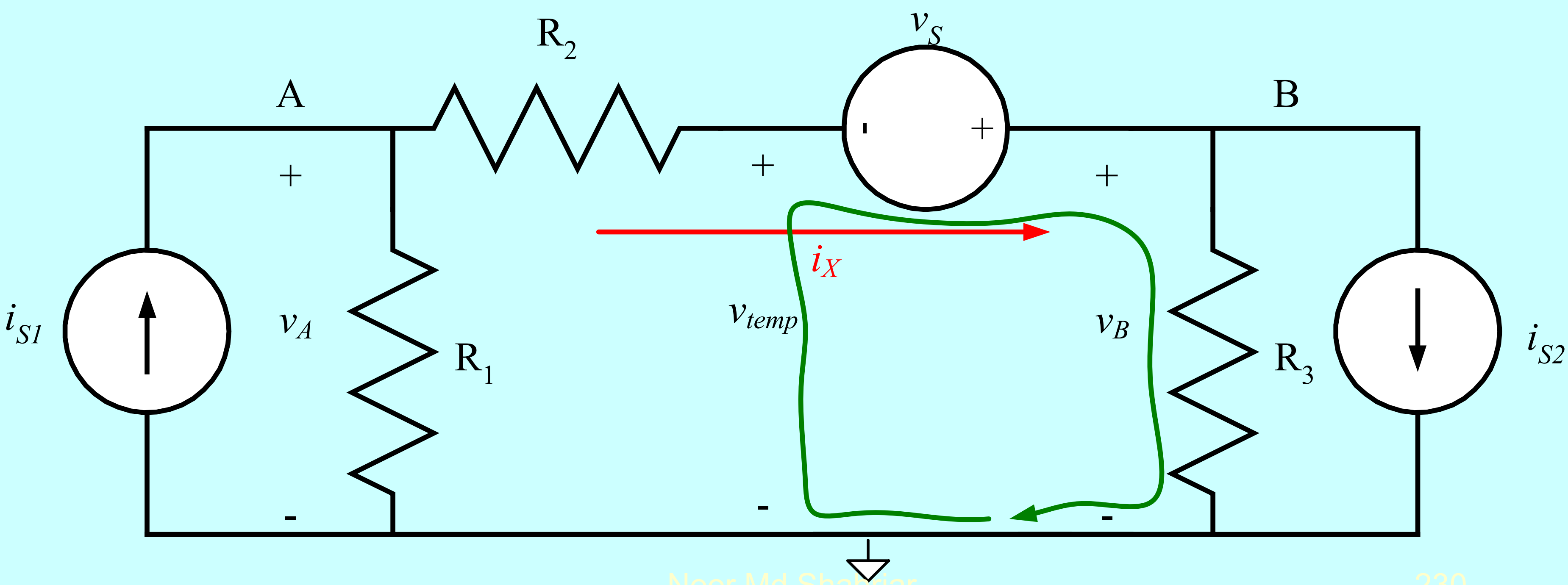


NVM – Voltage Source in Series Step 4 – Part 4

Using these results, we can write the two KCL relationships that we wanted.

A: $\frac{v_A}{R_1} - i_{S1} + \frac{v_A - (v_B - v_S)}{R_2} = 0$, and

B: $i_{S2} + \frac{v_B}{R_3} + \frac{(v_B - v_S) - v_A}{R_2} = 0.$



NVM – Voltage Source in Series Step 4 – Notes

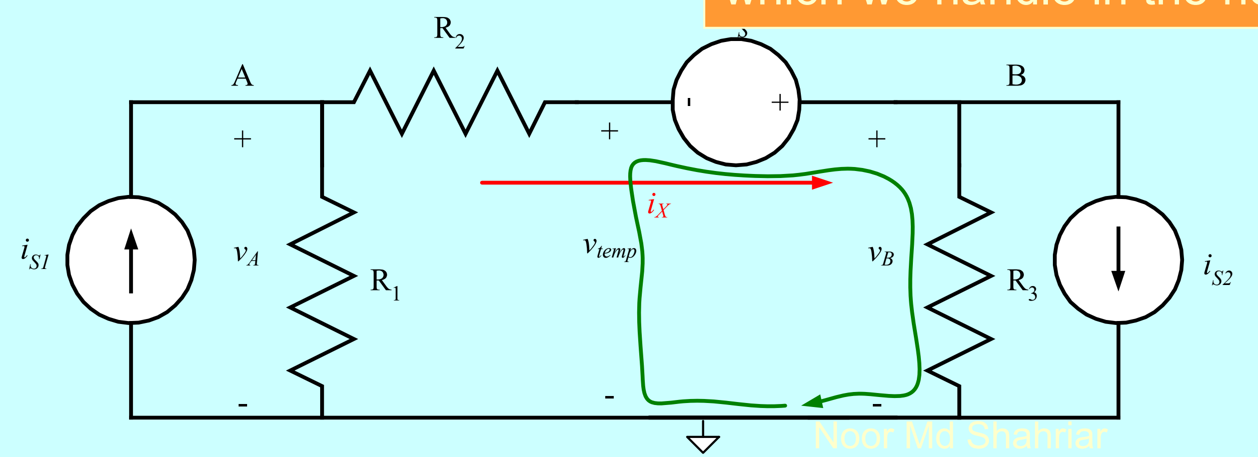
$$\text{A:}\quad \frac{v_A}{R_1} - i_{S1} + \frac{v_A - (v_B - v_S)}{R_2} = 0$$

$$\text{B:}\quad i_{S2} + \frac{v_B}{R_3} + \frac{(v_B - v_S) - v_A}{R_2} = 0$$

Note that this current is $-i_X$. This term is the current leaving node B, so the red term has a positive sign.

We have written what we wanted, two equations and two unknowns. While we could not write a current expression for the current through the voltage source directly, we were able to write one using the element in series with it.

If the element in series with the voltage source had been a current source, this would have been even easier; the current source determines the value of the current. If the element had been another voltage source, then the two voltage sources can be thought of as one voltage source between two essential nodes, which we handle in the next two cases.

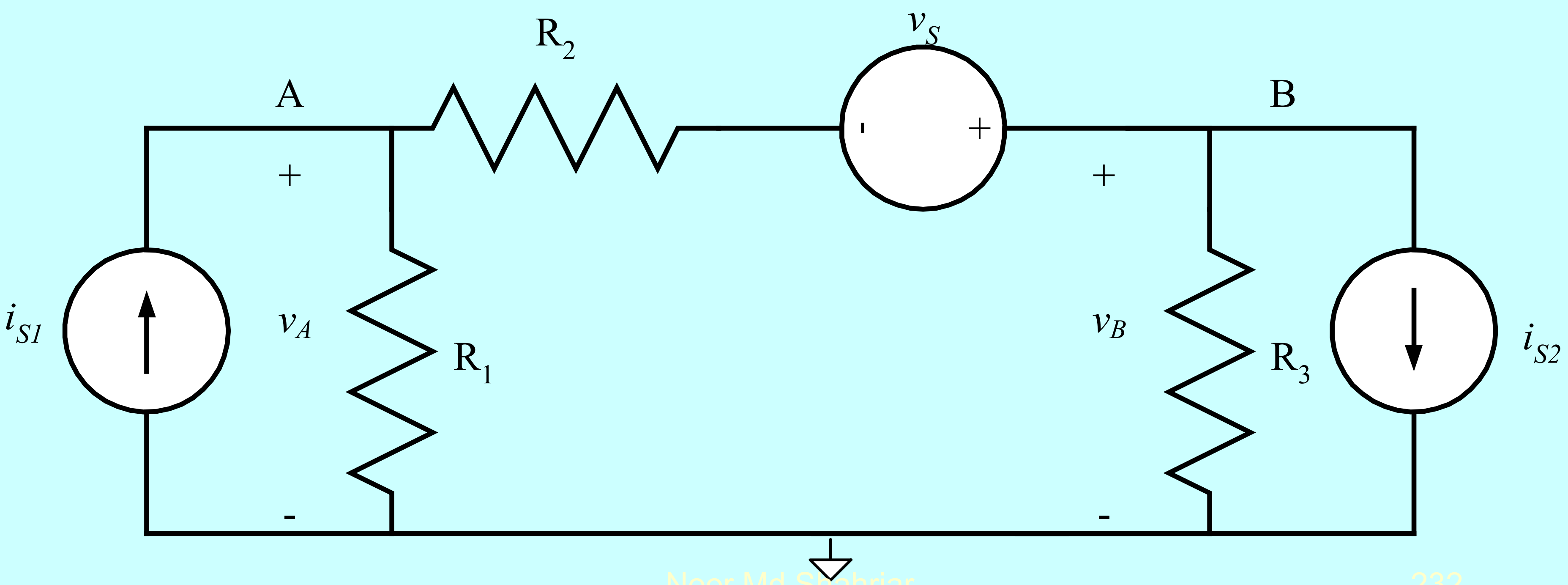


NVM – Voltage Source in Series Step 5

$$\text{A:}\quad \frac{v_A}{R_1} - i_{S1} + \frac{v_A - (v_B - v_S)}{R_2} = 0$$

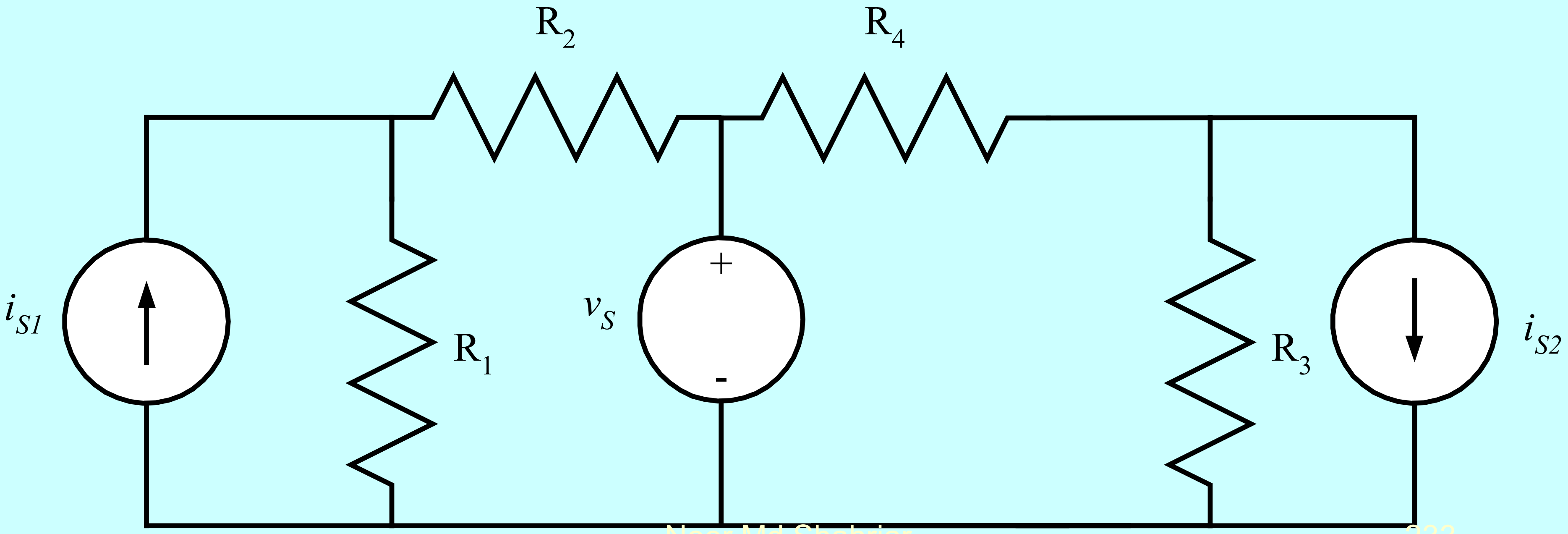
$$\text{B:}\quad i_{S2} + \frac{v_B}{R_3} + \frac{(v_B - v_S) - v_A}{R_2} = 0$$

Step 5 is not needed because there are no dependent sources in this circuit. We are done.



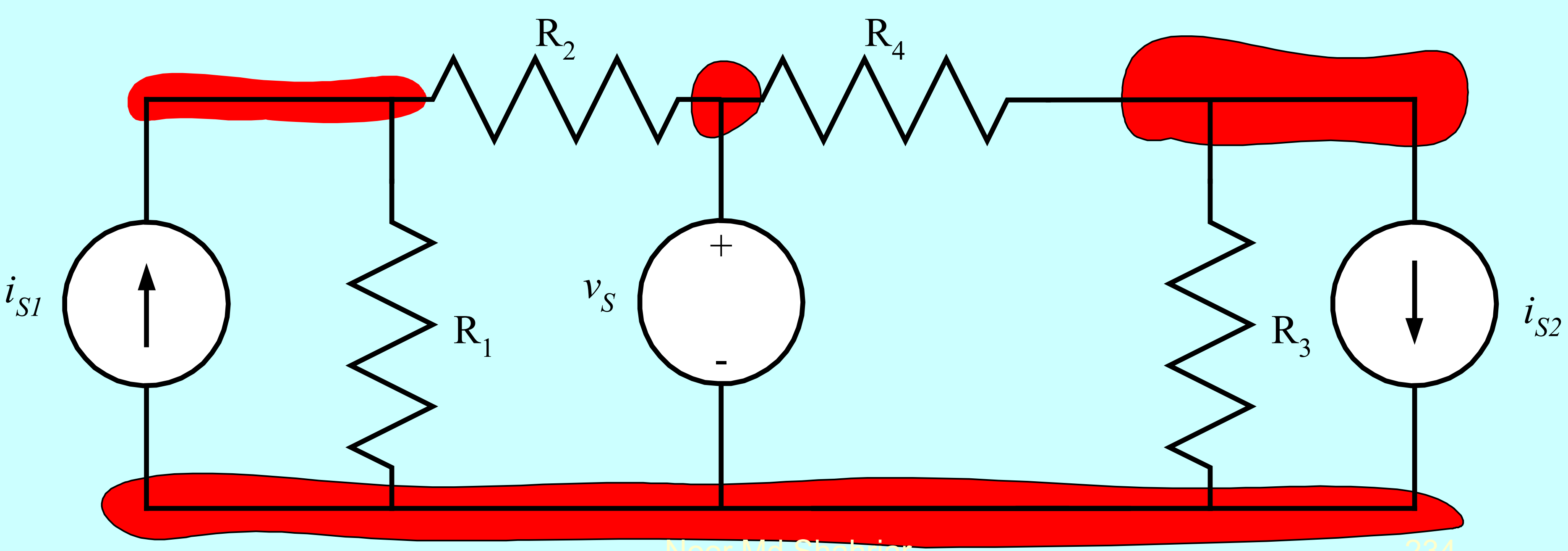
NVM – Voltage Source Between the Reference Node and Another Essential Node

Again, it seems to be best to study the NVM by doing examples. Our second example circuit is given here. We will go through the entire solution, but our emphasis will be on step 4. Note that here the voltage source v_S is between two essential nodes. We will pick one of them to be the reference node.



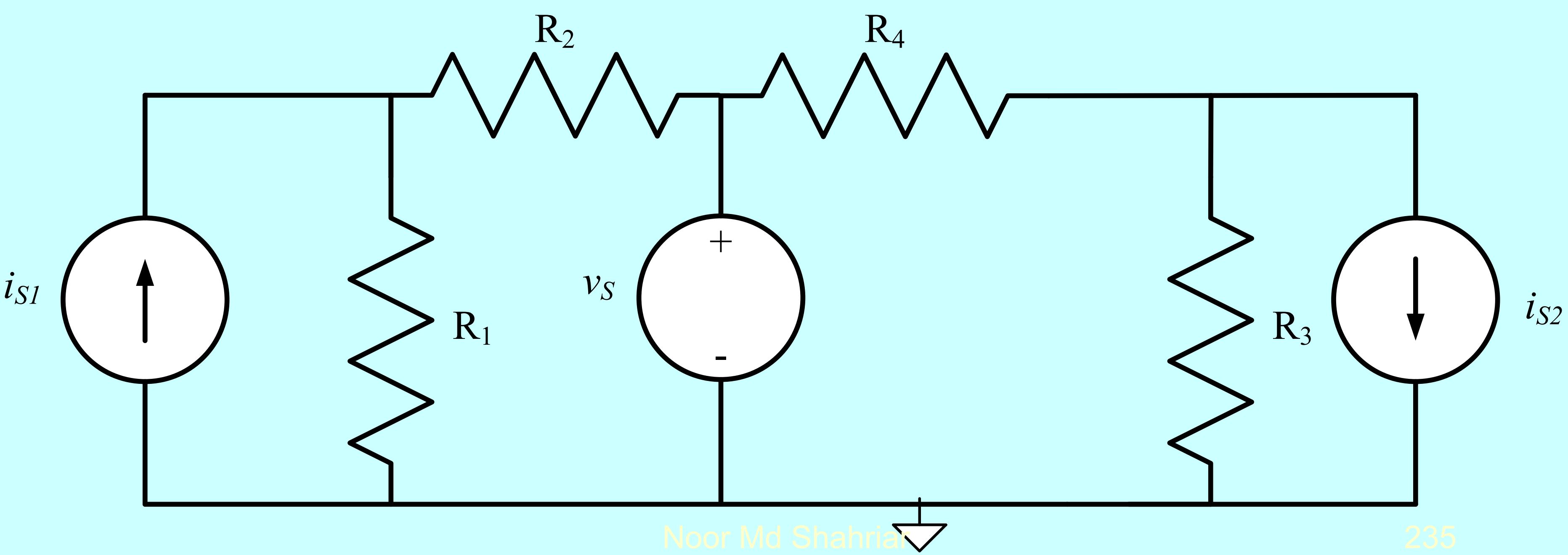
NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 1

The first step is to find the essential nodes. There are four of them here. They are shown in red.



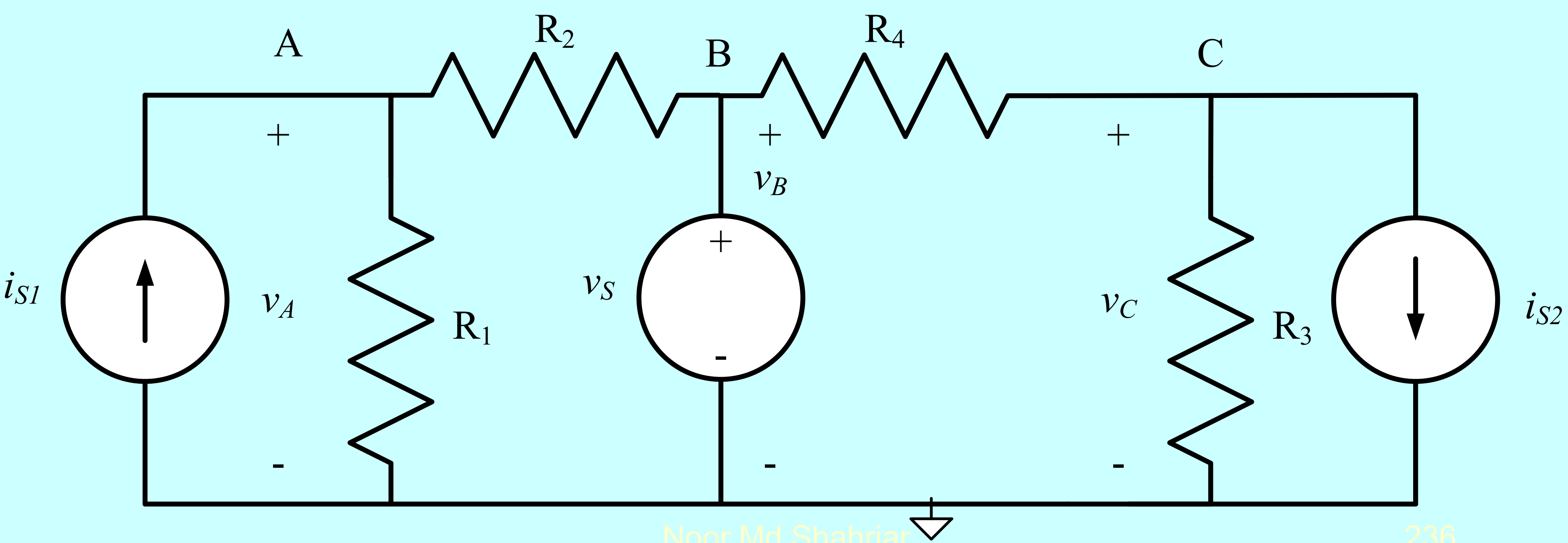
NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 2

The second step is to define the reference node. We will choose the bottom node again, because again it has the most connections.



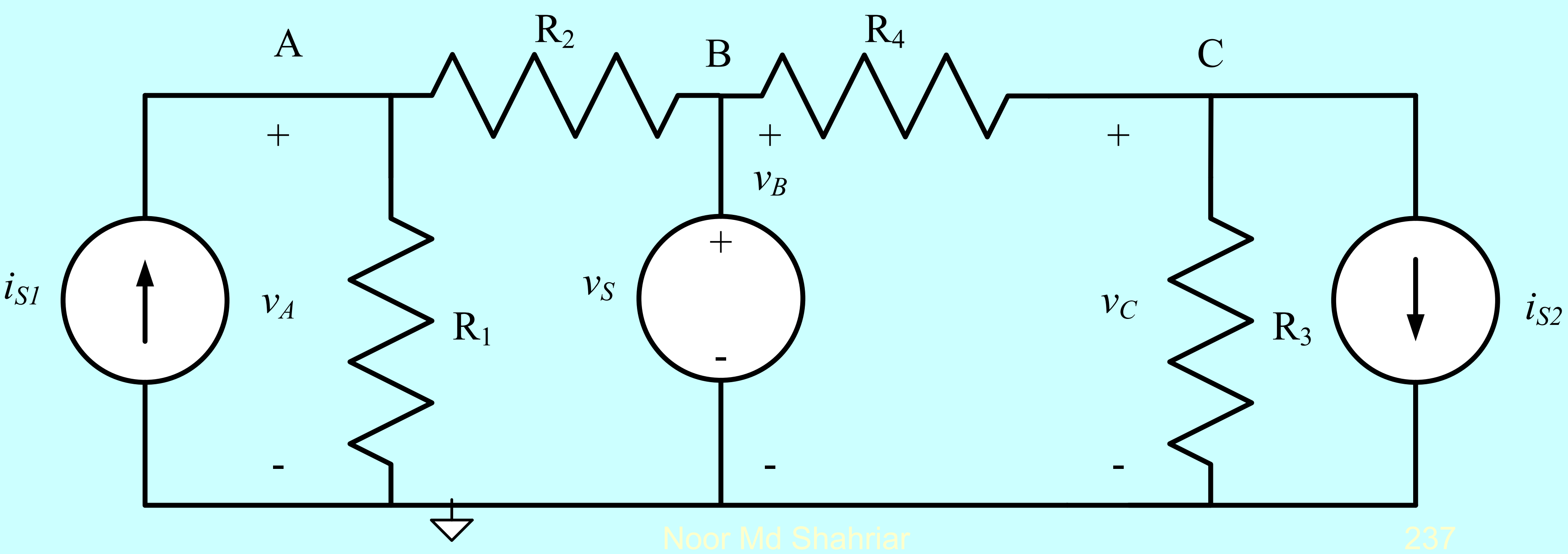
NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 3

The third step is to define the node voltages, and label them. We will also name the nodes at the same time.



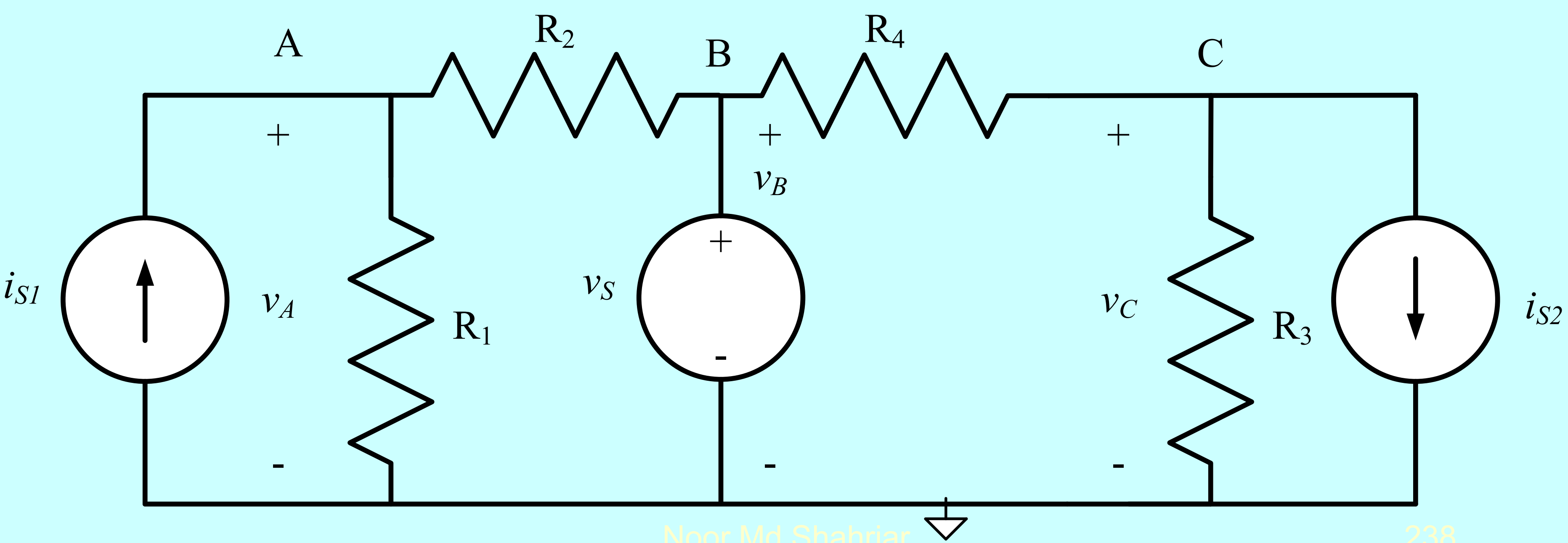
NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 4 – Part 1

The fourth step is to write KCL for nodes A, B, and C. We can write KCL equations for nodes A and C using the techniques we have already, but for B we will get into trouble since the current through the voltage source is not known, and cannot be easily given in terms of the node voltages.



NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 4 – Part 2

We can write KCL equations for nodes A and C using the techniques we had already, but for B we will get into trouble. However, we do know something useful; the voltage source determines the node voltage v_B. This can **be** our third equation.



NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 4 – Part 3

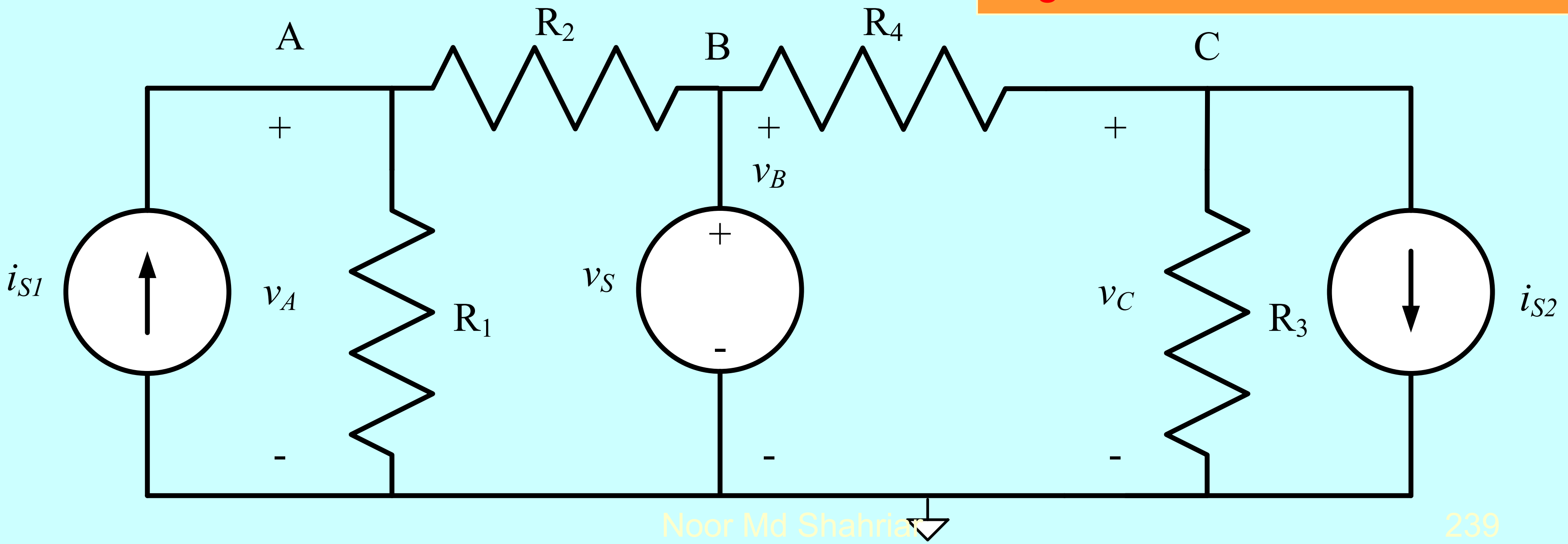
We can write the following equations:

$$\text{A:}\quad \frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0,$$

$$\text{B:}\quad v_B = v_S, \text{ and}$$

$$\text{C:}\quad i_{S2} + \frac{v_C}{R_3} + \frac{v_C - v_B}{R_4} = 0.$$

This equation indicates that the node-voltage v_B is equal to the voltage source. Take care about the signs in this equation. There is no minus sign here, because the polarities of v_S and v_B are aligned.



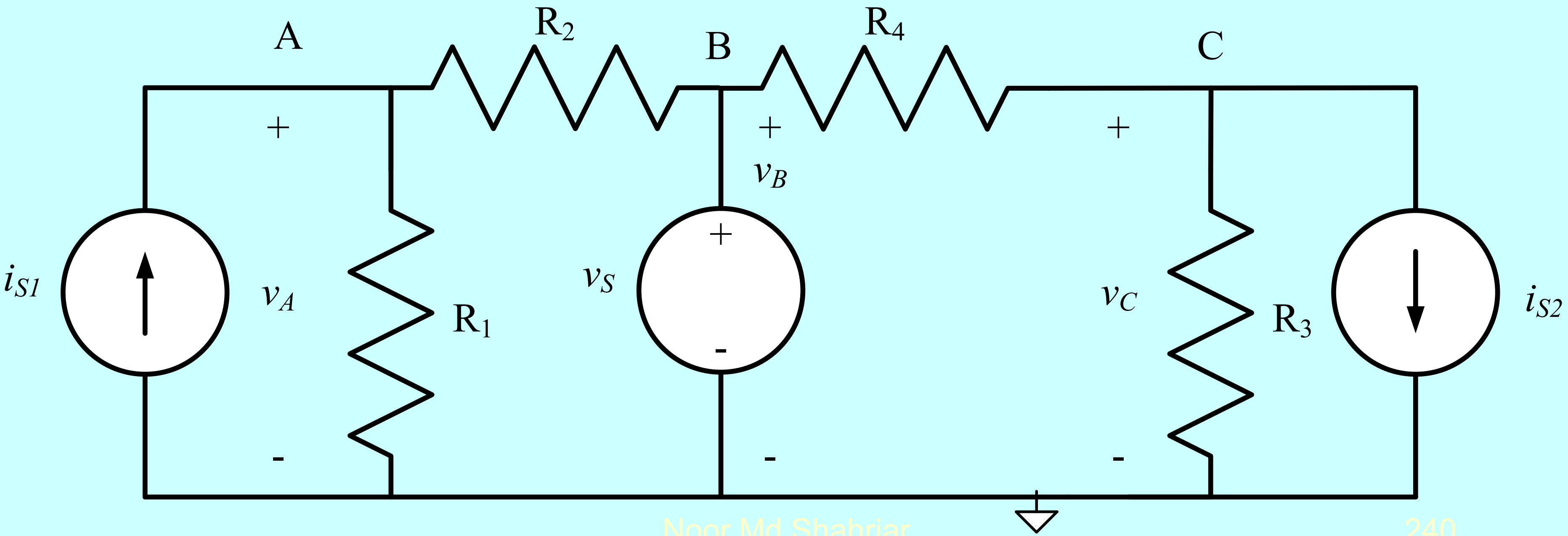
NVM – Voltage Source Between the Reference Node and Another Essential Node – Step 5

A: $\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$

B: $v_B = v_S$

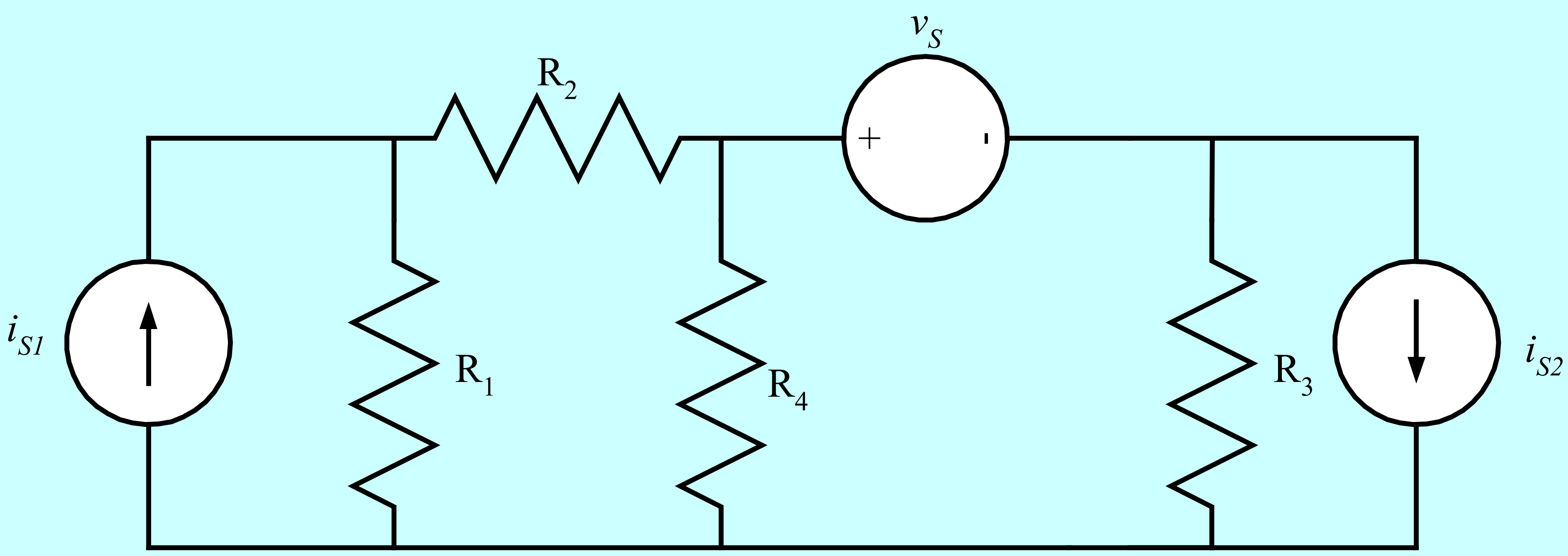
C: $i_{S2} + \frac{v_C}{R_3} + \frac{v_C - v_B}{R_4} = 0$

There are no dependent sources here, so we are done.



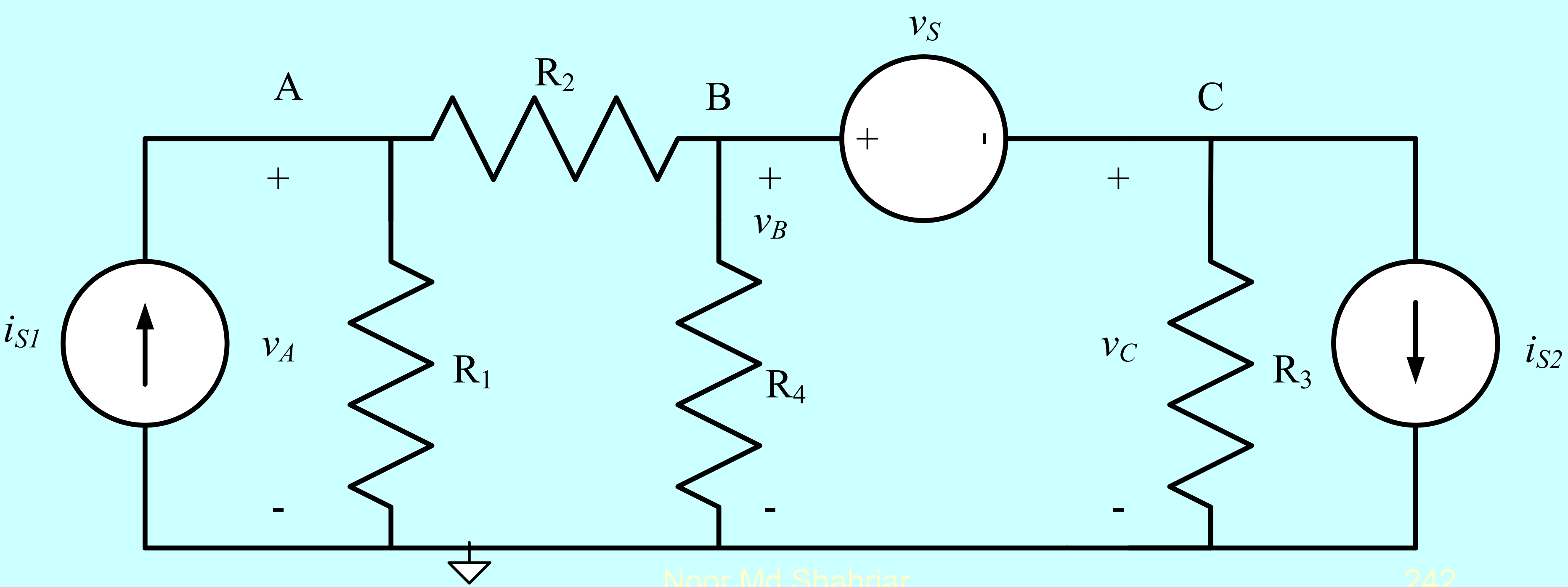
NVM – Voltage Source Between Two Non-Reference Essential Nodes

Again, it seems to be best to study the NVM by doing examples. Our third example circuit is given here. We will go through the entire solution, but our emphasis will be on step 4. Note that here the voltage source v_S is between two essential nodes. We will pick yet another essential node to be the reference node.



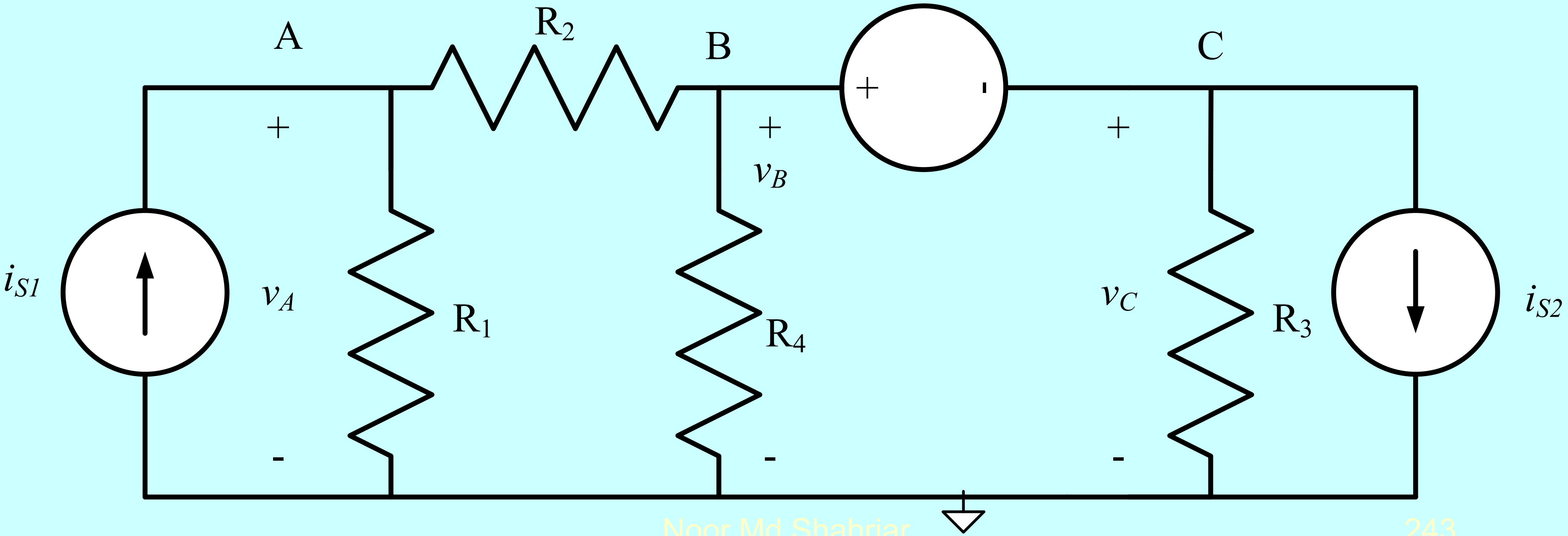
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Steps 1, 2, and 3

Since we have done similar circuits already, we have completed steps 1, 2, and 3 in this single slide. We identified four essential nodes, and picked the bottom node as reference, since it has five connections. We named the other three nodes, and labeled the node-voltages for each.



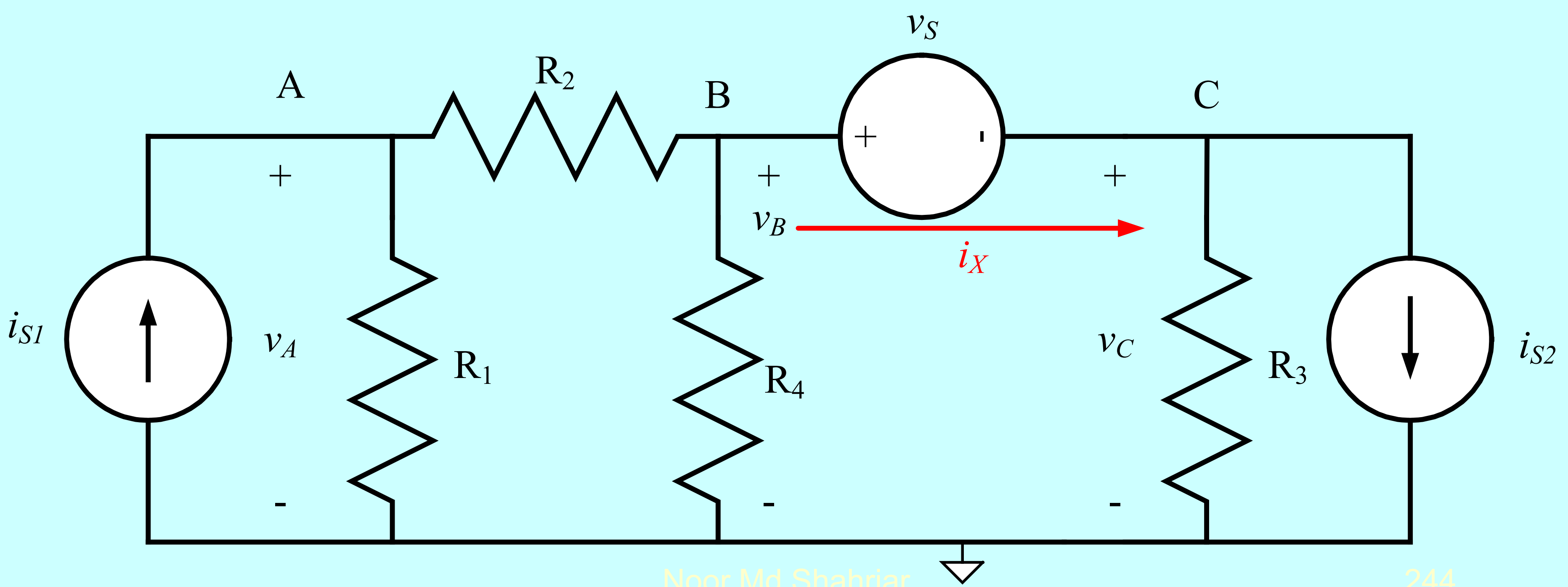
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 1

Now we want to write KCL equations for the three nodes, A, B, and C. However, we will have difficulties writing the equations for nodes B and C, because the voltage source can have any current through it. In addition, we note that v_S is not equal to v_B, nor is it equal to v_C. Thus, we cannot use the nice, simple KVL that we used when we had a voltage source between the reference node and another essential node.



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 2

We are going to take a very deliberate approach to this case, since many students find it difficult. To start, we will assume that we were willing to introduce an additional variable. (We will later show that we don't have to, but this is just to explain the technique.) We define the current through the voltage source to be i_X.

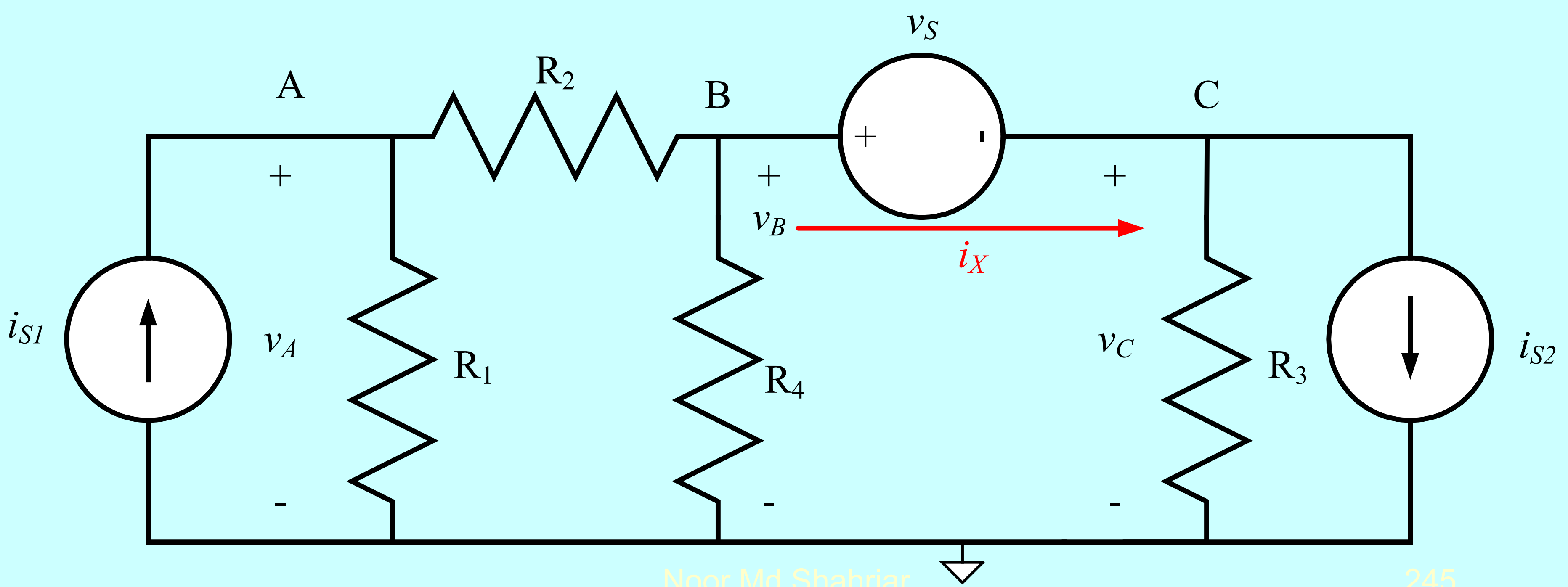


NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 3

Now, we can write KCL equations for nodes B and C, using i_X.

$$\text{B:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_X = 0, \text{ and}$$

$$\text{C:}\quad -i_X + i_{S2} + \frac{v_C}{R_3} = 0.$$



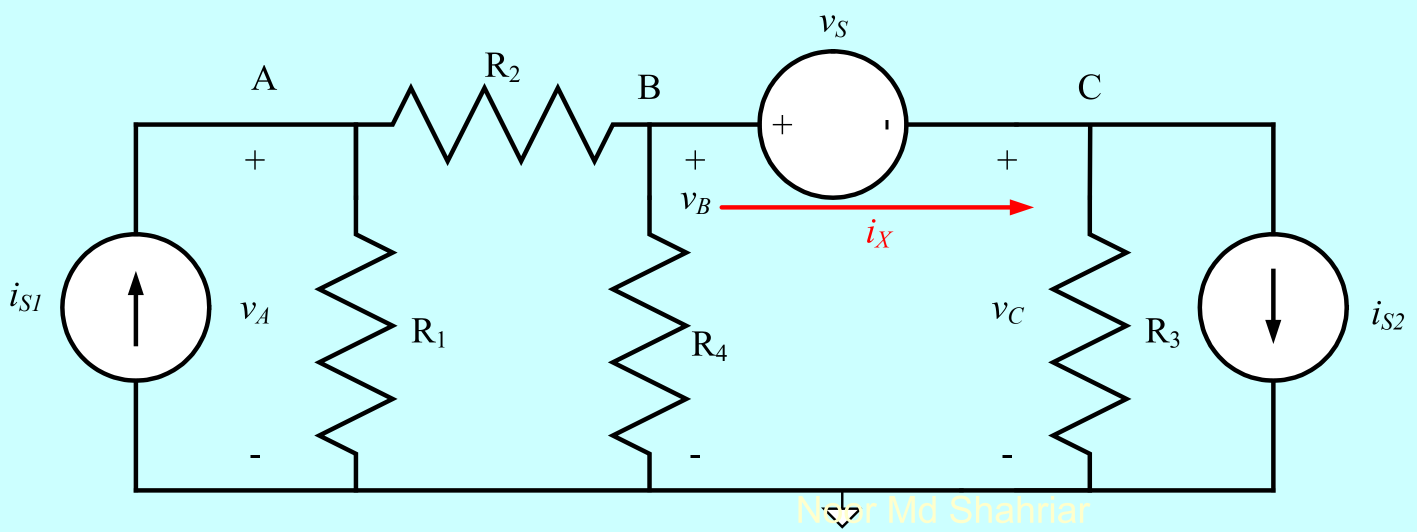
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 4

Now, remember that we did not want to use the variable i_X. If we examine the equations that we have just written, we note that we can eliminate i_X by adding the two equations together. We add the B equation to the C equation, and get:

$$\text{B:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_X = 0$$

$$\text{C:}\quad -i_X + i_{S2} + \frac{v_C}{R_3} = 0$$

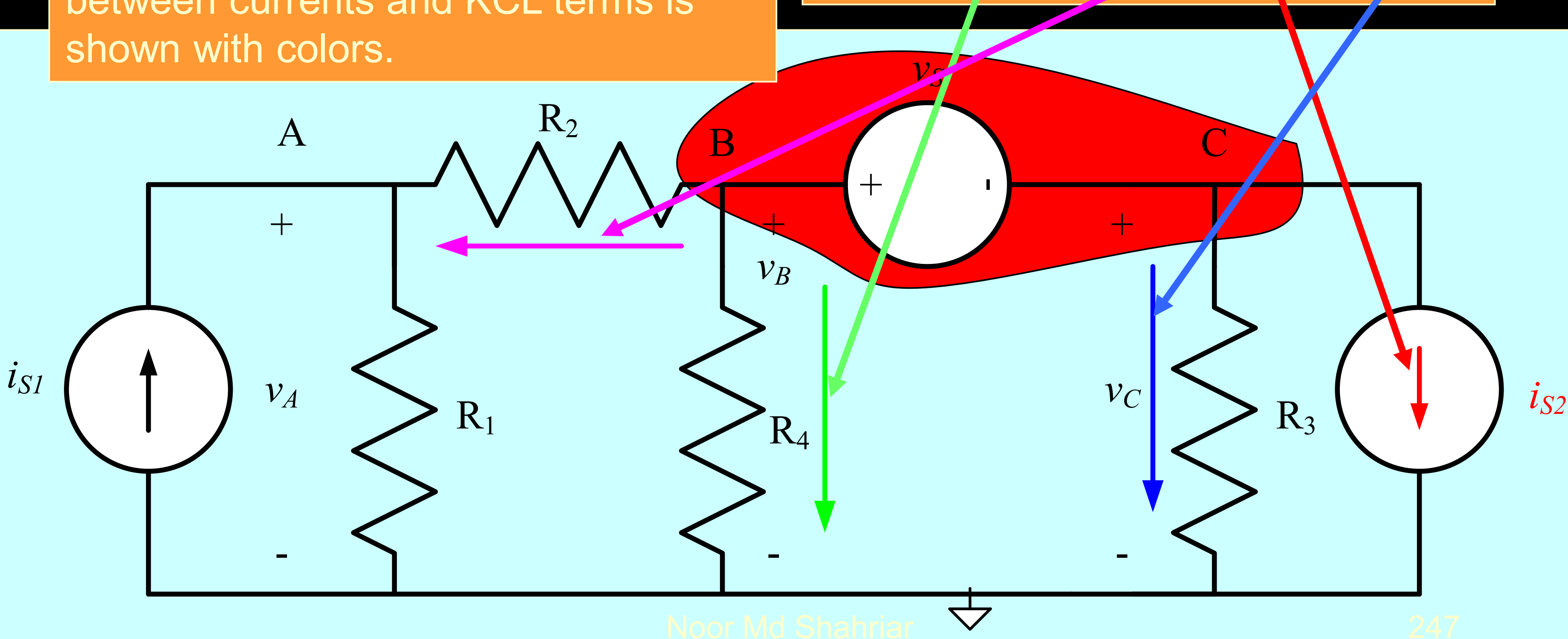
$$\text{B+C:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0.$$



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 5

Next, we examine this new equation that we have titled B+C. If we look at the circuit, this is just KCL applied to a closed surface that surrounds the voltage source. The correspondence between currents and KCL terms is shown with colors.

$$\text{B+C:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$$



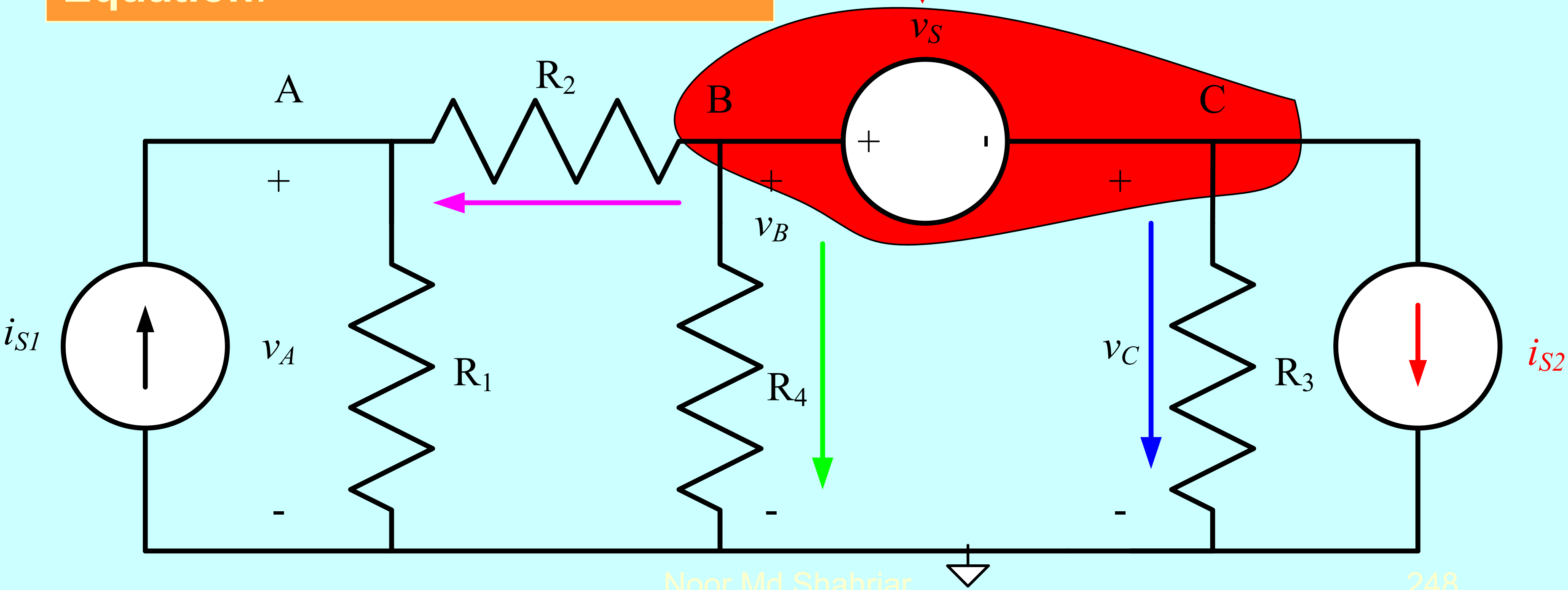
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 6

The large closed surface that includes the voltage source is called a **Supernode**. We will call the KCL equation that we write for this closed surface a **Supernode Equation**.

$$\text{B+C:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$$

Supernode

Supernode Equation



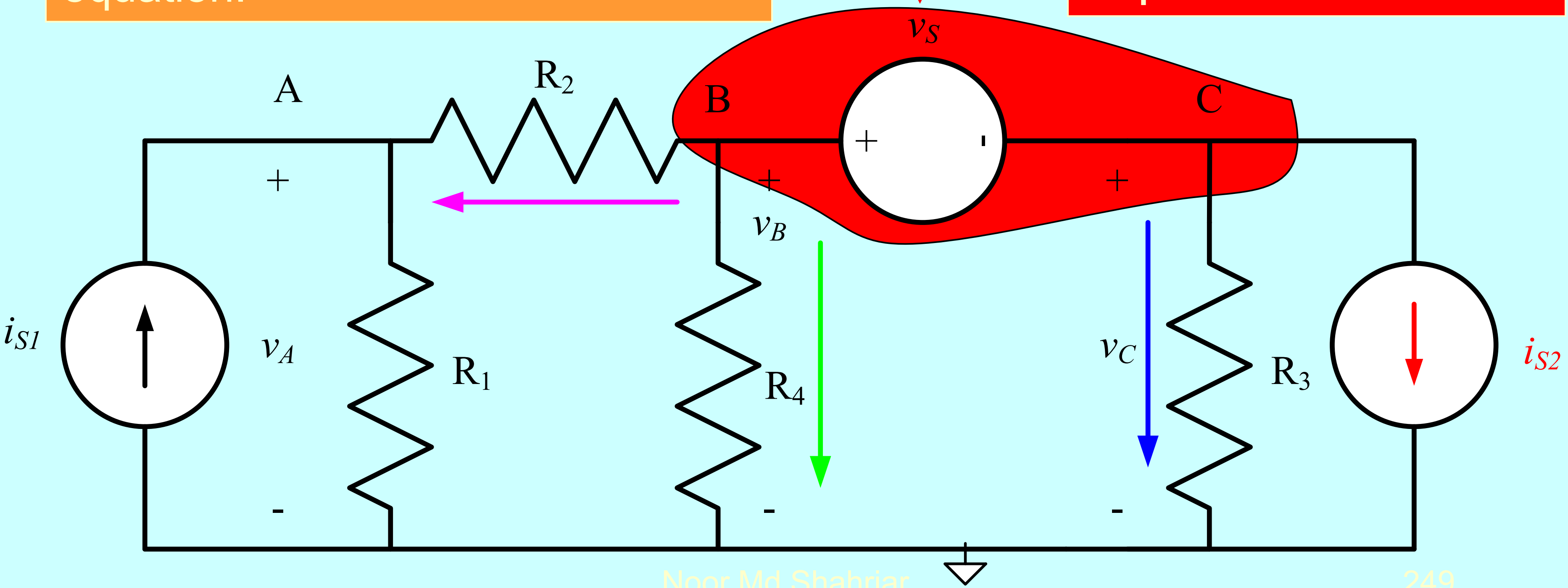
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 7

The Supernode Equation is fine, but it is not enough. With the equation for node A, we still only have two equations, and three unknowns. We need one more equation.

$$\text{B+C:} \quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$$

Supernode

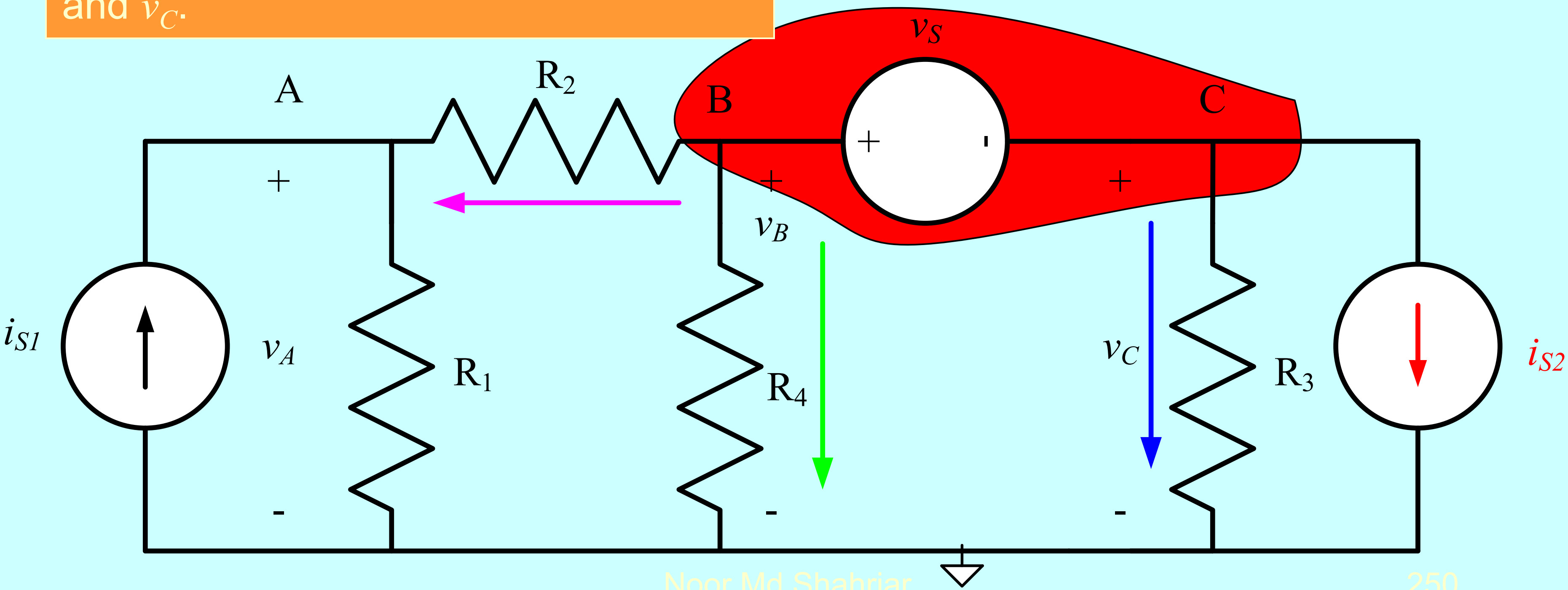
Supernode Equation



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 8

We need one more equation. We now note that we have not used the value of the voltage source, which we expect to influence the solution somehow. Note that the voltage source determines the difference between v_B and v_C.

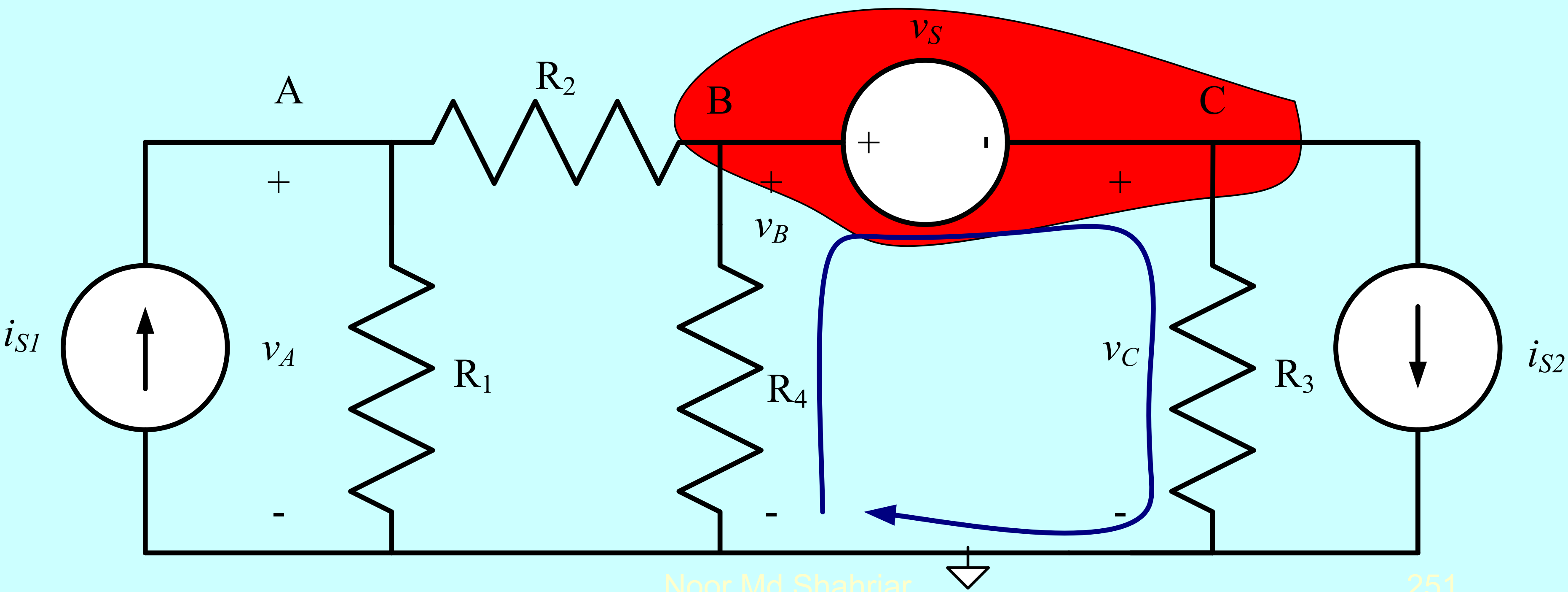
$$\text{B+C:} \quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$$



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 9

The voltage source determines the difference between v_B and v_C. We can use this to write the third equation we need. Using KVL around the dark blue loop in the circuit below, we write the following equation.

$$\text{B+C: } v_B - v_C = v_S$$



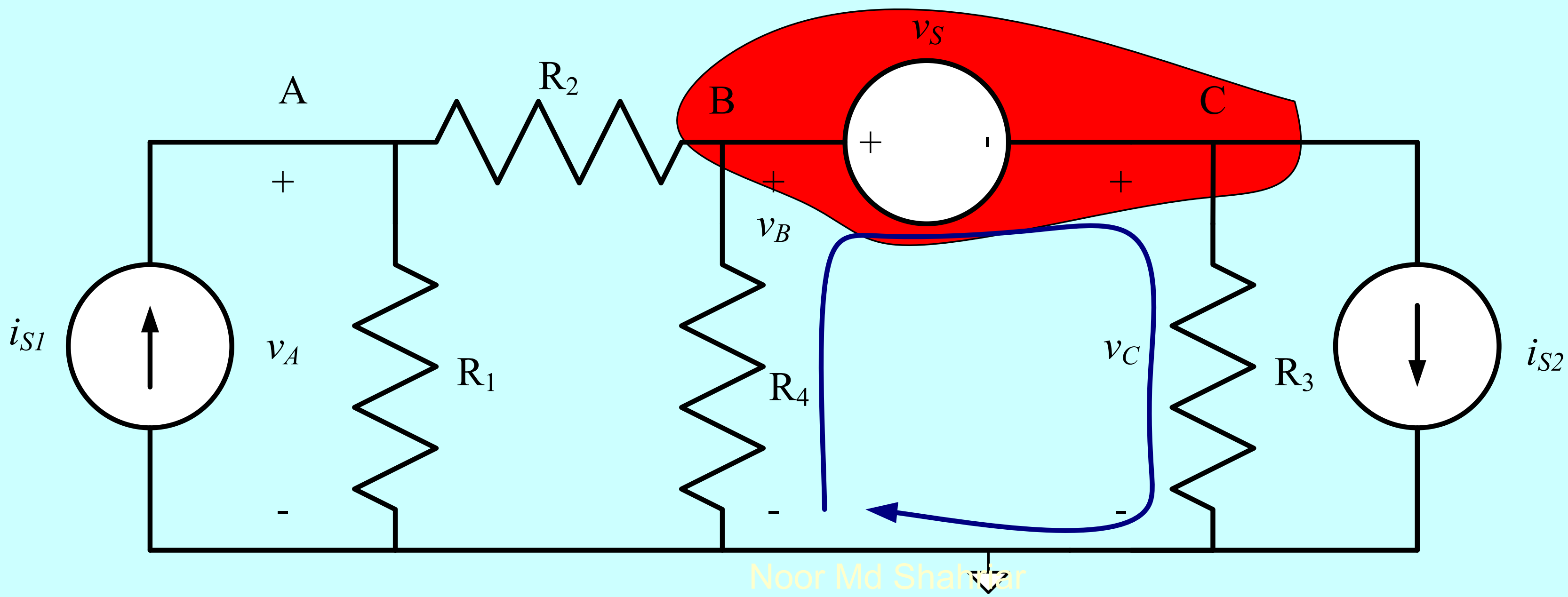
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 10

To complete the set of equations, we write the KCL equation for node A. That gives us three equations in three unknowns.

$$\text{A:}\quad \frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0,$$

$$\text{B+C:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0, \text{ and}$$

$$\text{B+C:}\quad v_B - v_C = v_S.$$



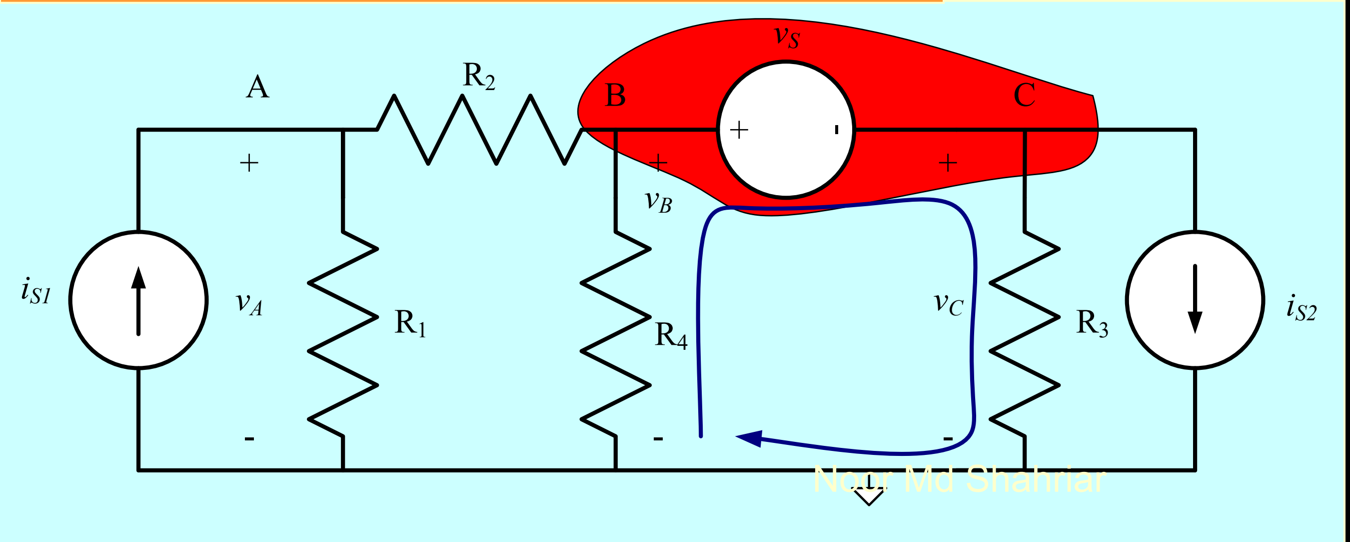
NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 11

To summarize our approach then, when we have a voltage source between two non-reference essential nodes, we:
- write one equation applying KCL to a supernode around the voltage source, and
- write a KVL using the voltage source to relate the two node voltages.

$$\text{A:}\quad \frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

$$\text{B+C:}\quad \frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$$

$$\text{B+C:}\quad v_B - v_C = v_S$$



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 4 – Part 12

We write:

- one equation applying KCL to a supernode around the voltage source, and
- one KVL using the voltage source to relate the two node voltages.

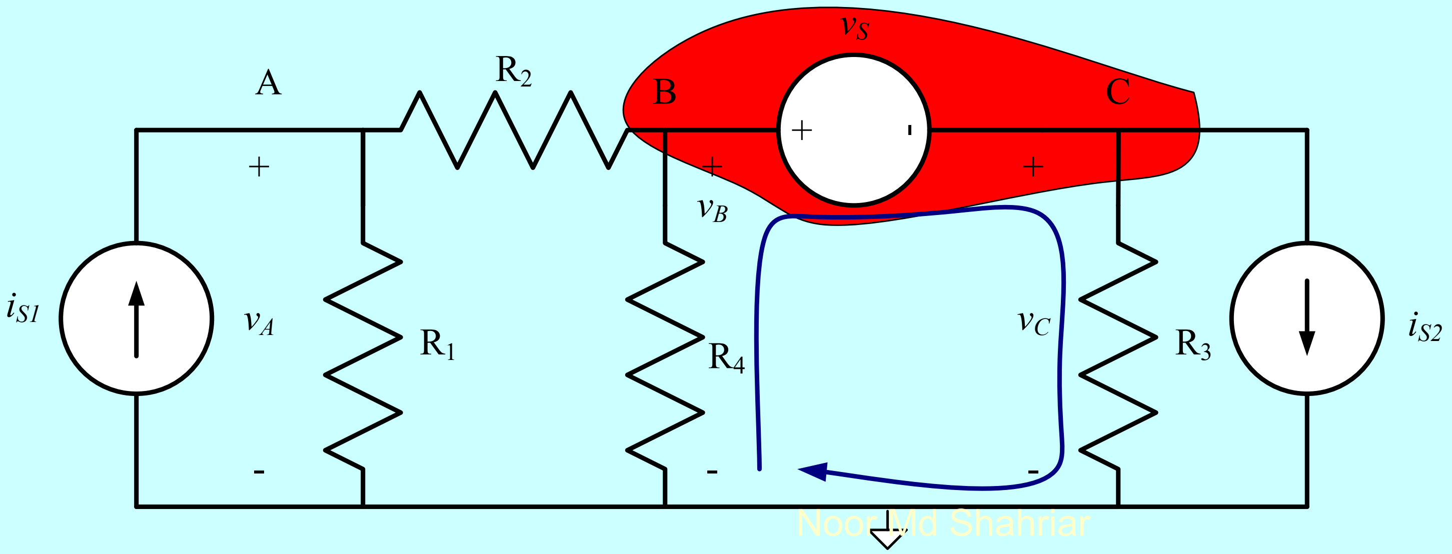
Supernode Equation

Constraint Equation

A: $\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$

B+C: $\frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$

B+C: $v_B - v_C = v_S$



NVM – Voltage Source Between Two Non-Reference Essential Nodes – Step 5

We write:
• one equation applying KCL to a supernode around the voltage source, and
• one KVL using the voltage source to relate the two node voltages.

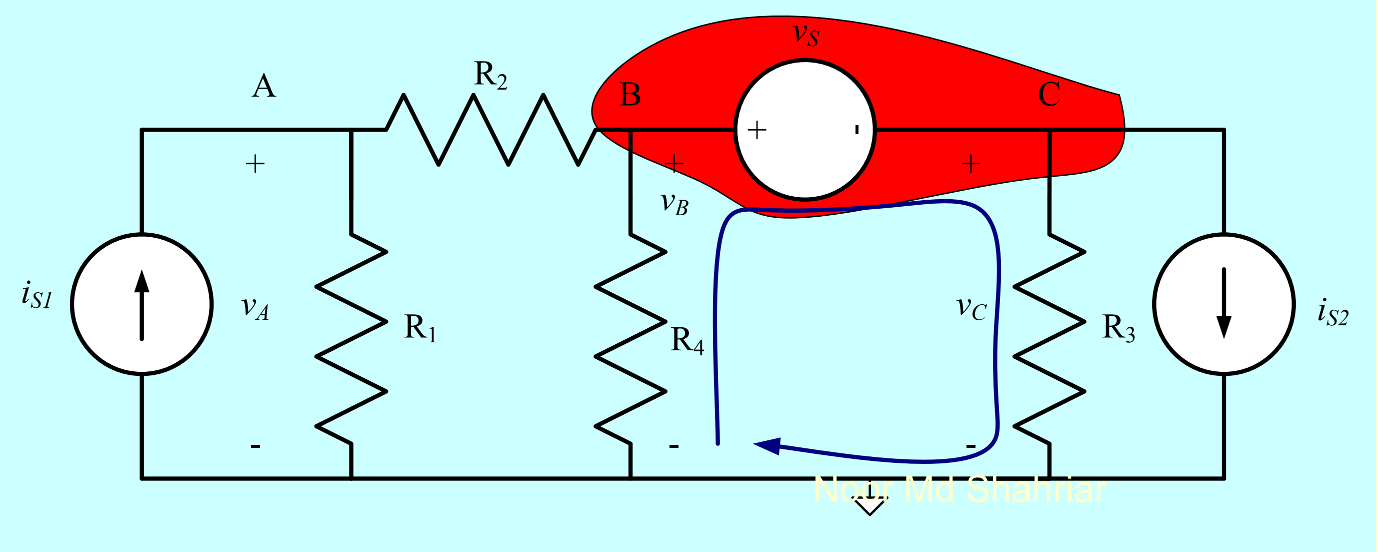
Supernode Equation

Constraint Equation

A: $\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$

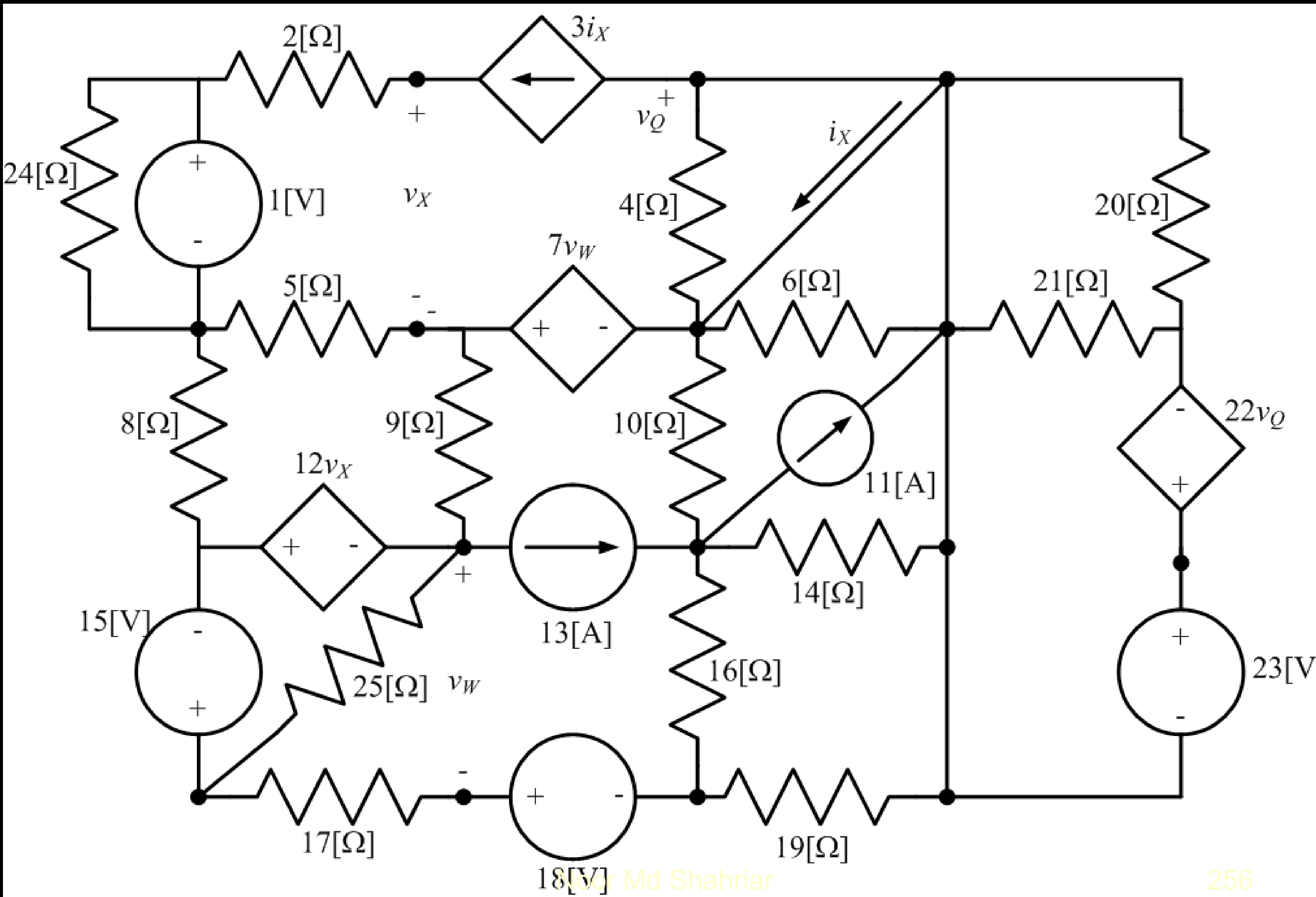
B+C: $\frac{v_B}{R_4} + \frac{v_B - v_A}{R_2} + i_{S2} + \frac{v_C}{R_3} = 0$

B+C: $v_B - v_C = v_S$



Step 5 is not needed in this problem since we do not have any dependent sources.

Example Problem: Use the node-voltage method to write a set of equations that could be used to solve the circuit below. Do not attempt to simplify the circuit. Do not attempt to solve the equations.

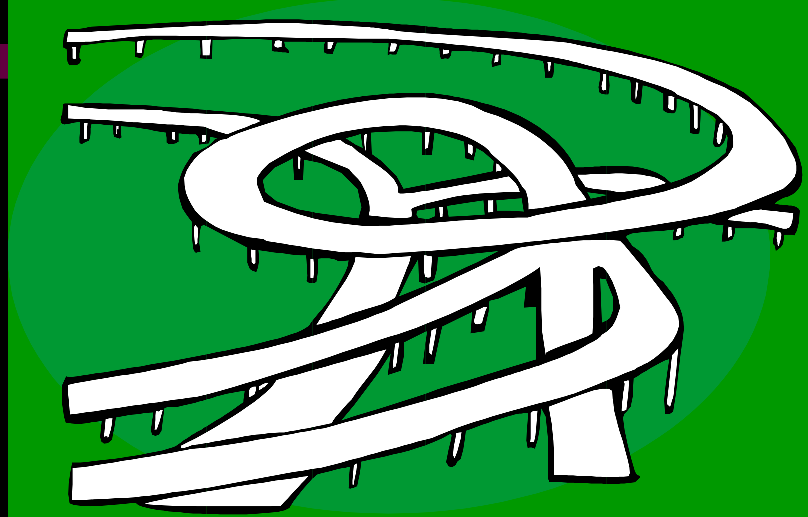


Week -10

Page- (258-335)

Some Basic Definitions

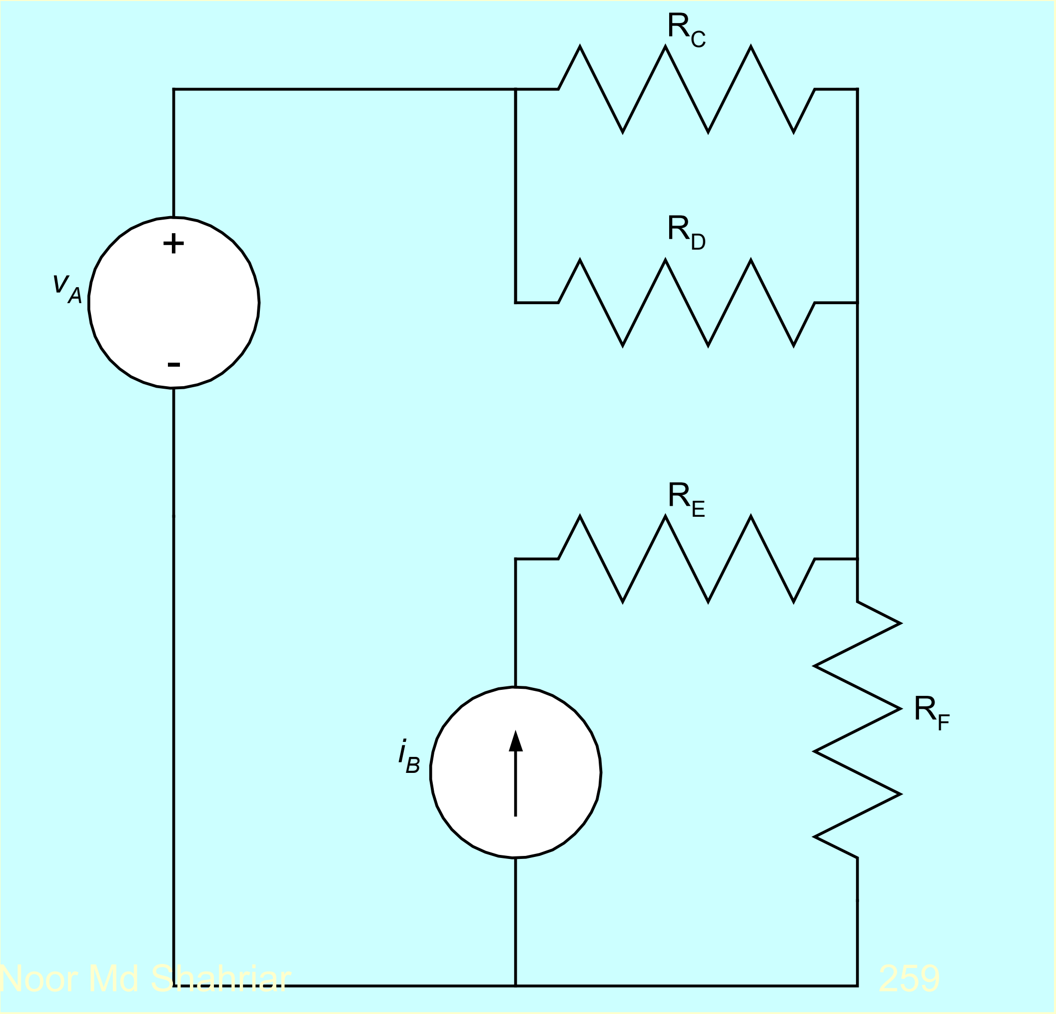
- **Closed Path** – a closed loop which follows components and wires. This definition effectively defines paths as following components and wires.
- **Mesh** – a closed path that does not enclose any other closed paths
- **Planar Circuit** – a circuit that can be drawn in a plane, that is, without wires that cross without touching



Different textbooks use slightly different definitions for these terms. If the difference is confusing, stick with your book. The key is to be able to find meshes, and most students find this to be fairly easy with practice.

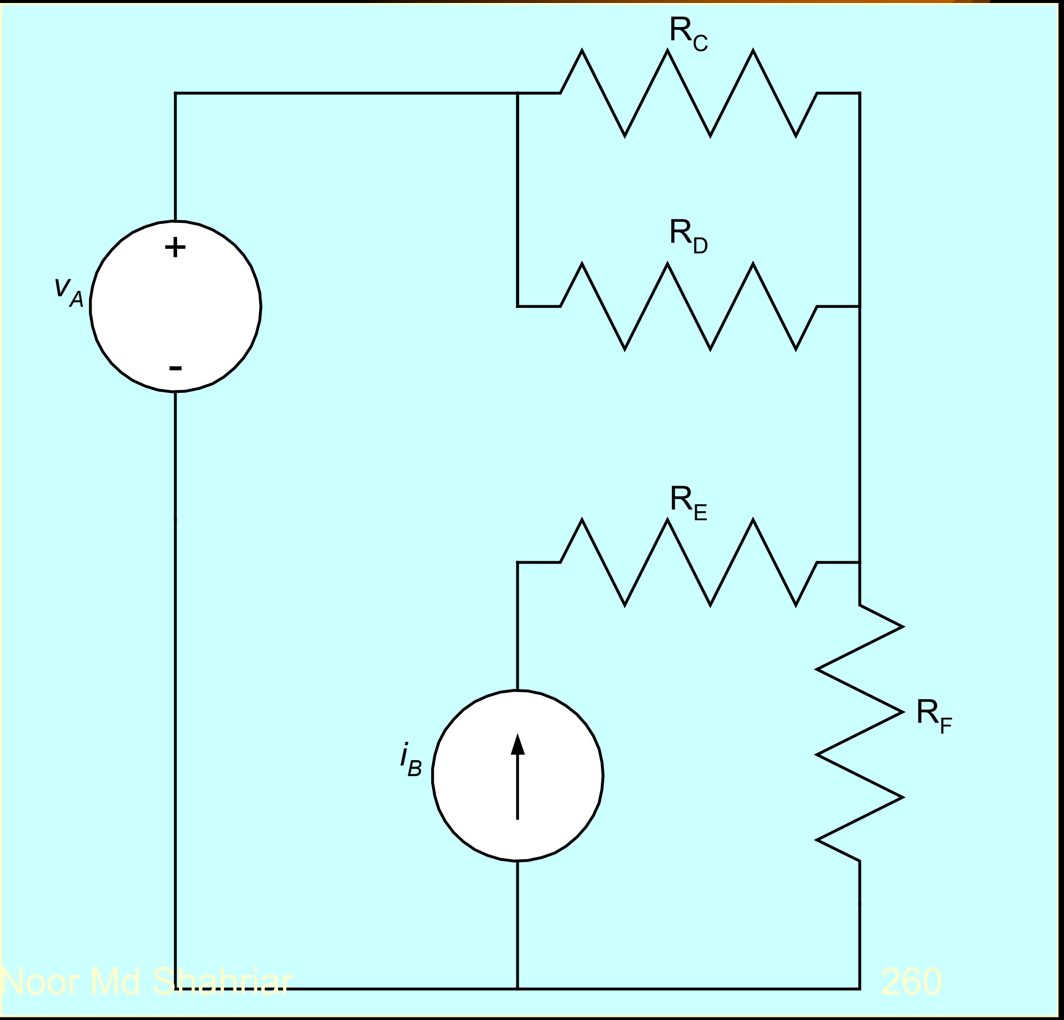
Some Review – Closed Loops

- A **closed loop** can be defined in this way: Start at any node and go in any direction and end up where you start. This is a closed loop.
- Note that this loop does not have to follow components and wires. It can jump across open space. Often we will follow components, but we will also have situations where we need to jump between nodes that have no connections.



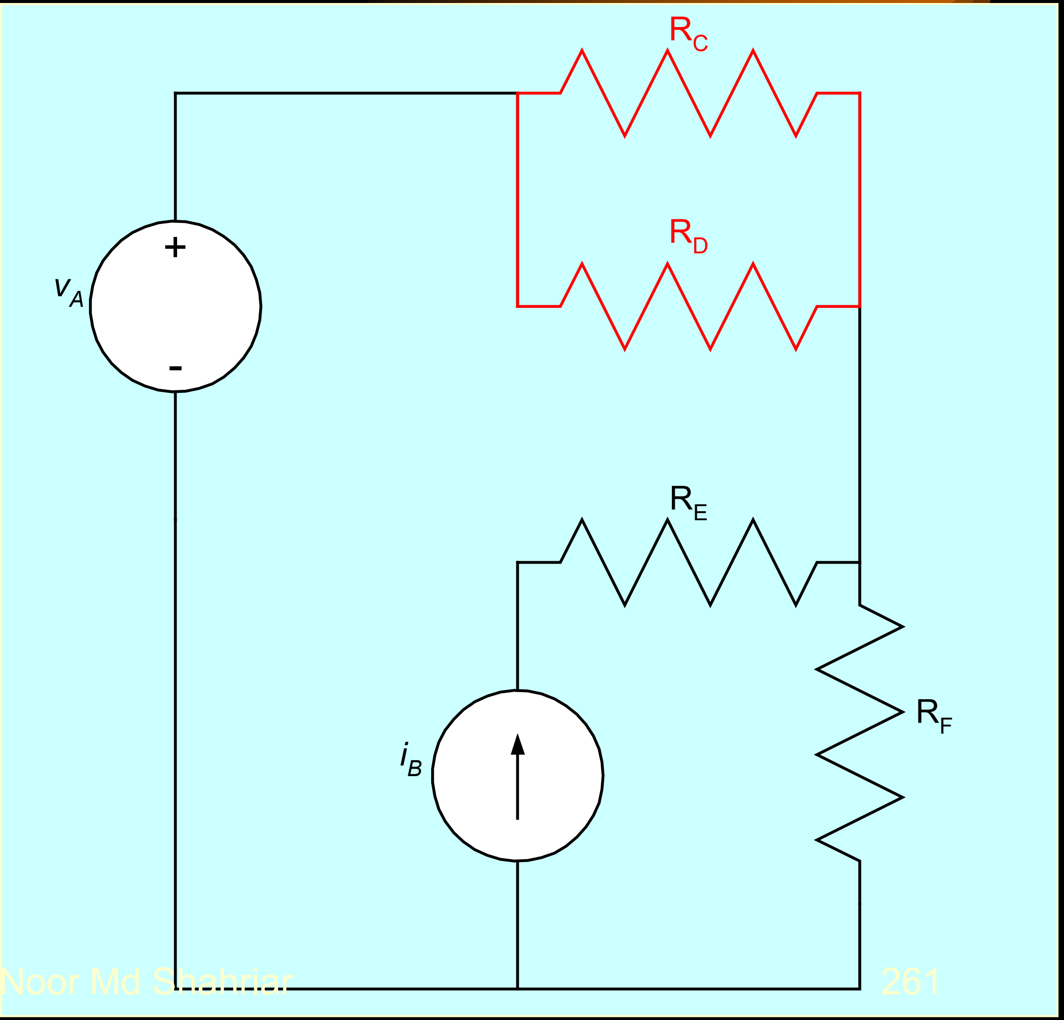
How Many Closed Paths? – 6

- How many closed paths are there following the elements shown?
- The answer is 6.
- We will show the closed paths on the following slides. Note which are meshes and which are not meshes.



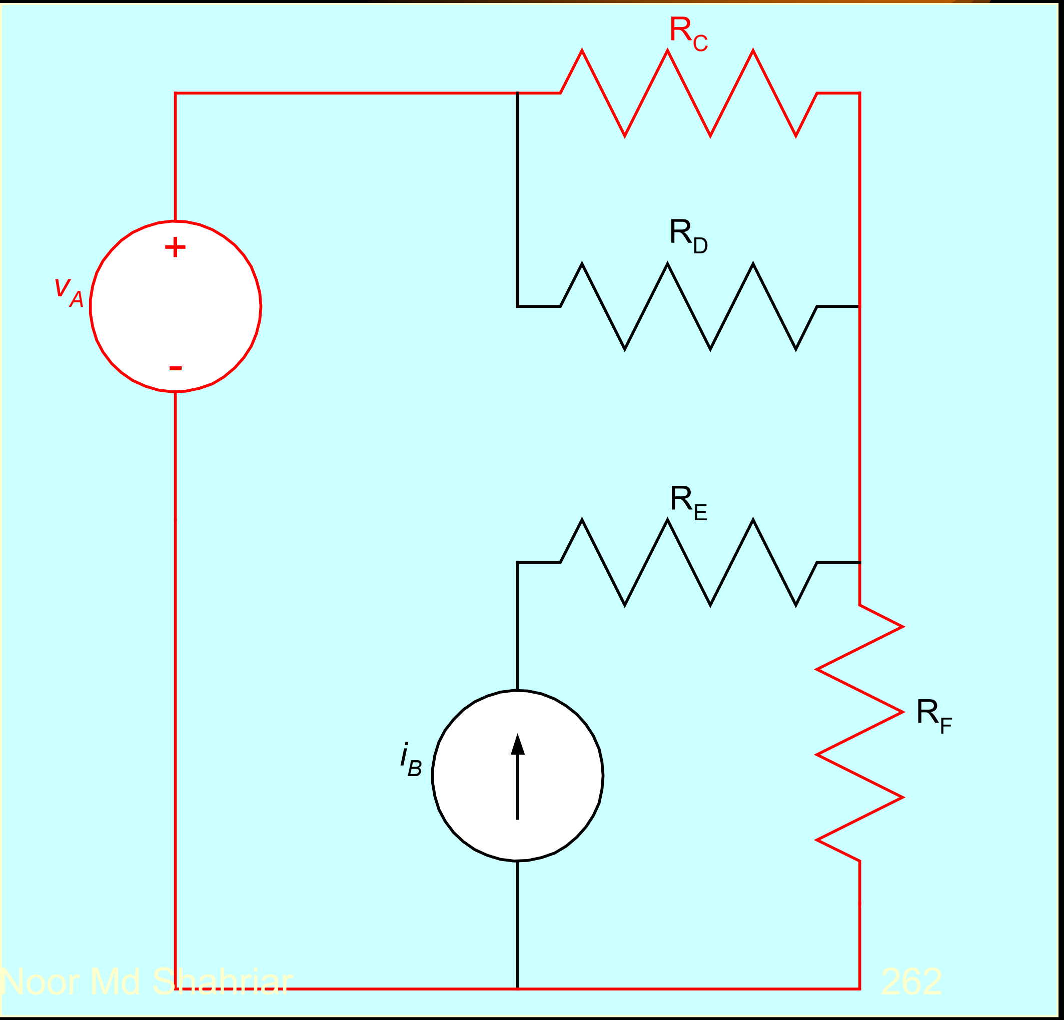
Closed Path #1

- Here is closed path #1. It is shown in red.
- It does not enclose any other closed paths. This is a mesh.



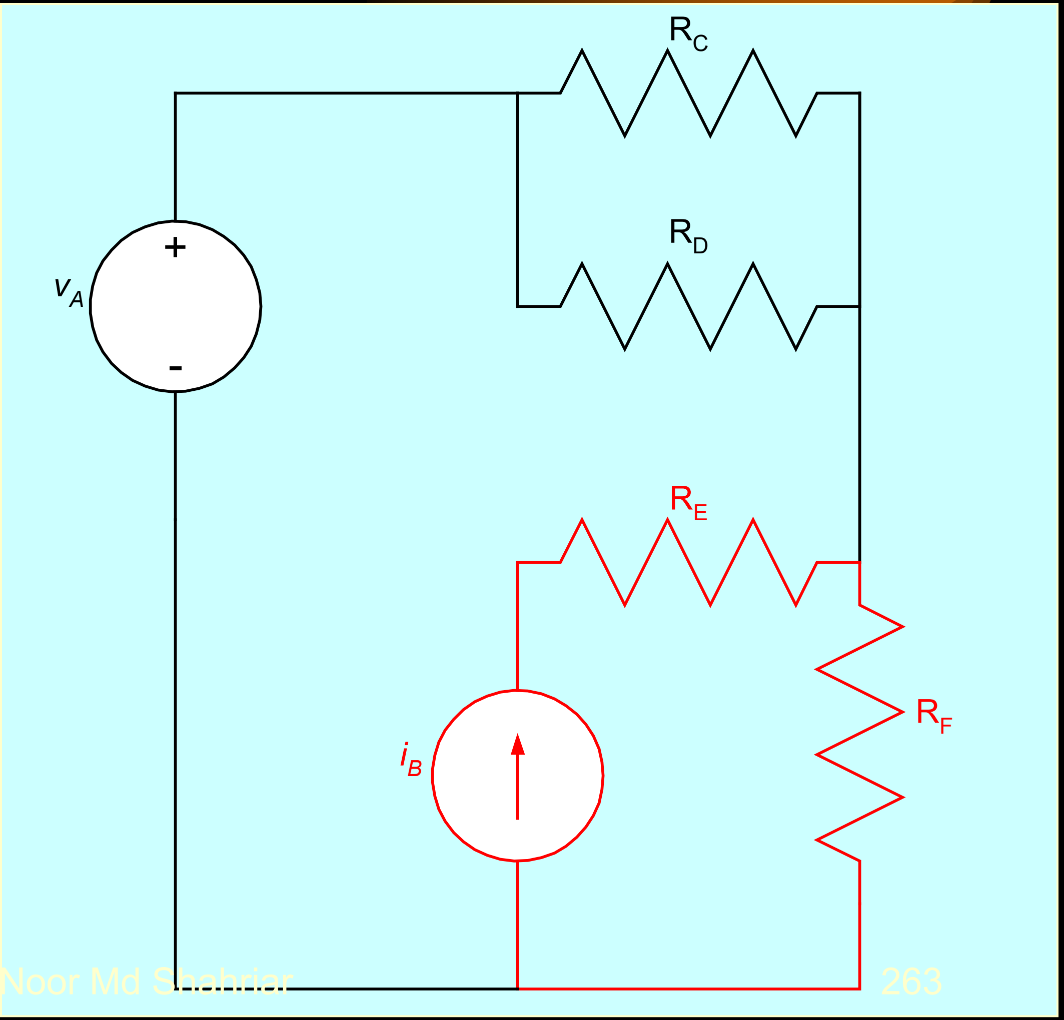
Closed Path #2

- Here is closed path #2. It is shown in red.
- It does enclose another closed path. (In fact, it encloses three meshes.) This is not a mesh.



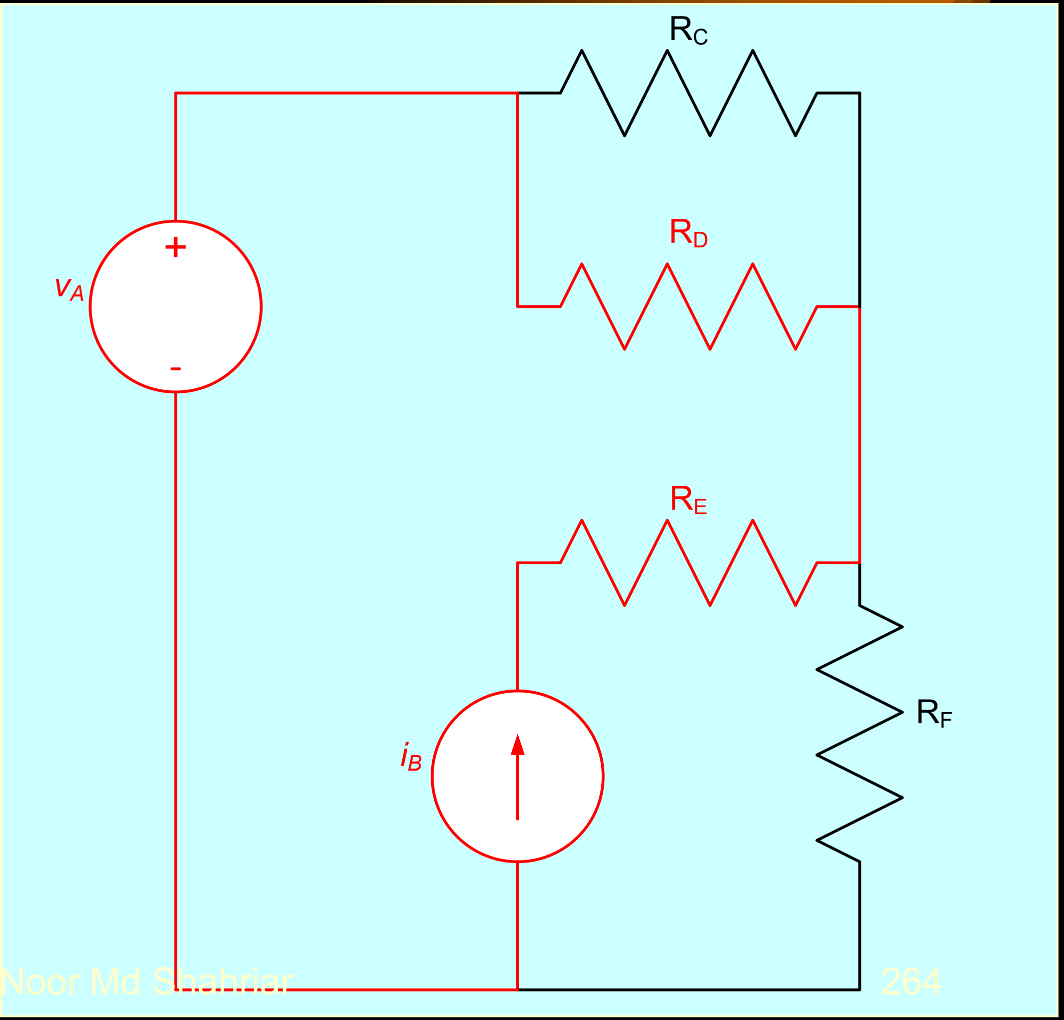
Closed Path #3

- Here is closed path #3. It is shown in red.
- It does not enclose any other closed paths. This is a mesh.



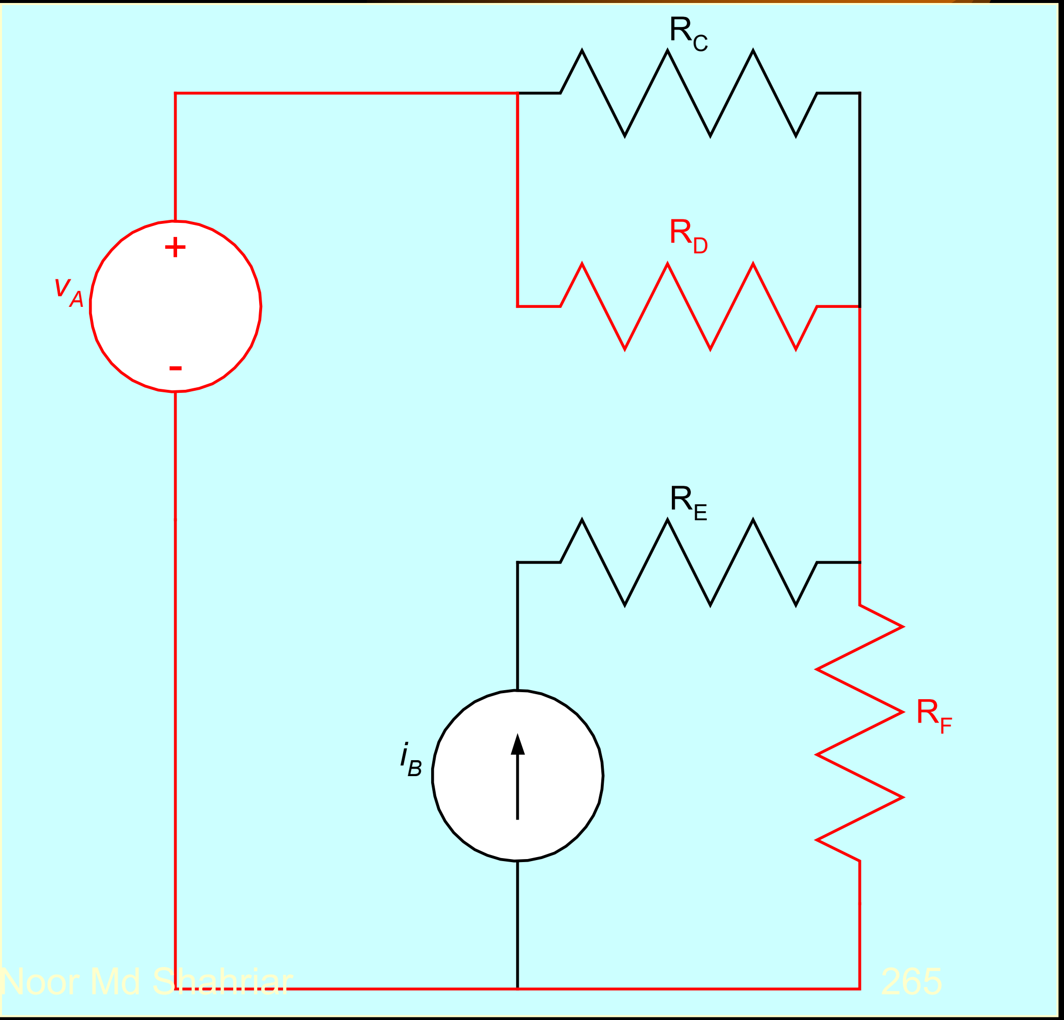
Closed Path #4

- Here is closed path #4. It is shown in red.
- It does not enclose any other closed paths. This is a mesh.



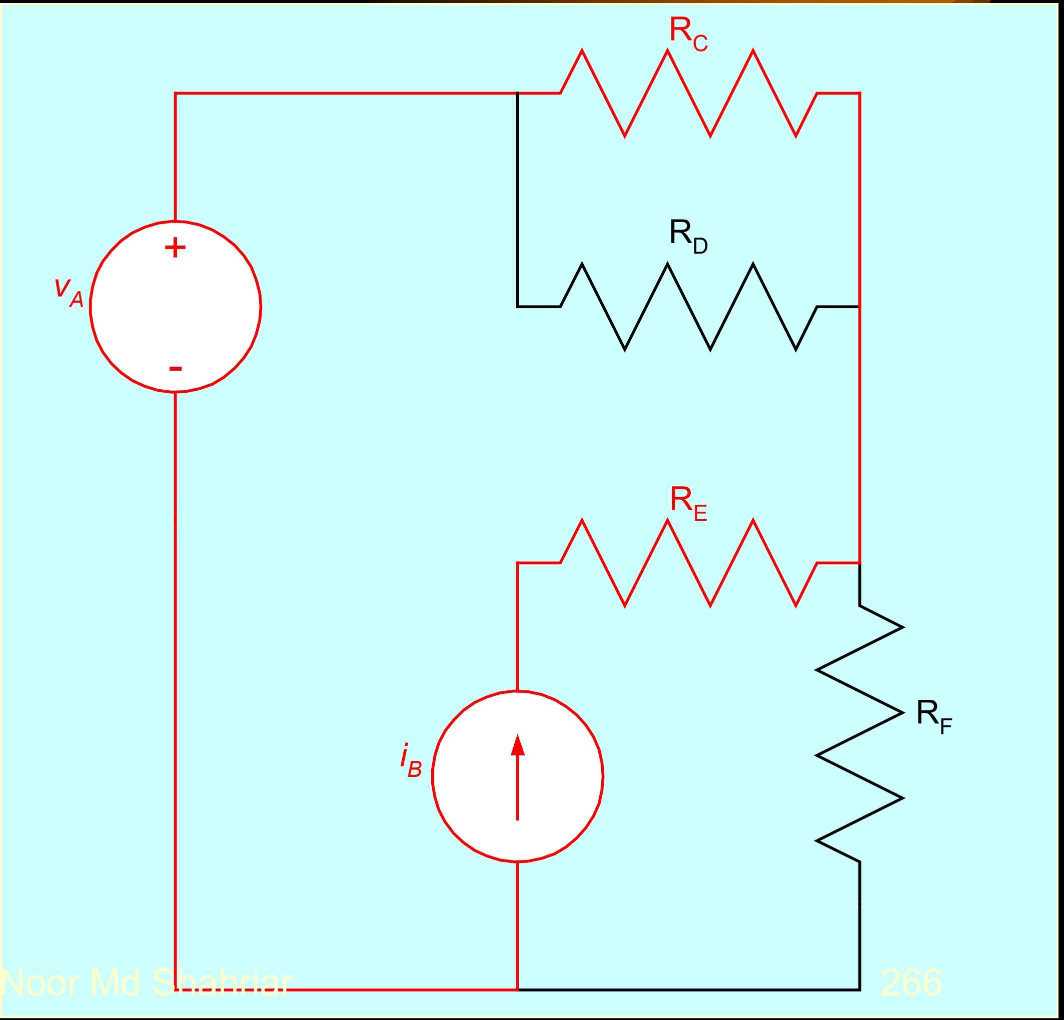
Closed Path #5

- Here is closed path #5. It is shown in red.
- It does enclose another closed path (in fact, two). This is not a mesh.



Closed Path #6

- Here is closed path #6. It is shown in red.
- It does enclose another closed path (in fact, it encloses two). This is not a mesh.
- In summary, we have three meshes in this circuit.

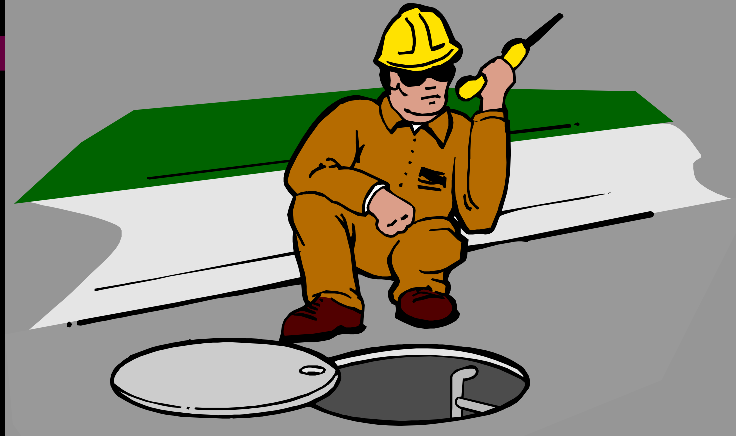


The Mesh-Current Method (MCM)

The Mesh-Current Method (MCM) is a systematic way to write all the equations needed to solve a circuit, and to write just the number of equations needed. It only works with planar circuits. The idea is that any other current or voltage can be found from these mesh-currents.

This method is not that important in very simple circuits, but in complicated circuits it gives us an approach that will get us all the equations that we need, and no extras.

It is also good practice for the writing of KCL and KVL equations. Many students believe that they know how to do this, but make errors in more complicated situations. Our work on the MCM will help correct some of those errors.



The Mesh-Current Method is a system. Like the sewer system here, the goal is be sure that everything is collected correctly. We want to write all the equations, the minimum number of equations, and nothing but **correct** equations. (However, we don't want to smell as bad!)

The Steps in the Mesh-Current Method (MCM)

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary. (If we cannot do this, we cannot use the MCM.)
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We will explain these steps by going through several examples.

Review KVL Equations Skip KVL Review

Kirchhoff's Voltage Law (KVL) – a Review

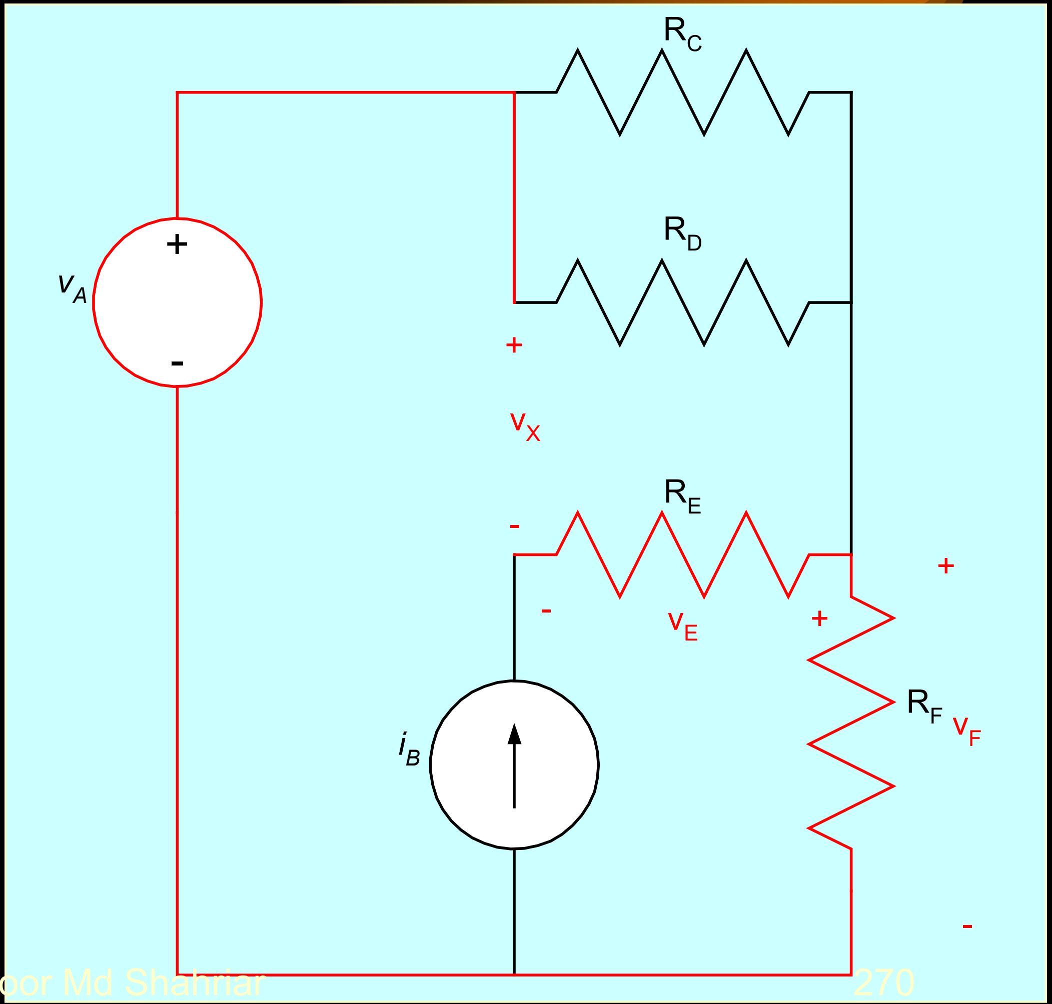
<u>The algebraic (or signed) summation of voltages around a closed loop must equal zero. Since a mesh is a closed loop, KVL will hold for meshes.</u>

For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a reference voltage drop, and a negative sign to a term that refers to a reference voltage rise.

Kirchhoff's Voltage Law (KVL) – an Example

- For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a voltage drop, and a negative sign to a term that refers to a voltage rise.
- In this example, we have already assigned reference polarities for all of the voltages for the loop indicated in red.
- For this circuit, and using our rule, starting at the bottom, we have the following equation:

$$-v_A + v_X - v_E + v_F = 0$$

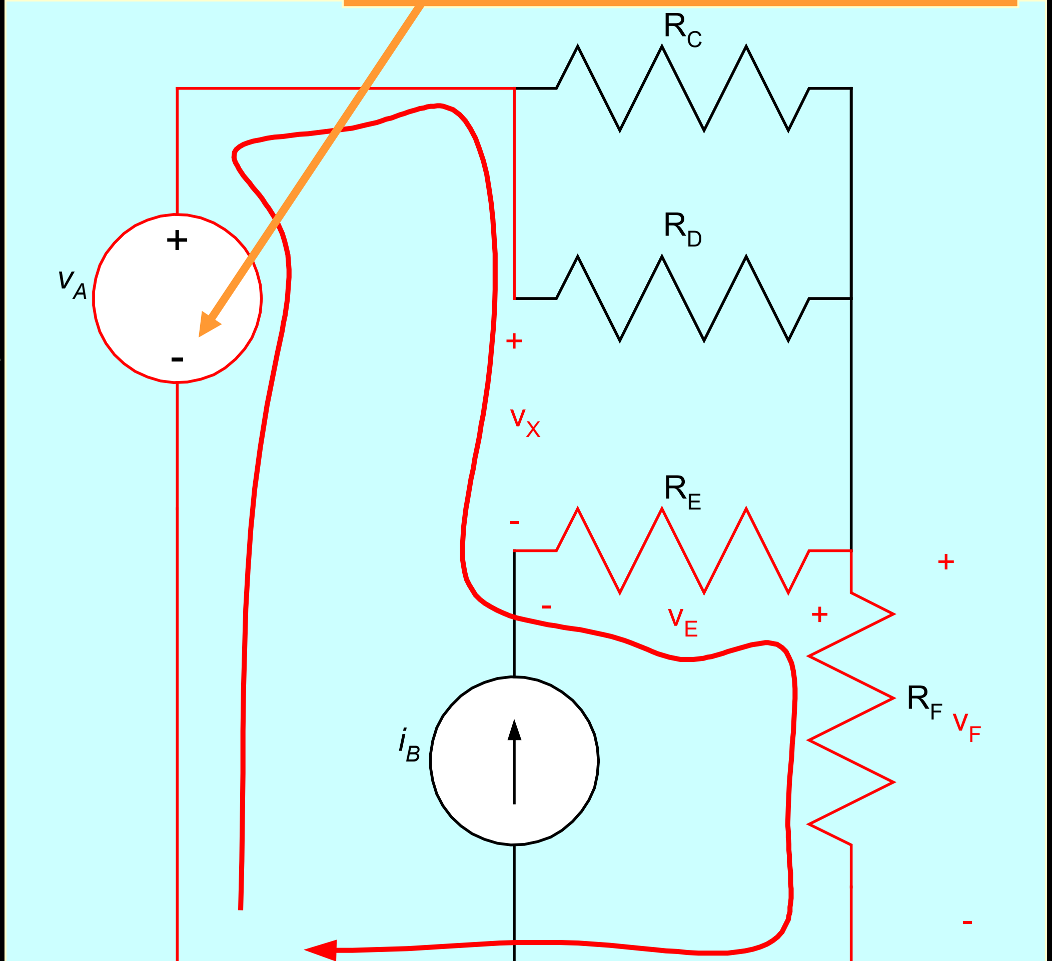


Kirchhoff's Voltage Law (KVL) – Notes

As we go up through the voltage source, we enter the negative sign first. Thus, v_A has a negative sign in the equation.

- For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a voltage drop, and a negative sign to a term that refers to a voltage rise.
- Some students like to use the following handy mnemonic device: Use the sign of the voltage that is on the side of the voltage that you enter. This amounts to the same thing.

$$-v_A + v_X - v_E + v_F = 0$$

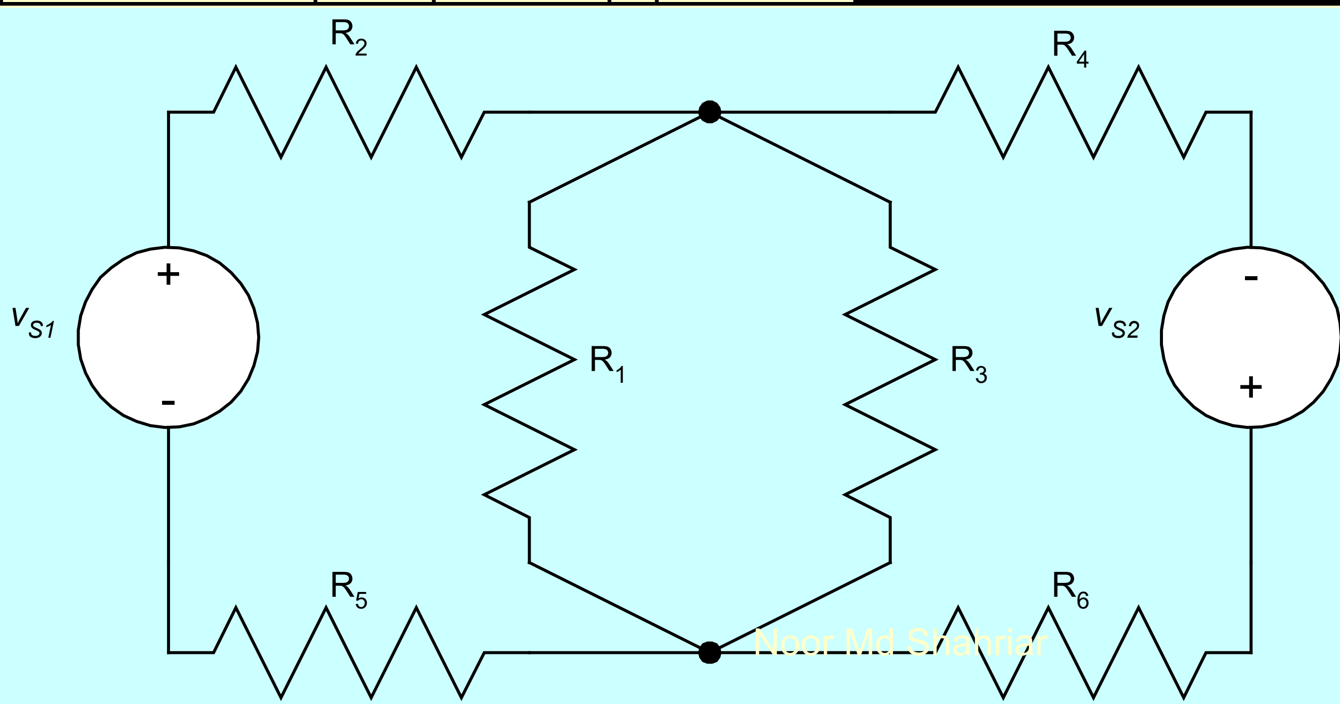


MCM – 1st Example

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent

For most students, it seems to be best to introduce the MCM by doing examples. We will start with simple examples, and work our way up to complicated examples. Our first example circuit is given here.

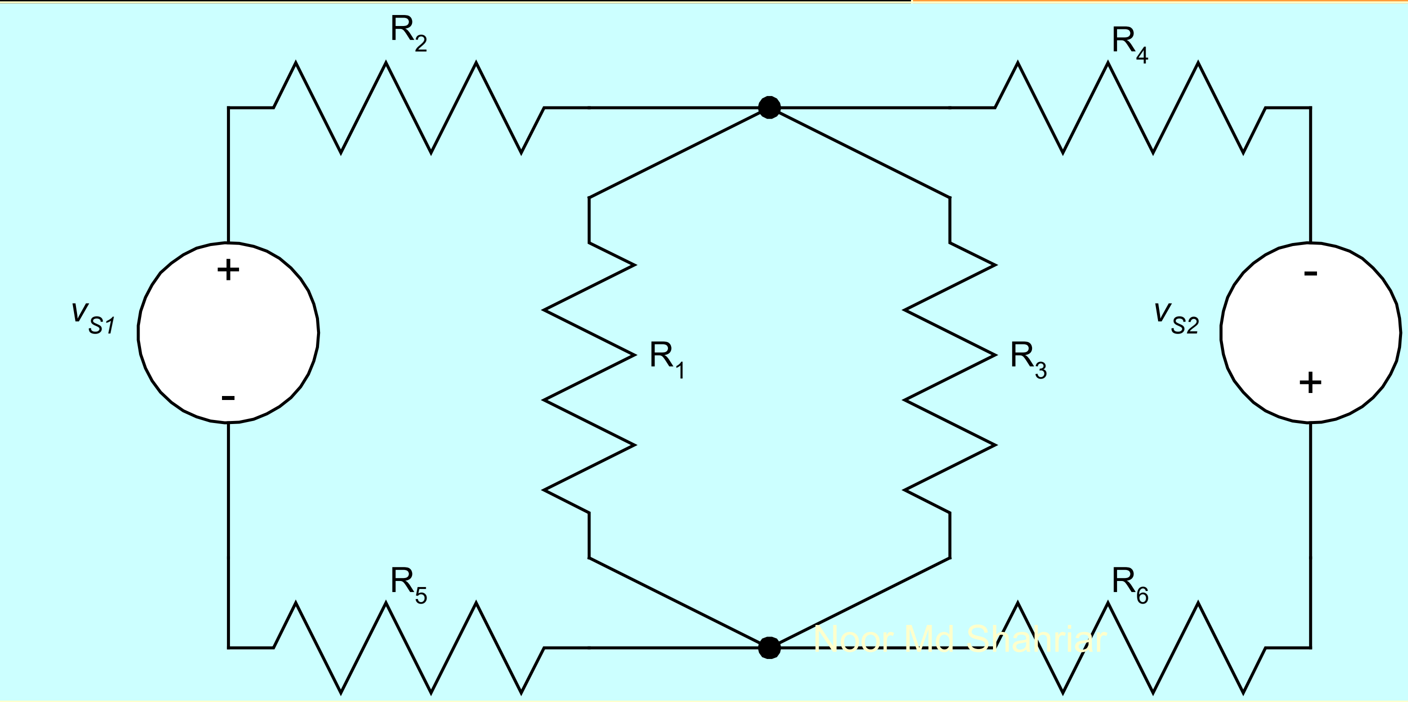


MCM – 1st Example – Step 1

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent

We need to redraw the circuit in planar form. This means that if we have any wires which are crossing without being connected, we need to move components around. This moving must not change any connections. We do this until all crossings are gone. If we cannot do this, we cannot use the MCM.



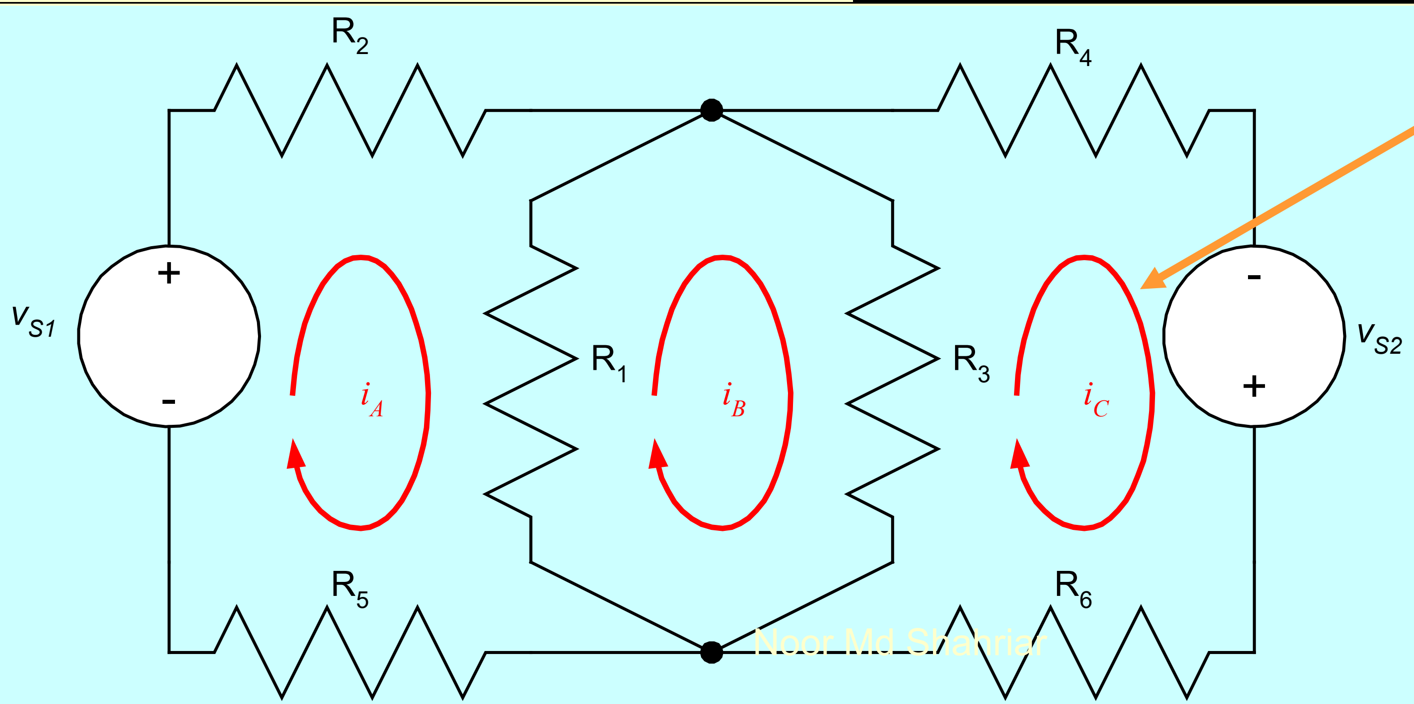
This circuit is already drawn in planar form. We can skip step 1 with this circuit.

MCM – 1st Example – Step 2

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We have assumed that it will be easy for you to identify the meshes in a circuit. Most students find this to be easy. They look for what some call “window panes” in the circuit. In this circuit, there are three meshes. We define the mesh currents by labeling them, showing the polarity using an arrow.



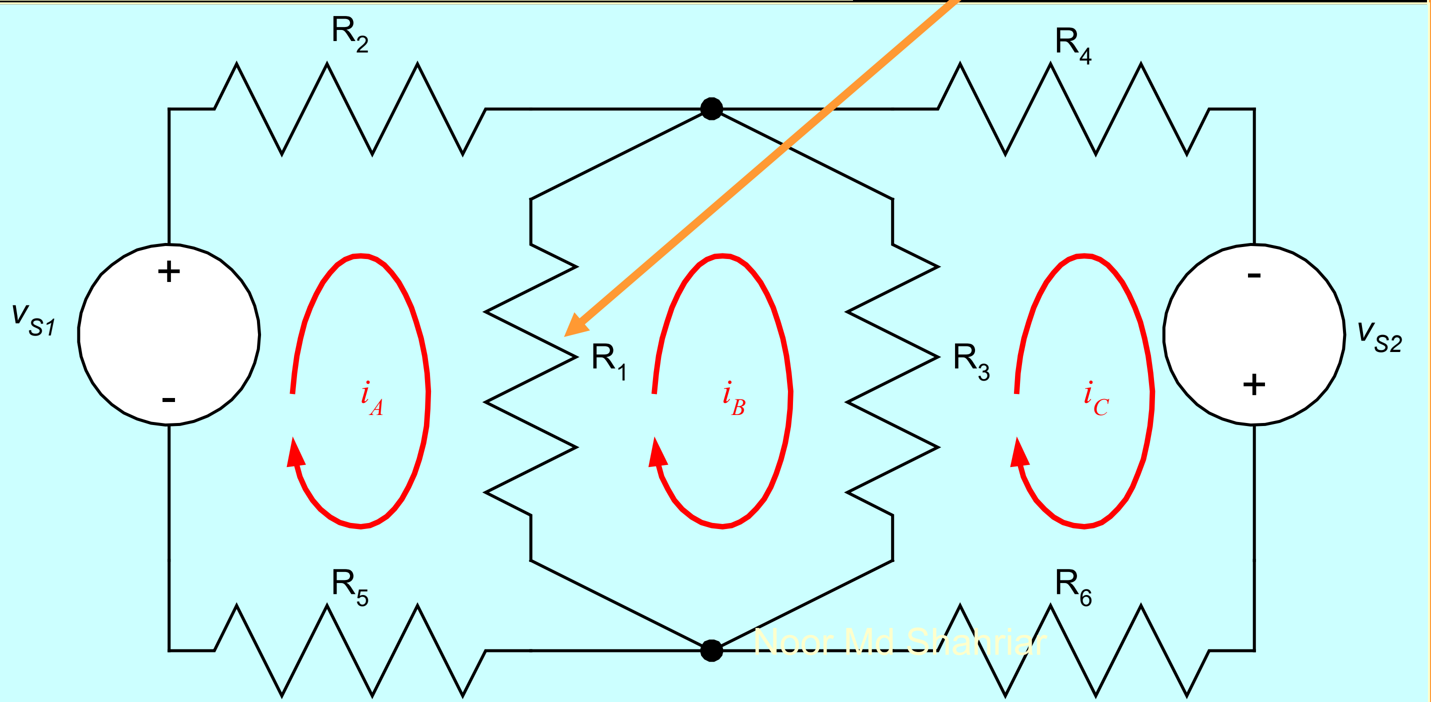
This symbol is used to designate the mesh current. We will choose a reference polarity which is clockwise in this material, but the choice is arbitrary.

MCM – 1st Example – Step 2 Note

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

A mesh current is thought of as a current that flows only around that mesh. The idea is that of charges which flow only in that subcircuit. In places where the meshes come together, both mesh currents flow simultaneously. In resistor R_1, two mesh currents, i_A and i_B, are flowing.



Some students ask whether mesh currents are real, or whether they exist. This depends upon your definition. If you define something as being real if you can measure it, then mesh currents are not real. The mesh current i_B in this circuit can not be measured directly.

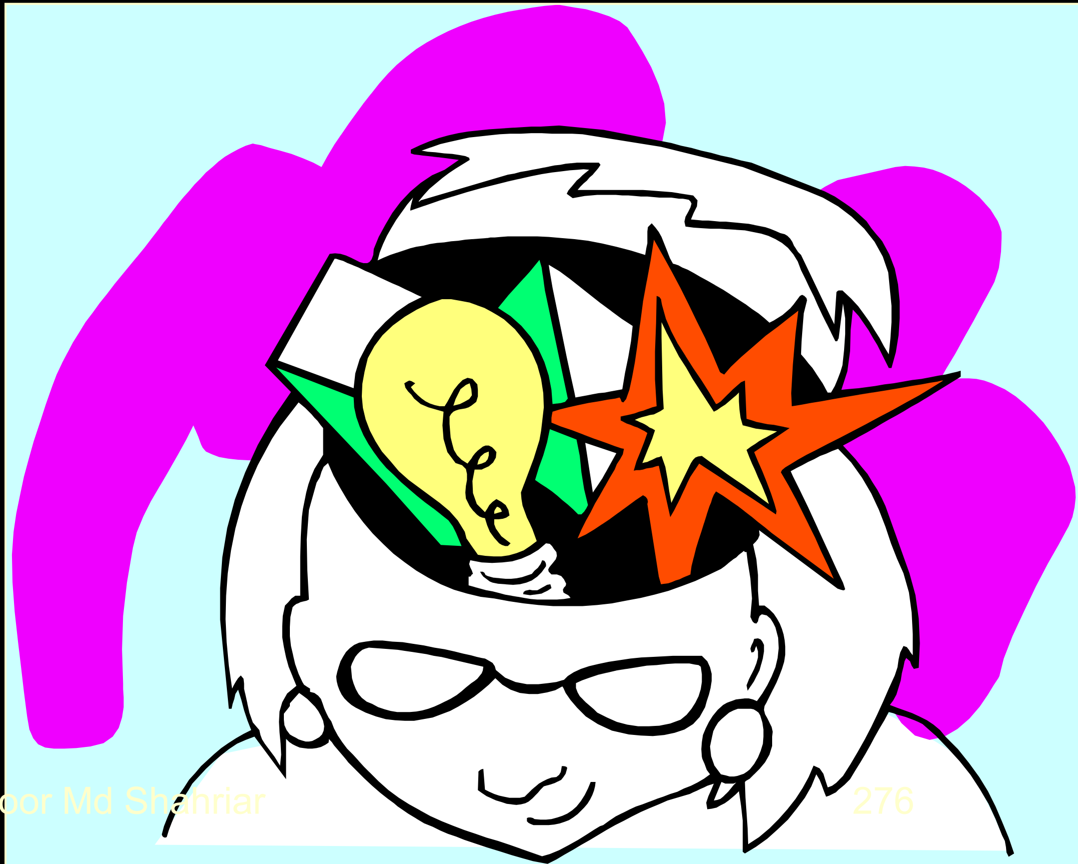
Mesh Currents Aren't Real???

Please don't let the issue of the reality of mesh currents bother you. One can debate whether they exist or not, but it is a moot point. It doesn't matter whether they exist or not. **We can use them to find real answers.** That is all that matters.

Think for a moment about the square root of minus one, or *j*, where

$$j = \sqrt{-1}.$$

There is no square root of negative numbers. There is no doubt that *j* does not exist, no doubt that it is imaginary. Still, we use it to solve for real answers. It is a tool. It does not matter whether it exists or not.



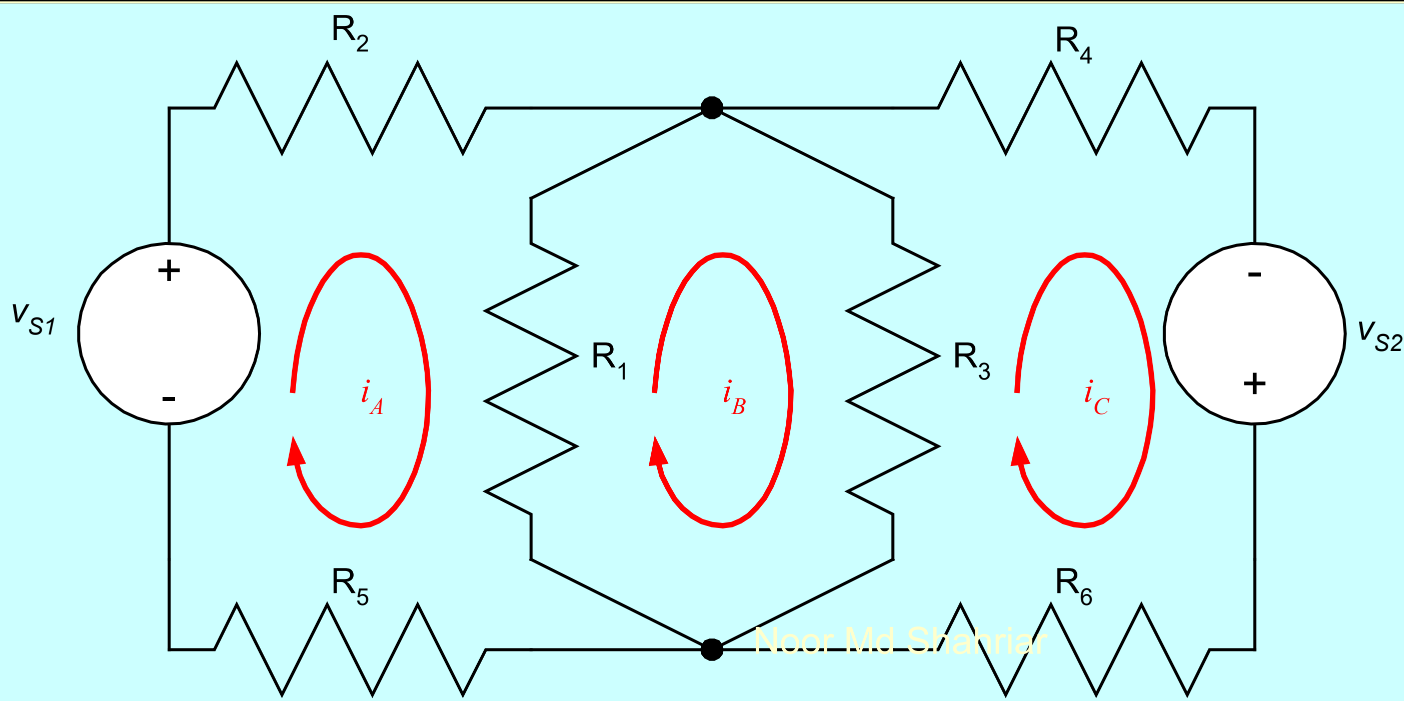
MCM – 1st Example – Step 3 – Part 1

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Now, we need to write a KVL equation for each mesh. That means three equations. Let's start with mesh A. The equation is:

$$\text{A:}\quad -v_{S1} + i_A R_2 + (i_A - i_B)R_1 + i_A R_5 = 0.$$

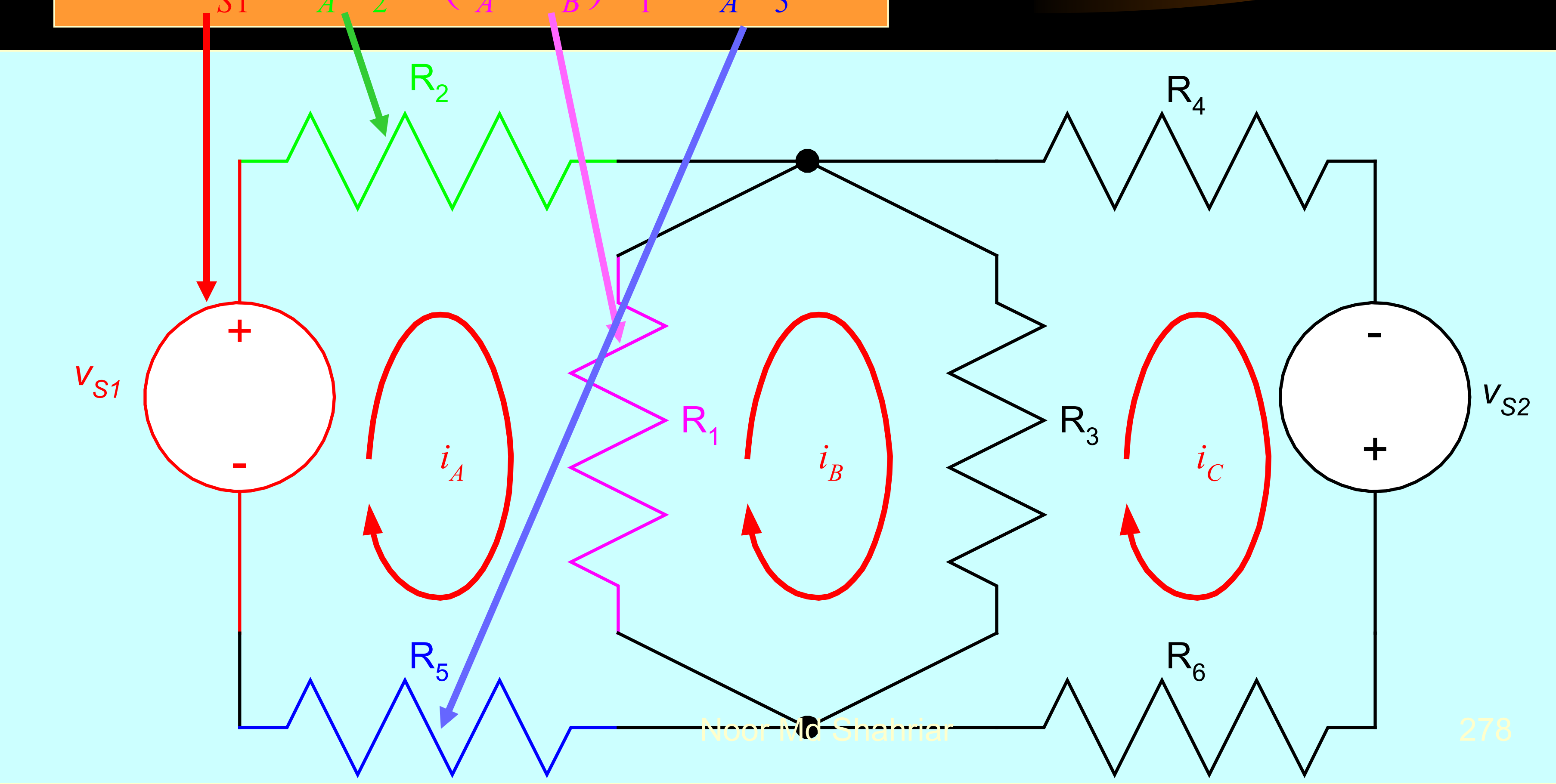


Do each of these voltage terms make sense? If not, go here to have them explained. If all four terms are clear to you, skip this explanation.

Step 3 – Part 1 – Explanation

Let's make sure that we understand where this equation comes from. As we go around the closed path that is the mesh, we have four voltages. Using Ohm's Law, we can show that the voltages across the resistors are functions of the currents through them.

A: $-v_{S1} + i_A R_2 + (i_A - i_B) R_1 + i_A R_5 = 0$

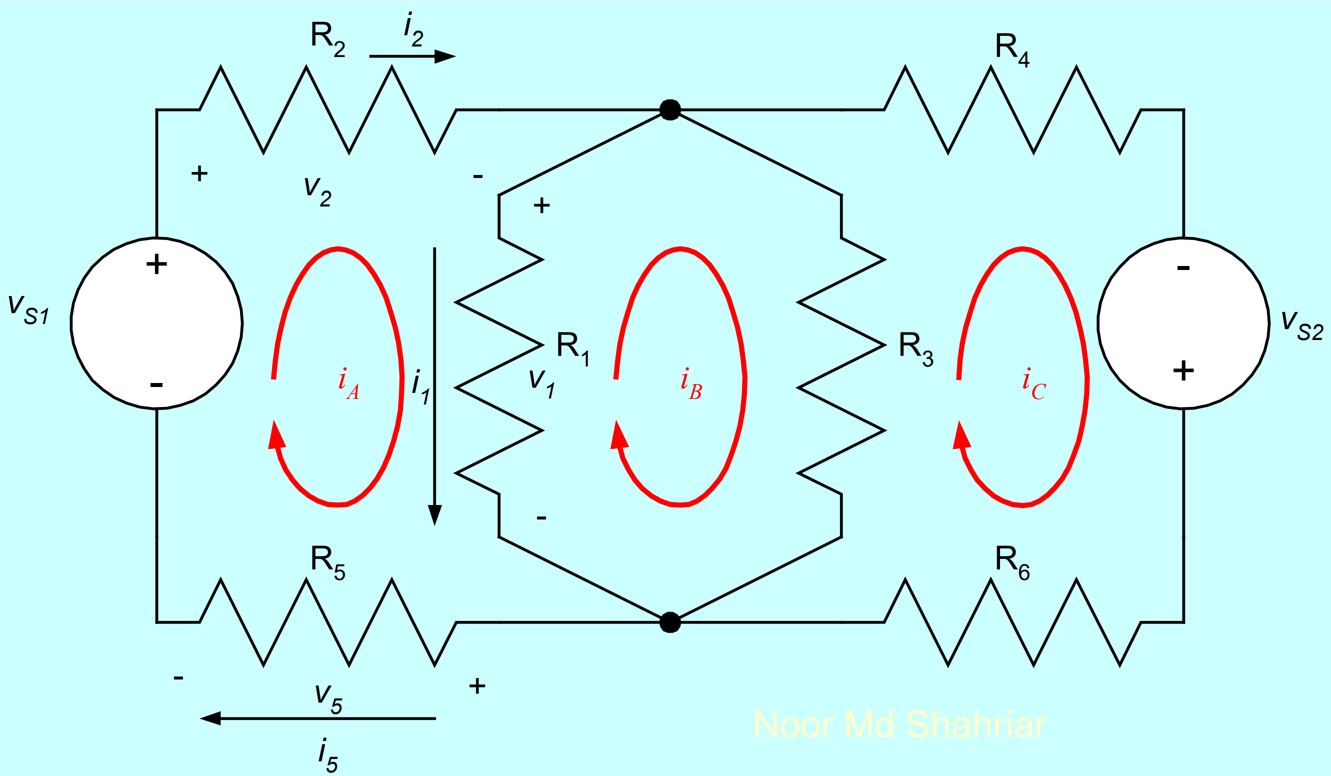


Step 3 – Part 1 – Explanation 2

Here, we have labeled the branch currents and voltages for each term of the equation. A **branch current** is the current in the component, which is the summation of the mesh currents that go through that branch, being careful about the signs. Note that in this circuit, $i_2 = i_A$, $i_1 = (i_A - i_B)$, and $i_5 = i_A$.

$$\text{A:} \quad -v_{S1} + v_2 + v_1 + v_5 = 0$$

$$\text{A:} \quad -v_{S1} + i_A R_2 + (i_A - i_B) R_1 + i_A R_5 = 0$$



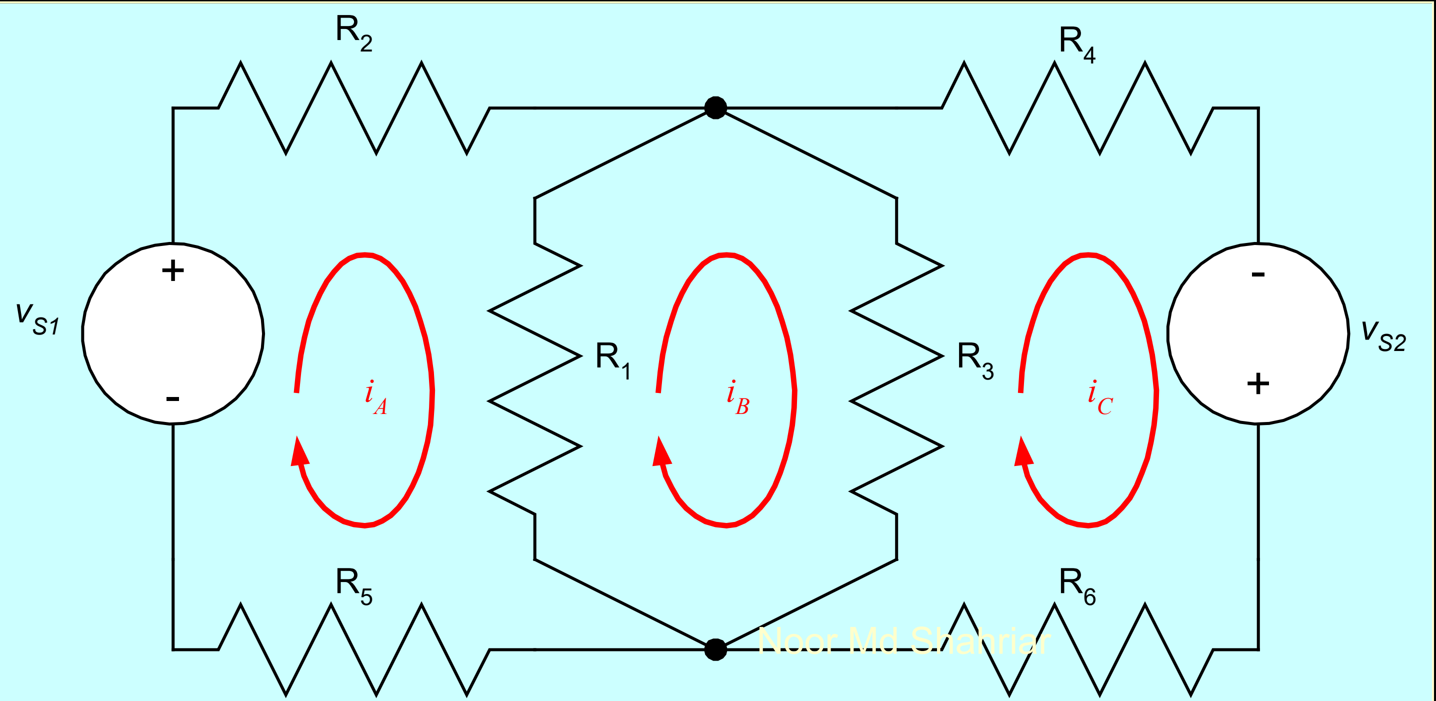
MCM – 1st Example – Step 3 – Part 2

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Next, let's write a KVL for mesh B. The equation is:

$$\text{B:}\;\; (i_B - i_A)R_1 + (i_B - i_C)R_3 = 0.$$



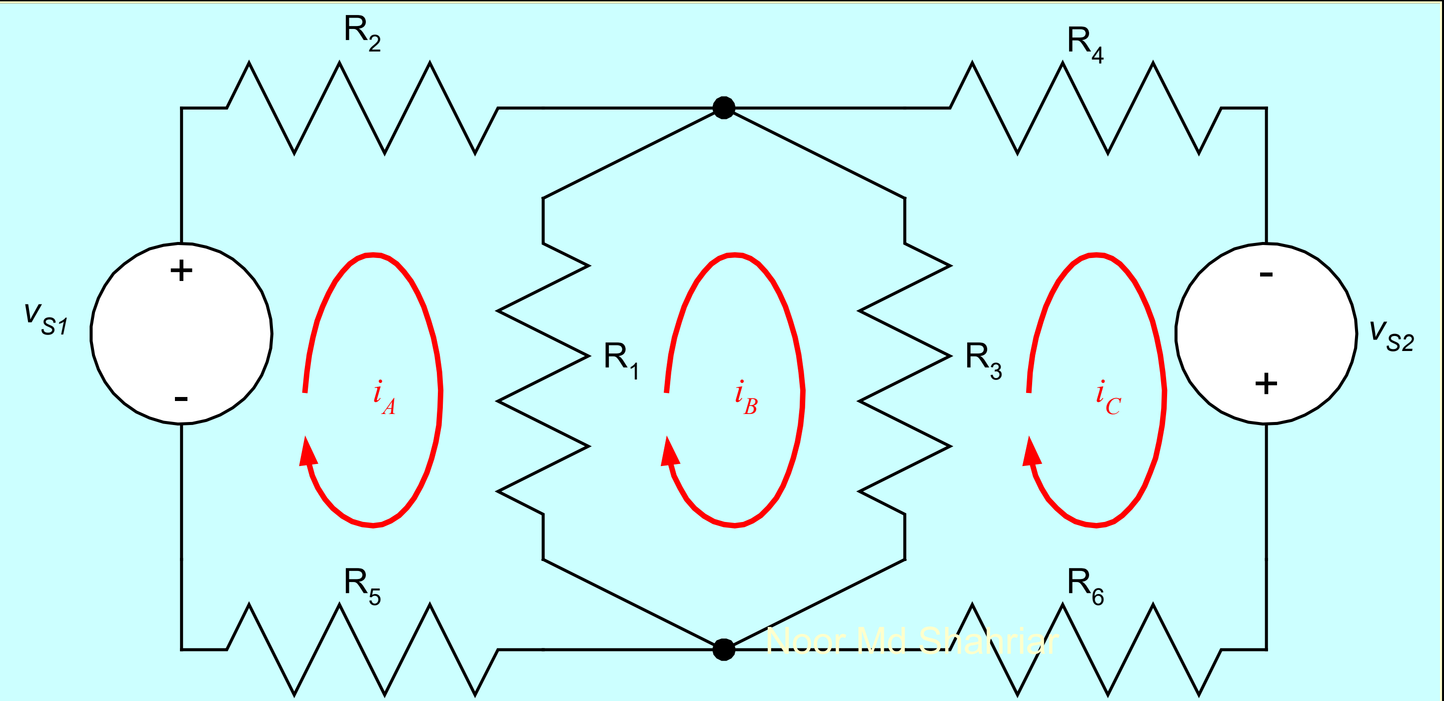
MCM – 1st Example – Step 3 – Part 3

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Finally, we write a KVL equation for mesh C. The equation is:

$$\text{C:}\ \ (i_C - i_B)R_3 + i_C R_4 - v_{S2} + i_C R_6 = 0.$$



MCM – 1st Example – Step 3 – Notes

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

Some notes that may be helpful:

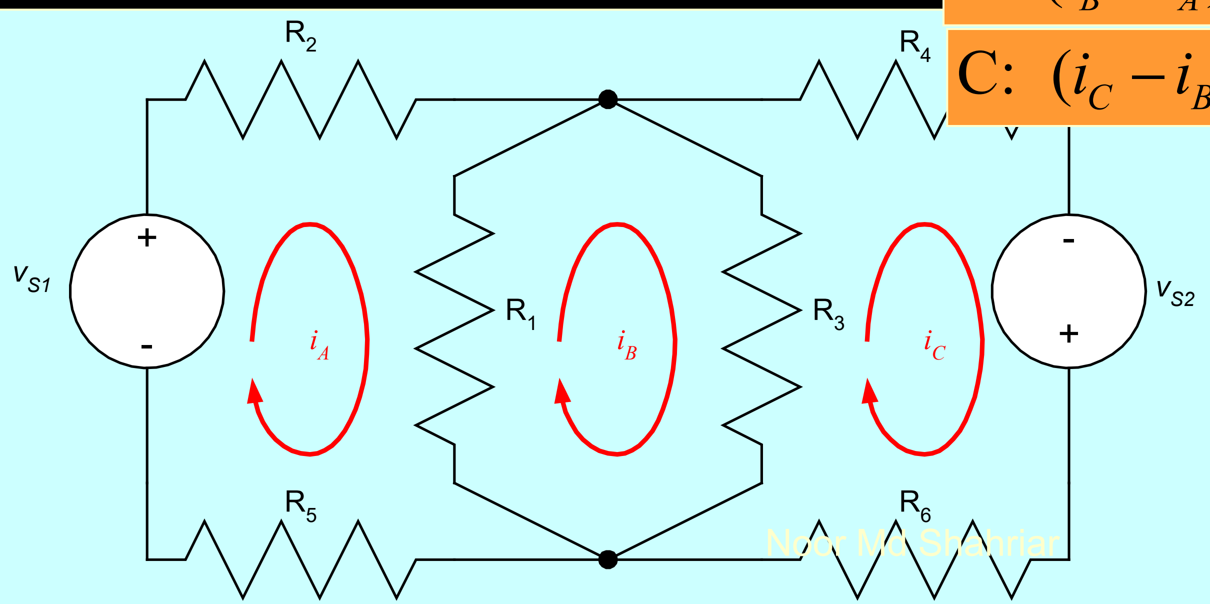
a) We have named the meshes for the mesh currents that are in them.

b) When we write these equations using the conventions we picked, the A mesh equation has a positive sign associated with all the terms with i_A, and a negative sign with all other mesh-current terms. This is a good way to check your equations.

$$\text{A:} \quad -v_{S1} + i_A R_2 + (i_A - i_B) R_1 + i_A R_5 = 0$$

$$\text{B:} \quad (i_B - i_A) R_1 + (i_B - i_C) R_3 = 0$$

$$\text{C:} \quad (i_C - i_B) R_3 + i_C R_4 - v_{S2} + i_C R_6 = 0$$



MCM – 1st Example – Step 4

The Mesh-Current Method steps are:

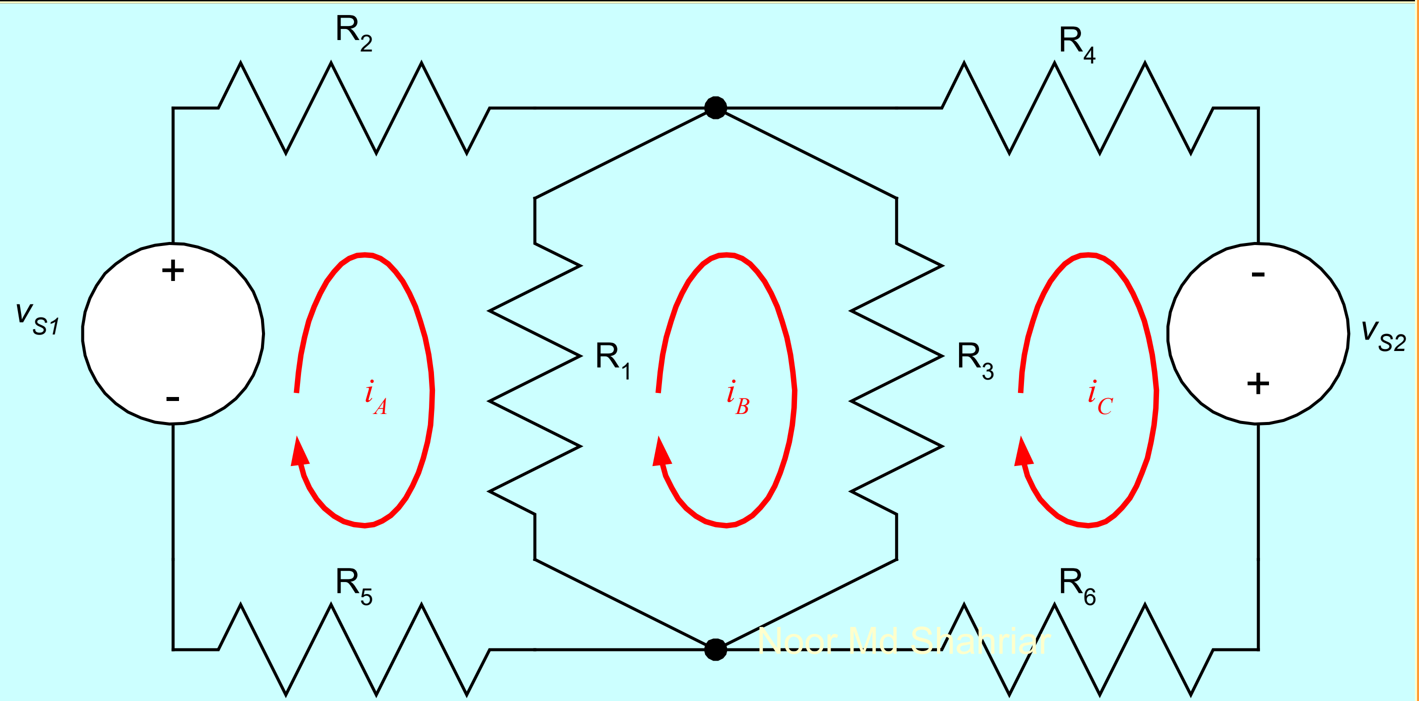
1. Redraw the circuit in planar form, if necessary.
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

There are no dependent sources in this circuit, so we can skip step 4. We should now have the same number of equations (3) as unknowns (3), and we can solve.

$$\text{A:}\quad -v_{S1} + i_A R_2 + (i_A - i_B)R_1 + i_A R_5 = 0$$

$$\text{B:}\quad (i_B - i_A)R_1 + (i_B - i_C)R_3 = 0$$

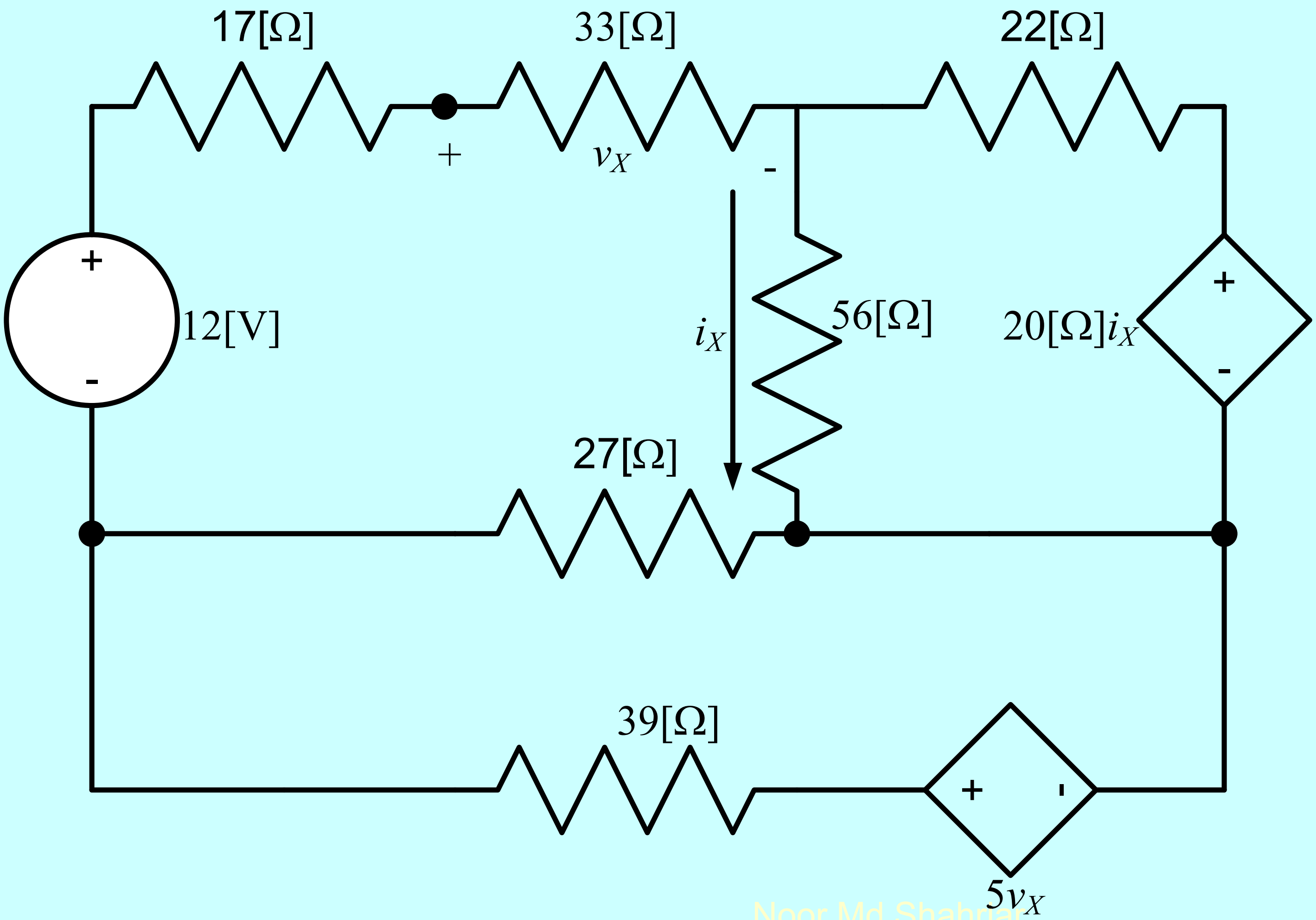
$$\text{C:}\quad (i_C - i_B)R_3 + i_C R_4 - v_{S2} + i_C R_6 = 0$$



Note that we have assumed that all the values of the resistors and sources have been given. If not, we need more information before we can solve.

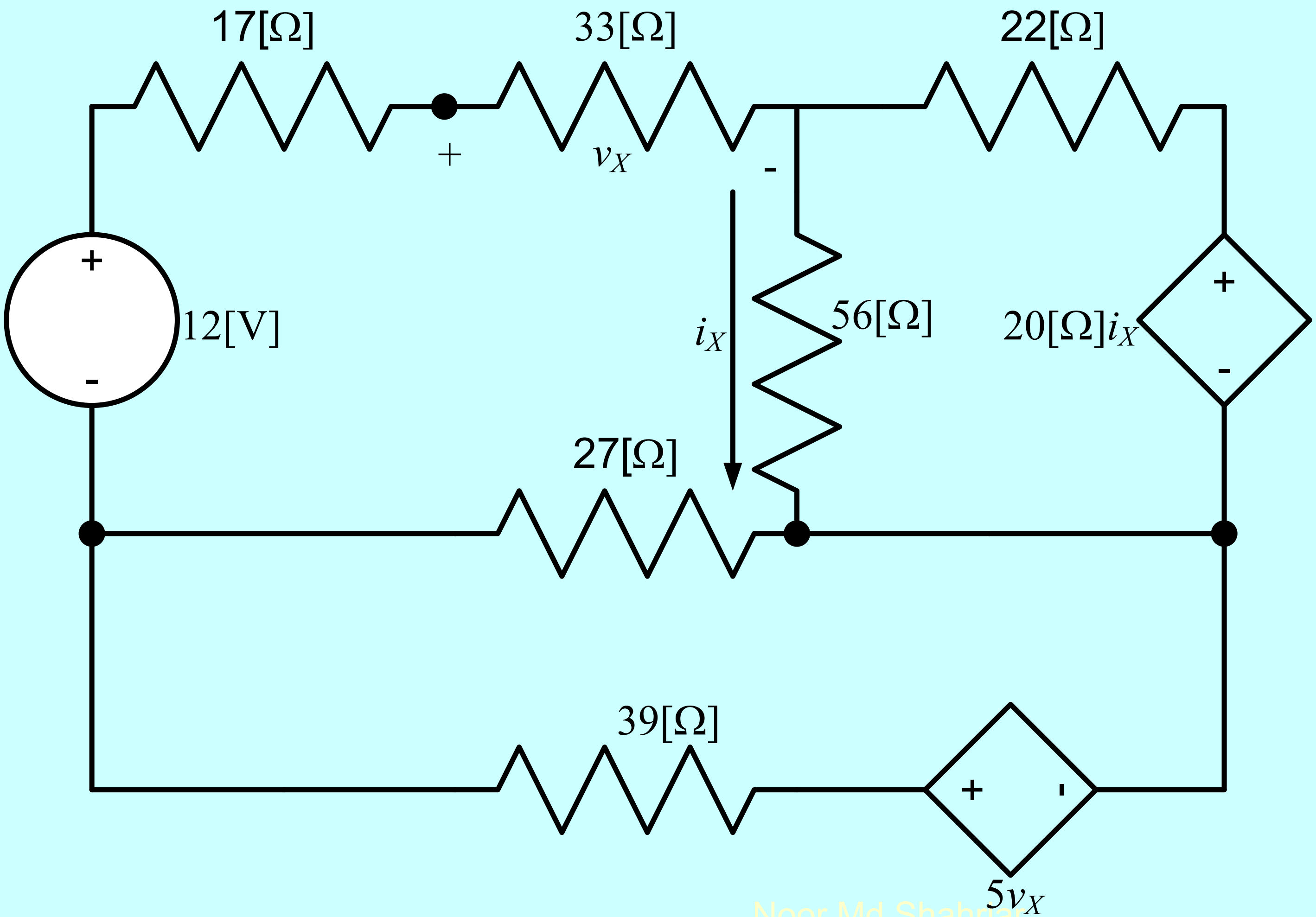
MCM – 2nd Example

Our second example circuit is given here. Numerical values are given in this example. Let's find the current i_X shown, using the Mesh-Current Method.



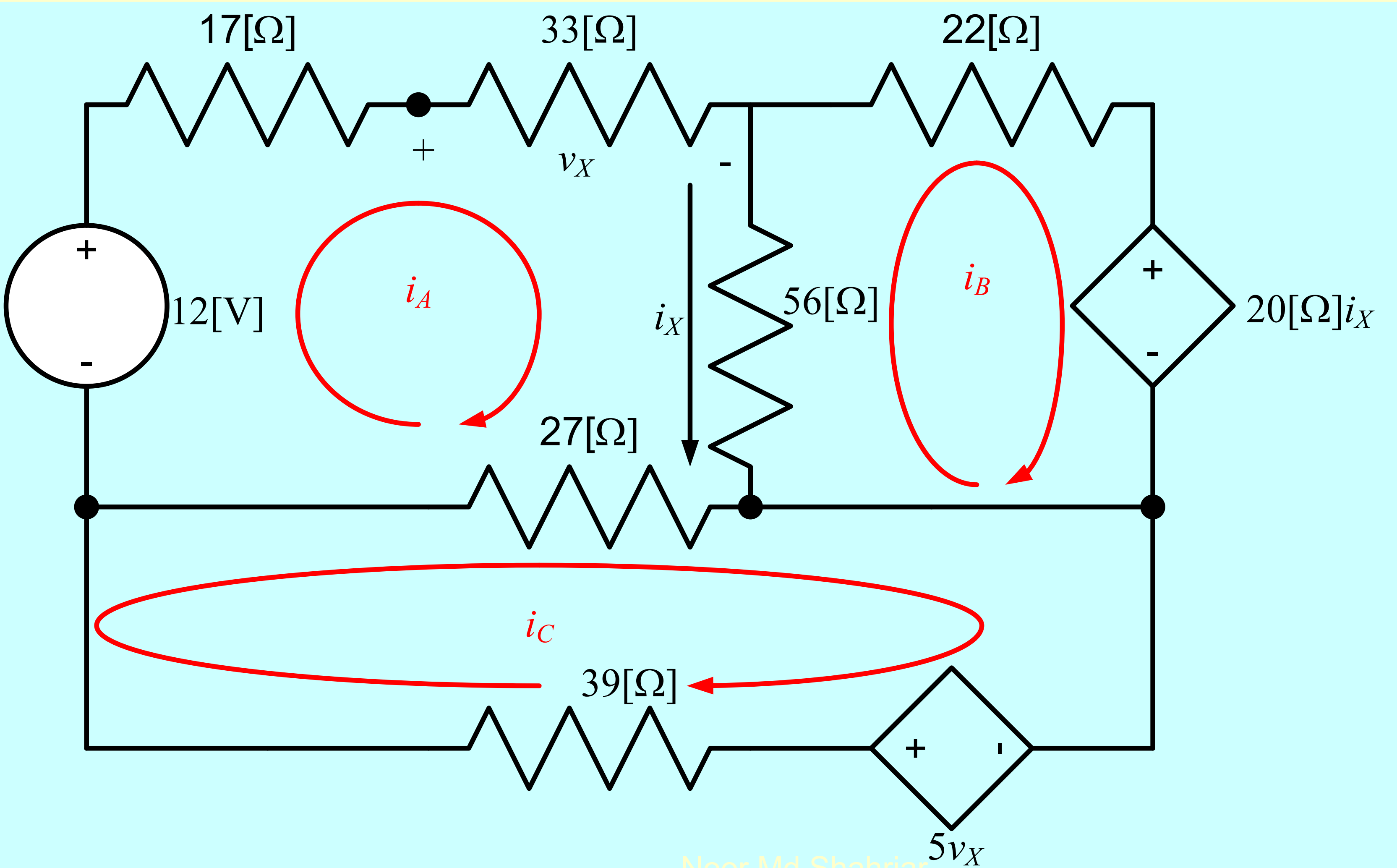
MCM – 2nd Example – Step 1

This circuit is already drawn in planar form, so we may skip step 1.



MCM – 2nd Example – Step 2

We have defined the mesh currents for the three meshes in this circuit. As is our practice, we have defined them to be clockwise.

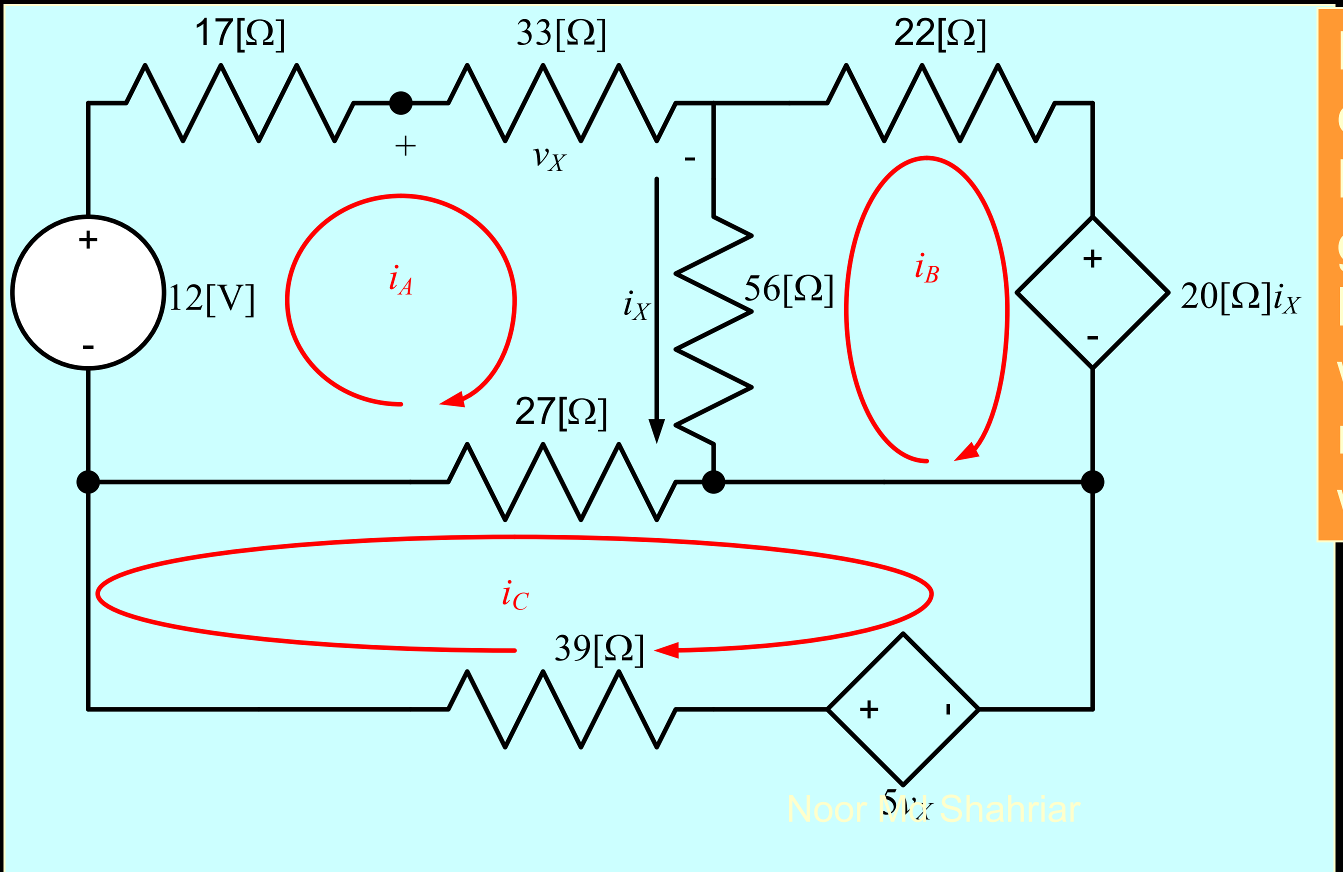


MCM – 2nd Example – Step 3

A: $-12[V]+i_A 17[\Omega]+i_A 33[\Omega]+(i_A-i_B)56[\Omega]+(i_A-i_C)27[\Omega]=0,$

B: $(i_B-i_A)56[\Omega]+i_B 22[\Omega]+20[\Omega]i_X=0,$ and

C: $(i_C-i_A)27[\Omega]-5v_X+i_C 39[\Omega]=0.$



Now, we write KVL equations for nodes A, B, and C. These are given here. We have labeled each equation with the name of the mesh for which it was written.

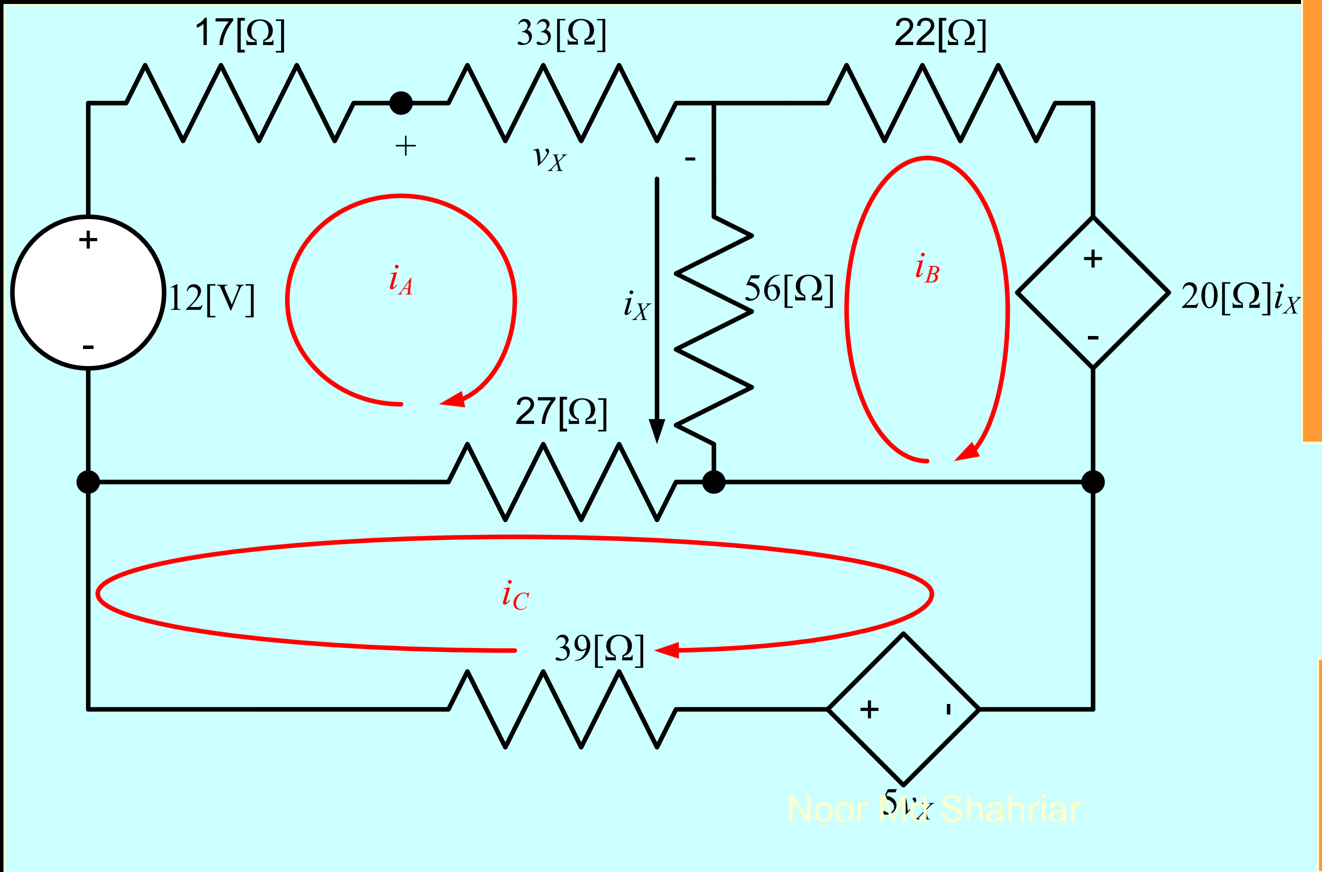
MCM – 2nd Example – Step 4

Hopefully, it is now clear why we needed step 4. Until this point, we have 3 equations and 5 unknowns. We need two more equations.

A: $-12[\text{V}]+i_A 17[\Omega]+i_A 33[\Omega]+(i_A-i_B)56[\Omega]+(i_A-i_C)27[\Omega]=0$

B: $(i_B-i_A)56[\Omega]+i_B 22[\Omega]+20[\Omega]i_X=0$

C: $(i_C-i_A)27[\Omega]-5v_X+i_C 39[\Omega]=0$



We get these equations by writing equations for i_X and v_X, using KCL, KVL and Ohm's Law, and using the mesh currents already defined. Let's write the two equations we need:

$$i_X = i_A - i_B, \text{ and}$$
$$v_X = i_A 33[\Omega].$$

Now, we have 5 equations and 5 unknowns.

MCM – 2nd Example – Solution

We have the following equations.

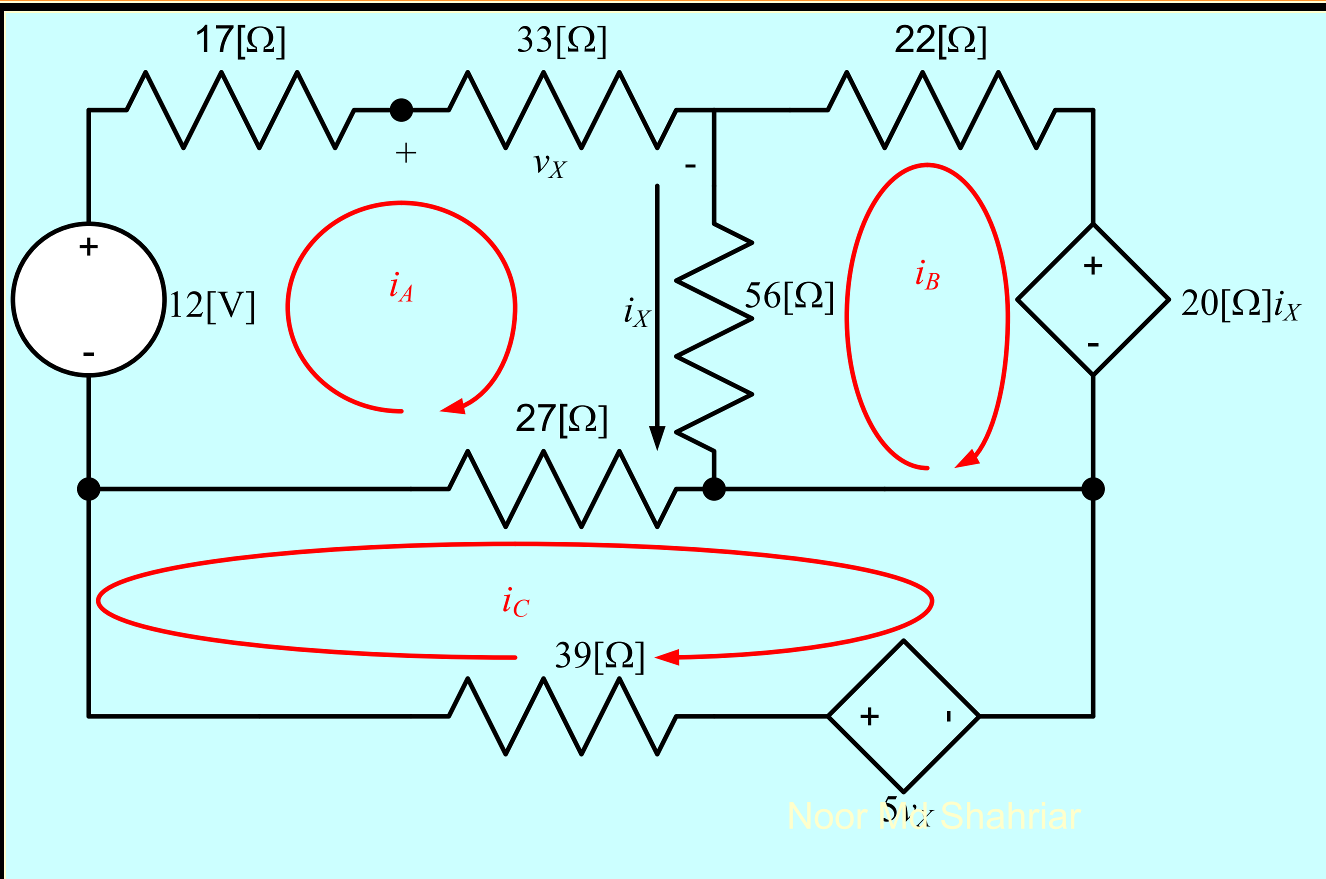
$$\text{A:}\quad -12[\text{V}]+i_A 17[\Omega]+i_A 33[\Omega]+(i_A-i_B)56[\Omega]+(i_A-i_C)27[\Omega]=0$$

$$\text{B:}\quad (i_B-i_A)56[\Omega]+i_B 22[\Omega]+20[\Omega]i_X=0$$

$$\text{C:}\quad (i_C-i_A)27[\Omega]-5v_X+i_C 39[\Omega]=0$$

$$i_X=i_A-i_B$$

$$v_X=i_A 33[\Omega]$$



The solution is:

$$i_A=0.6093[\text{A}]$$

$$i_B=0.3782[\text{A}]$$

$$i_C=1.772[\text{A}]$$

$$i_X=0.2311[\text{A}]$$

$$v_X=20.11[\text{V}]$$

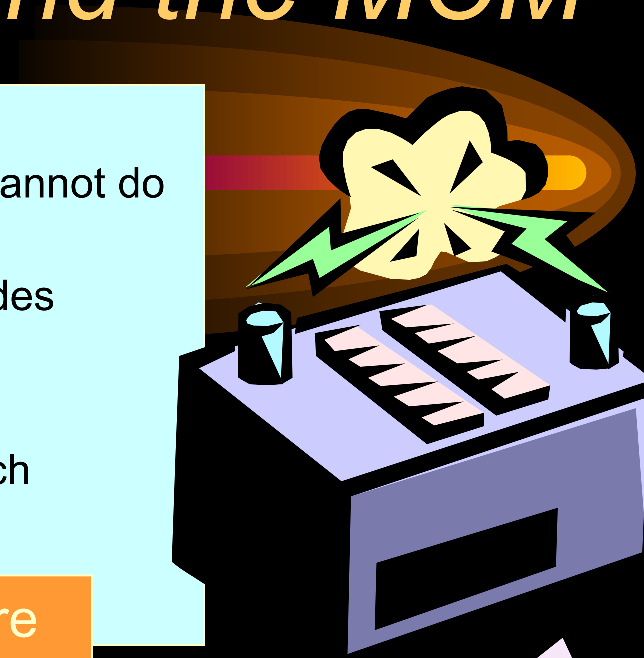
The Steps in the Mesh-Current Method (MCM)

The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary. (If we cannot do this, we cannot use the MCM.)
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

We will explain these steps by going through several examples.

Current Sources and the MCM



The Mesh-Current Method steps are:

1. Redraw the circuit in planar form, if necessary. (If we cannot do this, we cannot use the MCM.)
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

A problem arises when using the MCM when there are current sources present. The problem is in Step 3. The voltage across a current source can be anything; the voltage depends on what the current source is **connected** to. Therefore, it is not clear what to write for the KVL expression. We could introduce a new voltage variable, but we would rather not introduce another variable. In addition, if all we do is directly write KVL equations, we cannot include the value of the current source.

Current Sources and the MCM – Solution

The Mesh-Current Method steps are:

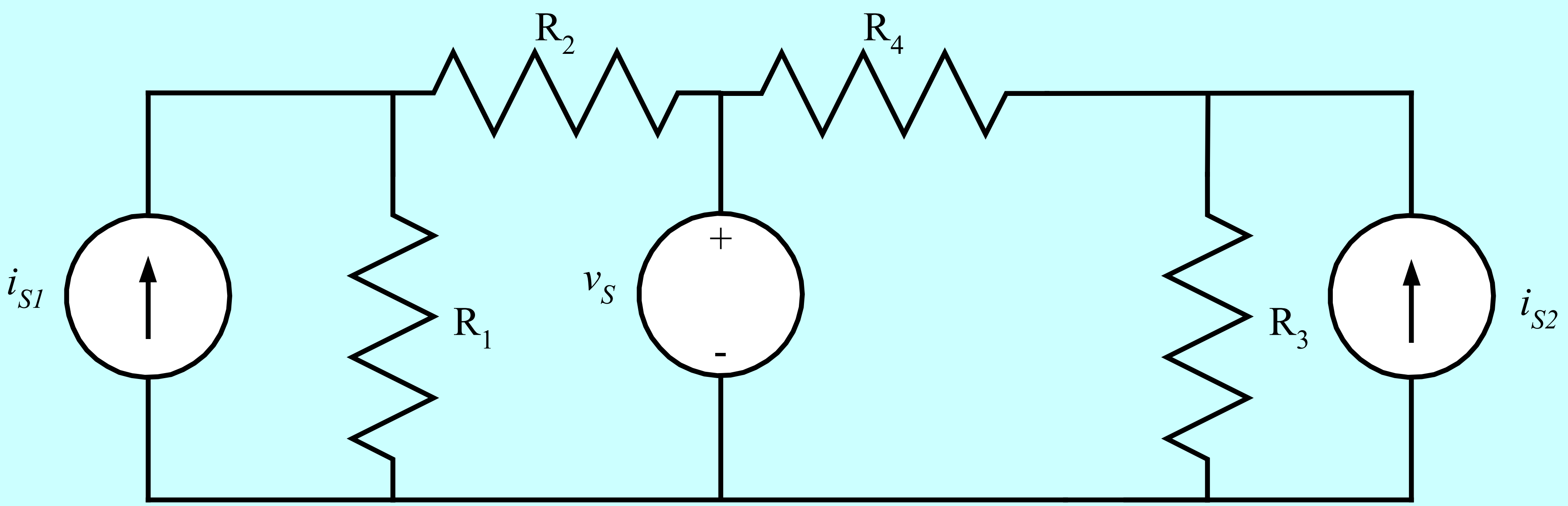
1. Redraw the circuit in planar form, if necessary. (If we cannot do this, we cannot use the MCM.)
2. Define the mesh currents, by labeling them. This includes showing the polarity of each mesh current.
3. Apply KVL for each mesh.
4. Write an equation for each current or voltage upon which dependent sources depend, as needed.

The solution for what to do when there is a current source present depends on how it appears. There are two possibilities. We will handle each of them in turn. The two possibilities are:

1. A current source as a part of only one mesh
2. A current source as a part of two meshes

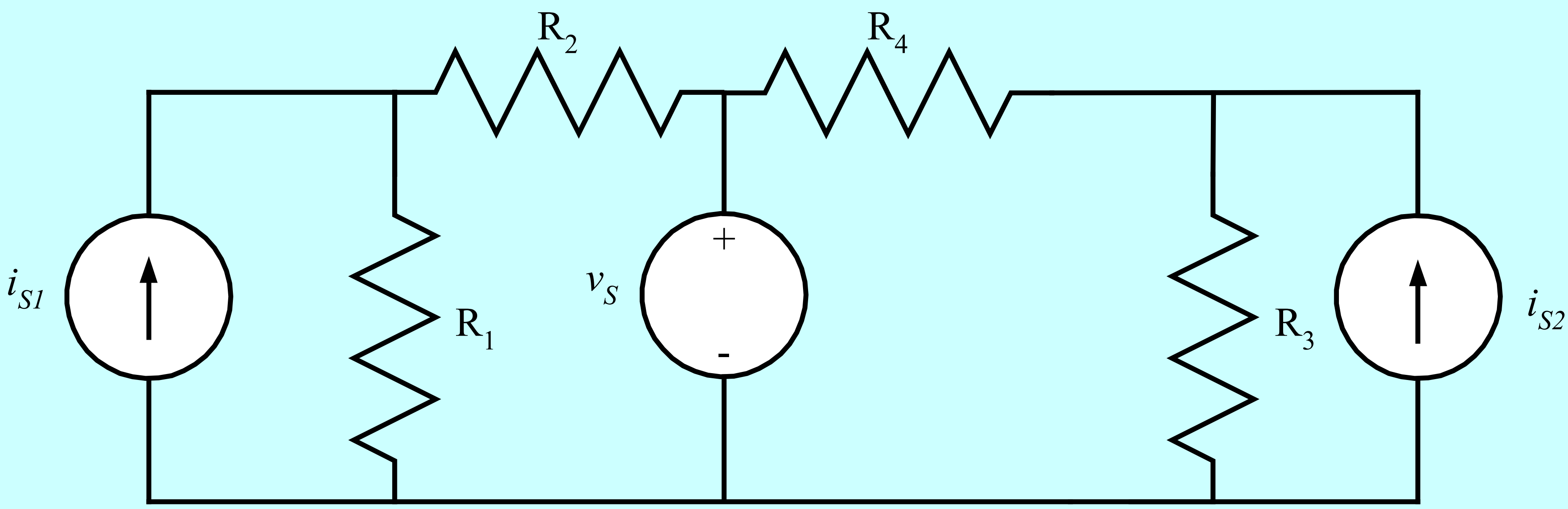
MCM – Current Source as a Part of Only One Mesh

Again, it seems to be best to study the MCM by doing examples. Our next example circuit is given here. We will go through the entire solution, but our emphasis will be on step 3. Note that here the current sources i_{S1} and i_{S2} are each a part of only one mesh.



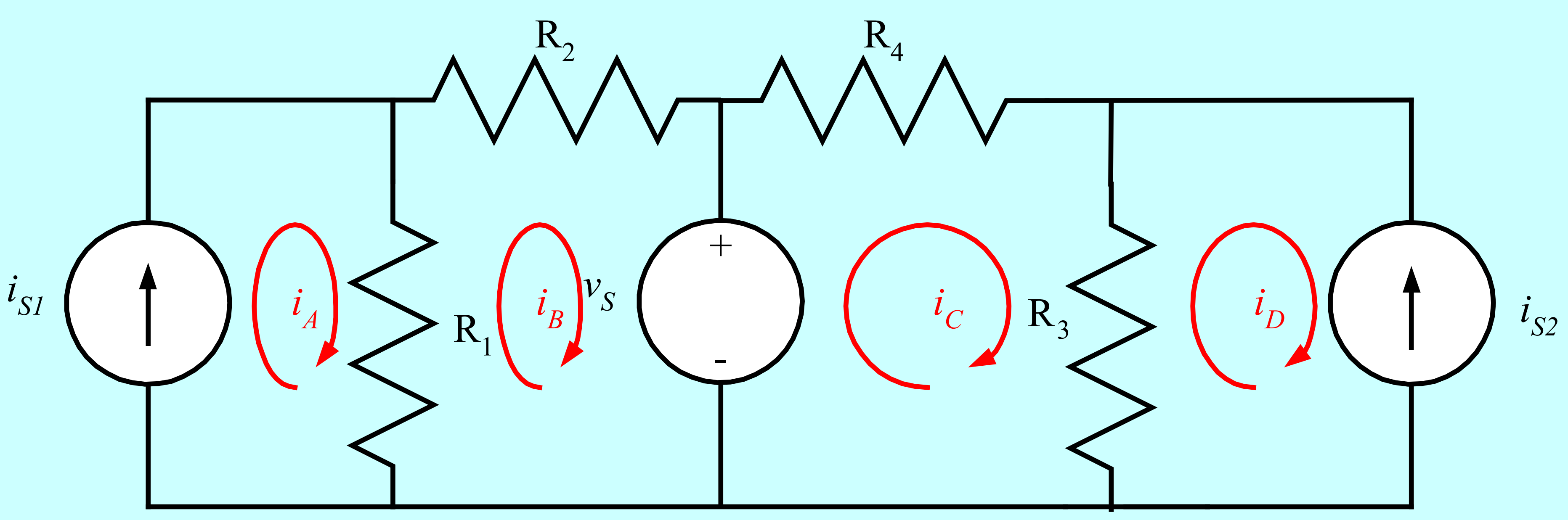
MCM – Current Source as a Part of Only One Mesh – Step 1

The first step is to redraw the circuit in planar form. This circuit is already in planar form, and this step can be skipped for this circuit.



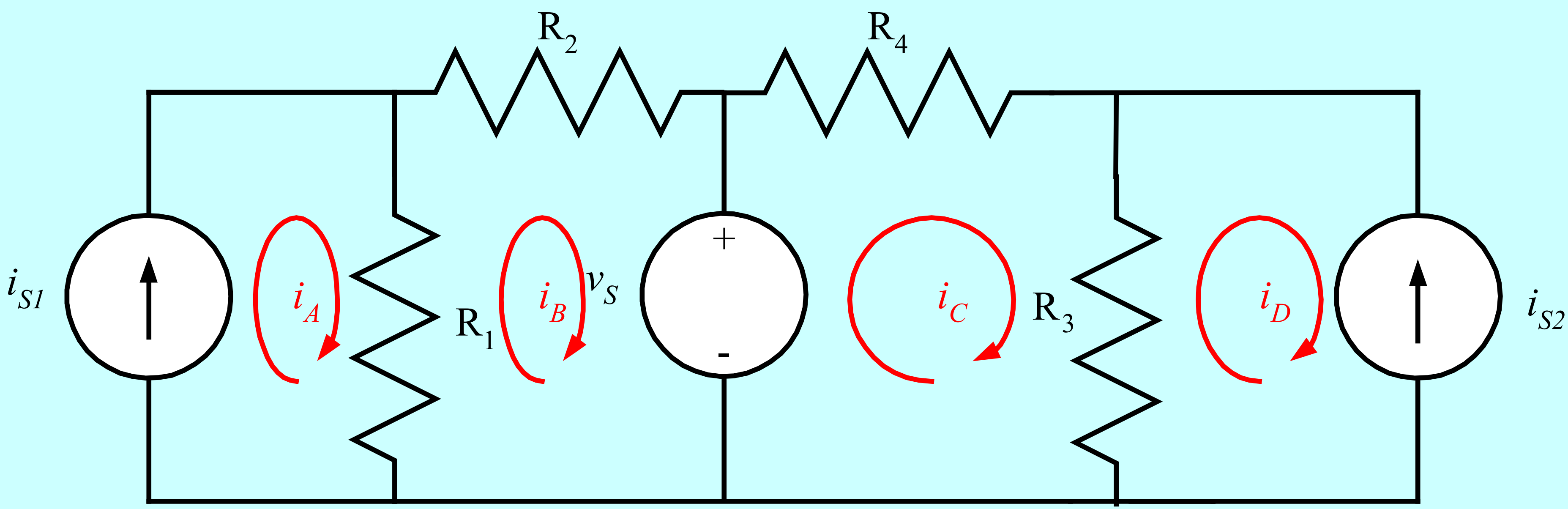
MCM – Current Source as a Part of Only One Mesh – Step 2

The second step is to define the mesh currents. This has been done in the circuit below.



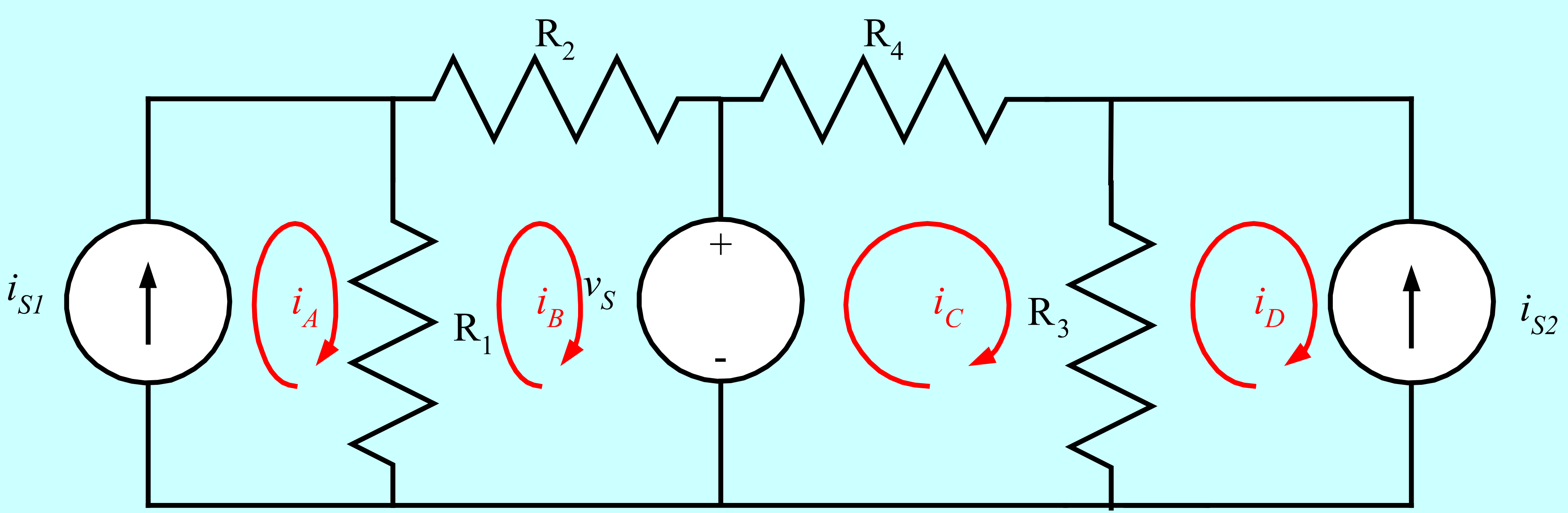
MCM – Current Source as a Part of Only One Mesh – Step 3 – Part 1

The third step is to write KVL for meshes A, B, C, and D. We can write KVL equations for meshes B and C using the techniques we have already, but for A and D we will get into trouble since the voltages across the current sources are not known, and cannot be easily given in terms of the mesh currents.



MCM – Current Source as a Part of Only One Mesh – Step 3 – Part 2

We can write KVL equations for meshes B and C using the techniques we had already, but for meshes A and D we will get into trouble. However, we do know something useful; the current sources determine each of the mesh currents in meshes A and D. This can be used to get the equations we need.



MCM – Current Source as a Part of Only One Mesh – Step 3 – Part 3

We can write the following equations:

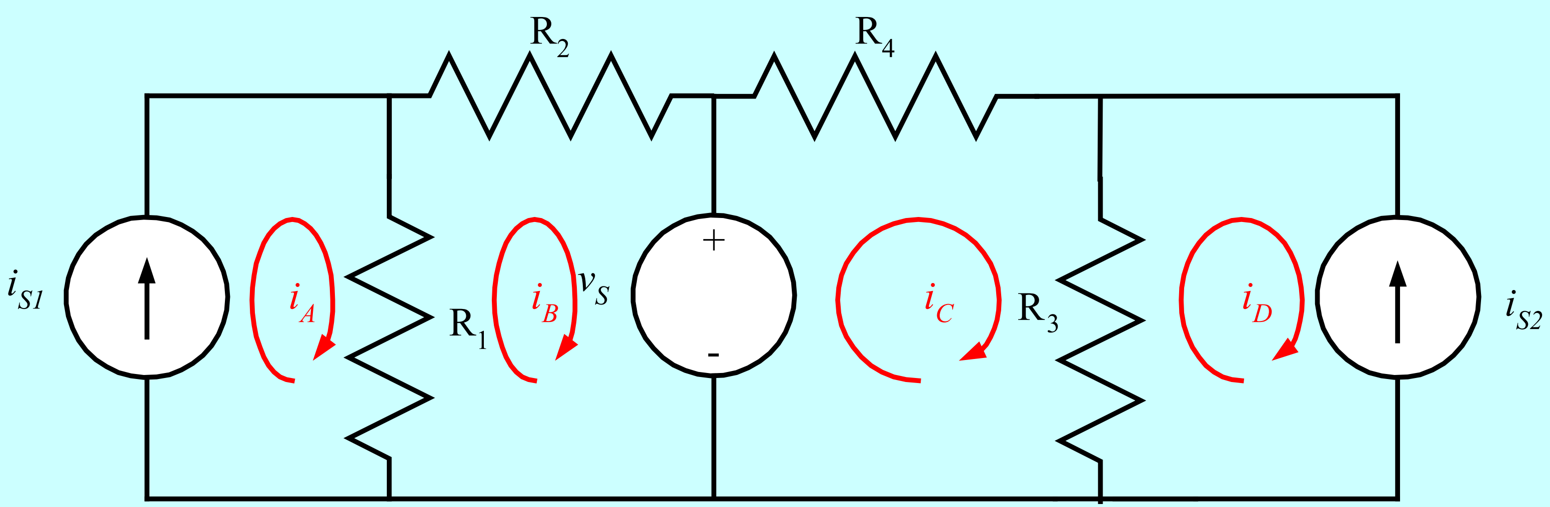
A: $i_A = i_{S1}$

B: $(i_B - i_A)R_1 + i_B R_2 + v_S = 0$

C: $-v_S + i_C R_4 + (i_C - i_D)R_3 = 0$

D: $i_D = -i_{S2}$

These equations indicate that the mesh current i_A is equal to the current source i_{S1}, and that the mesh current i_D is equal to but opposite in sign of the current source i_{S2}. Take care about the signs in these equations.



MCM – Current Source as a Part of Only One Mesh – Step 4

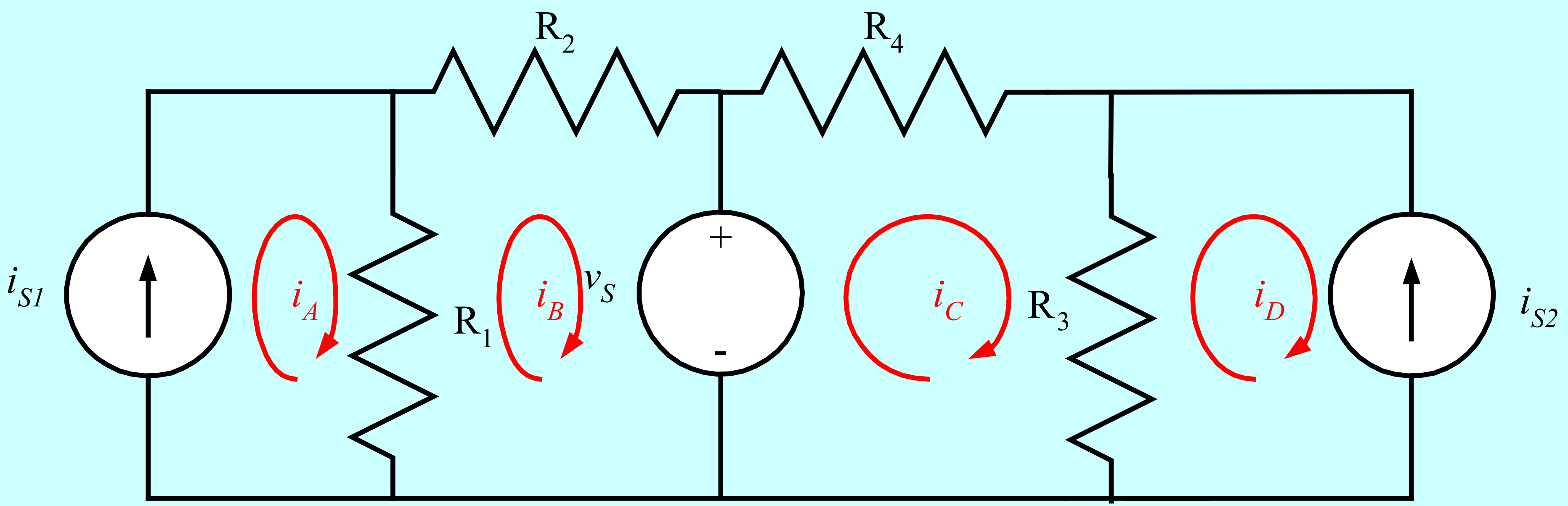
A: $i_A = i_{S1}$

B: $(i_B - i_A)R_1 + i_B R_2 + v_S = 0$

C: $-v_S + i_C R_4 + (i_C - i_D)R_3 = 0$

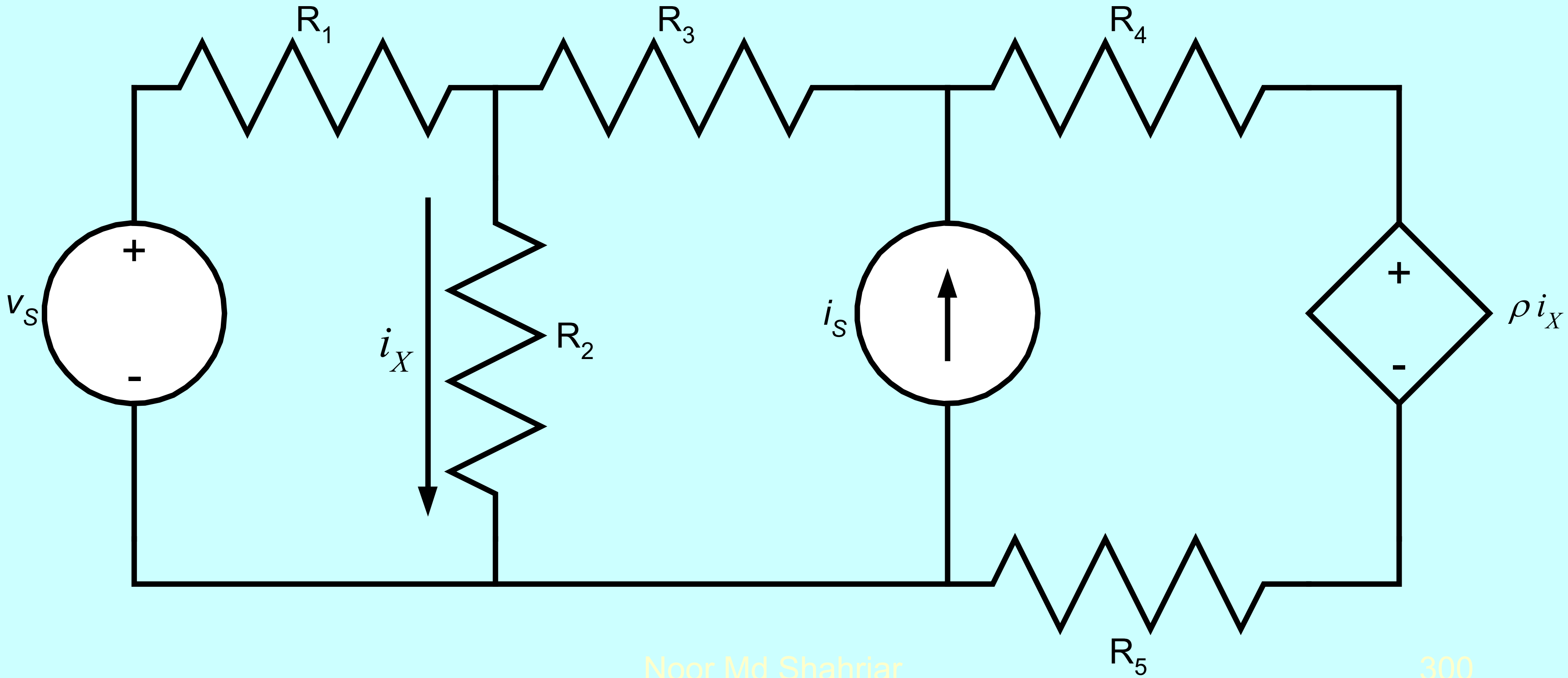
D: $i_D = -i_{s2}$

There are no dependent sources here, so we are done.



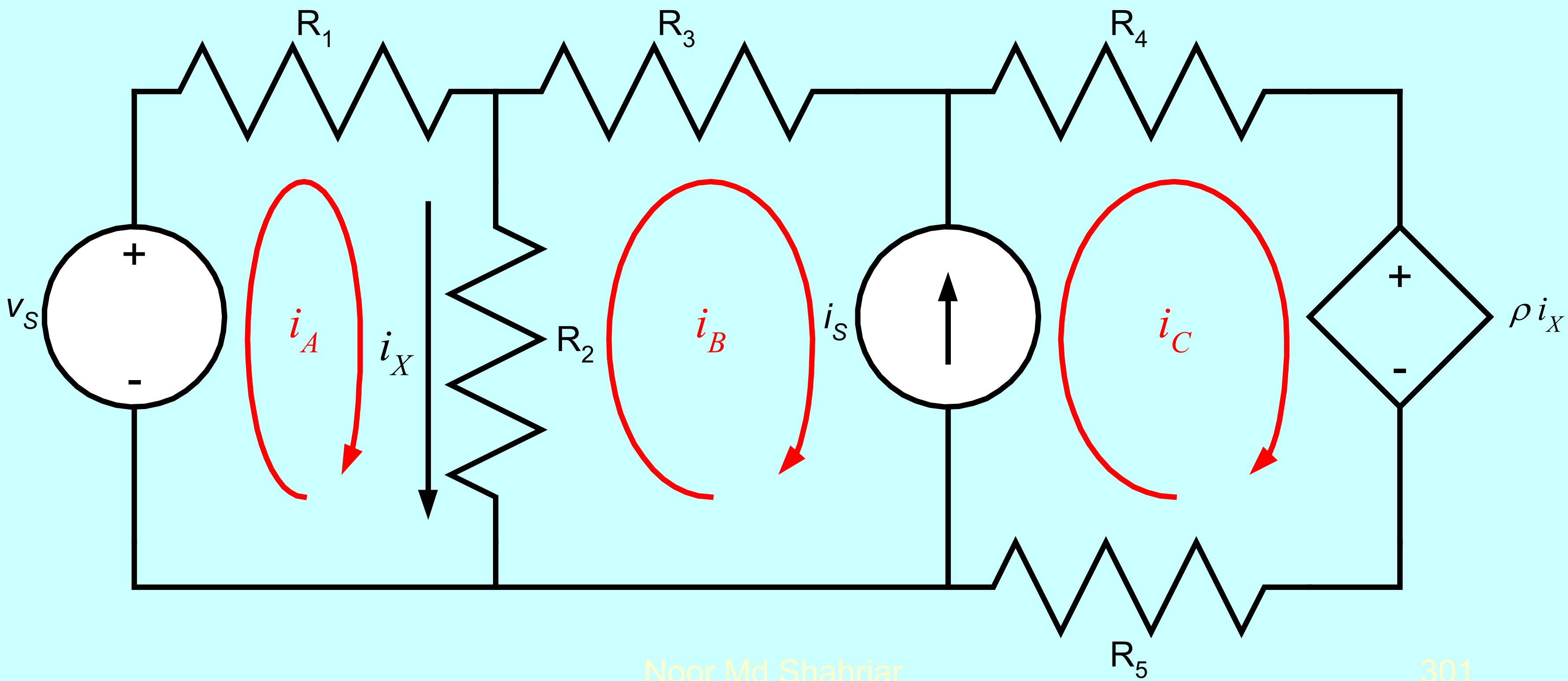
MCM – Current Source as a Part of Two Meshes

Again, it seems to be best to study the MCM by doing examples. Our next example circuit is given here. We will go through the entire solution, but our emphasis will be on step 3. Note that here the current source i_S is a part of two meshes.



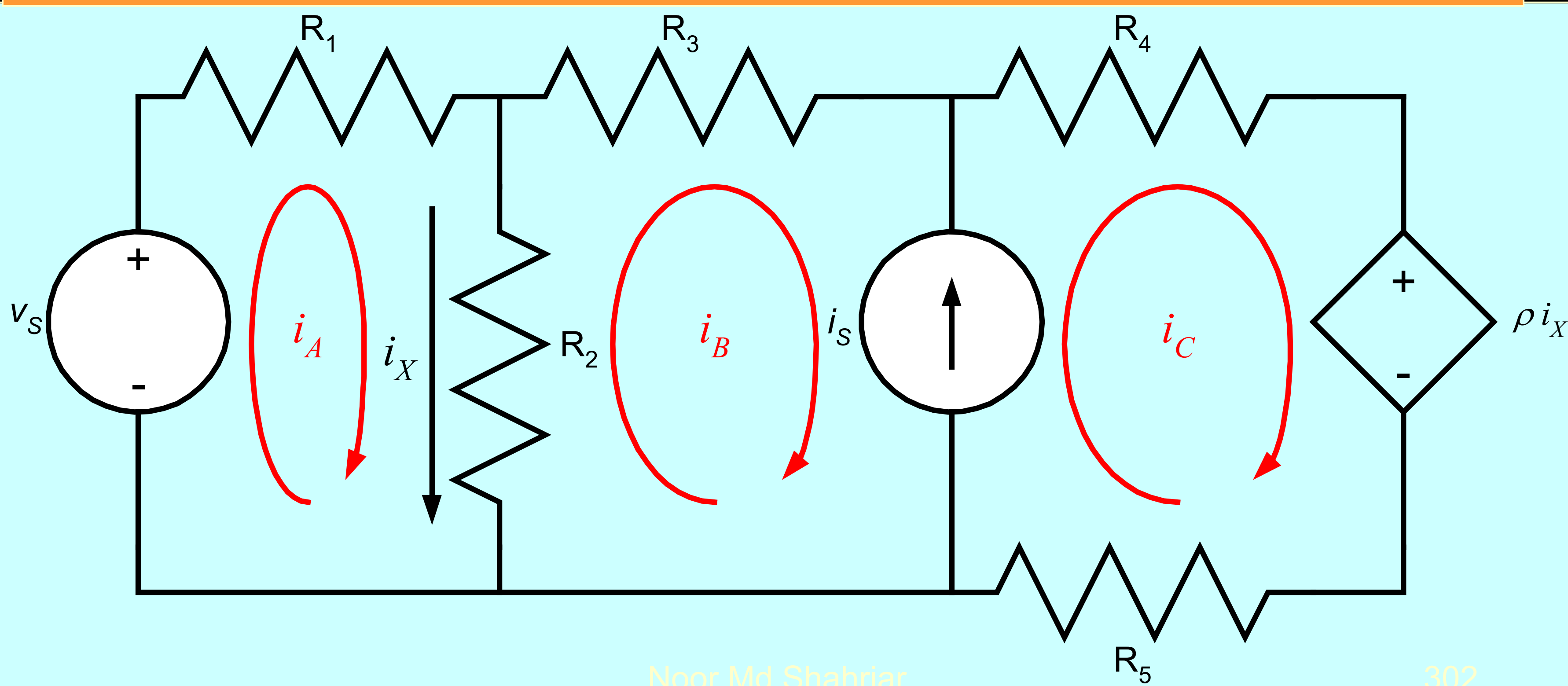
MCM – Current Source as a Part of Two Meshes – Steps 1 and 2

Since we have done similar circuits already, we have completed steps 1 and 2 in this single slide. It was already drawn in planar form. We defined three mesh currents.



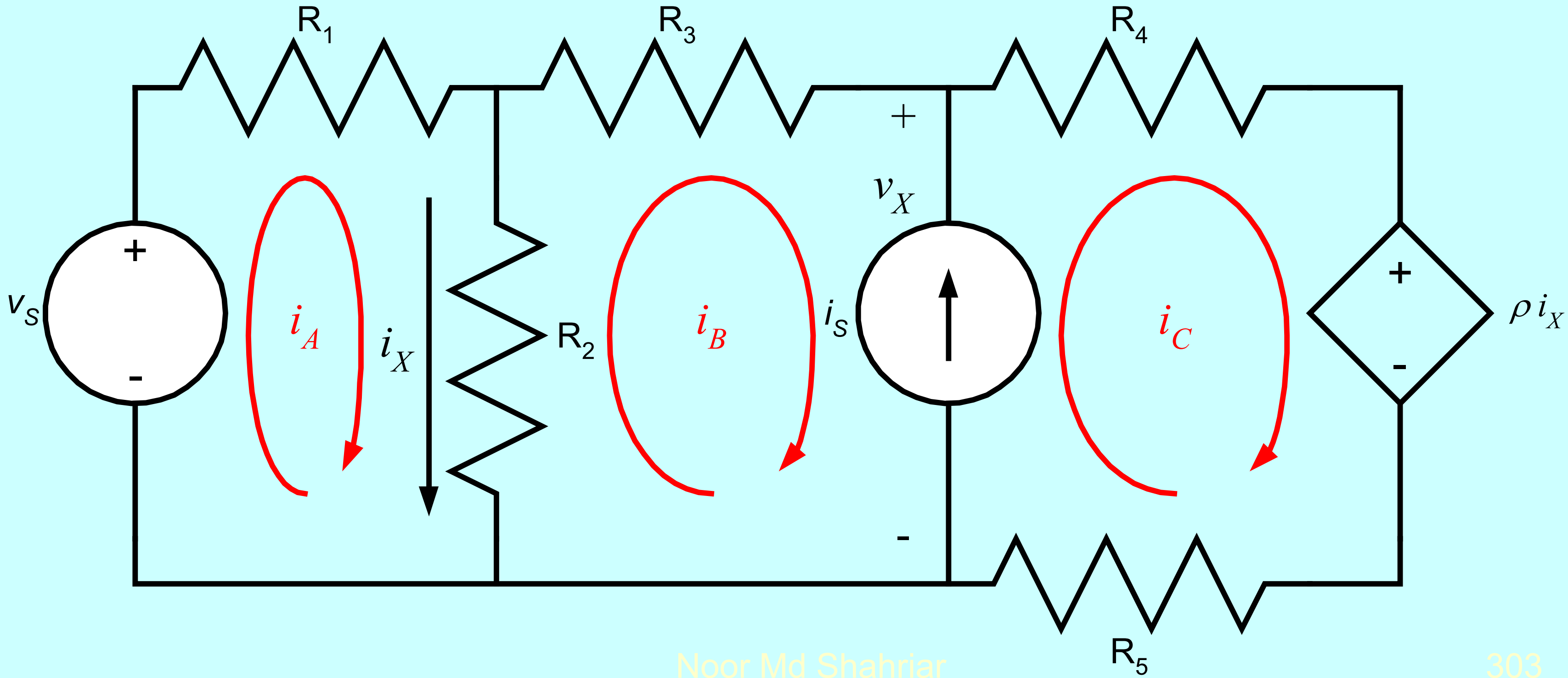
MCM – Current Source as a Part of Two Meshes – Step 3 – Part 1

Now we want to write KVL equations for the three meshes, A, B, and C. However, we will have difficulties writing the equations for meshes B and C, because the current source can have any voltage across it. In addition, we note that i_S is not equal to i_B, nor to $-i_B$, nor is it equal to i_C.



MCM – Current Source as a Part of Two Meshes – Step 3 – Part 2

We are going to take a very deliberate approach to this case, since many students find it difficult. To start, we will assume that we were willing to introduce an additional variable. (We will later show that we do not have to, but this is just to explain the technique.) We define the voltage across the current source to be v_X.

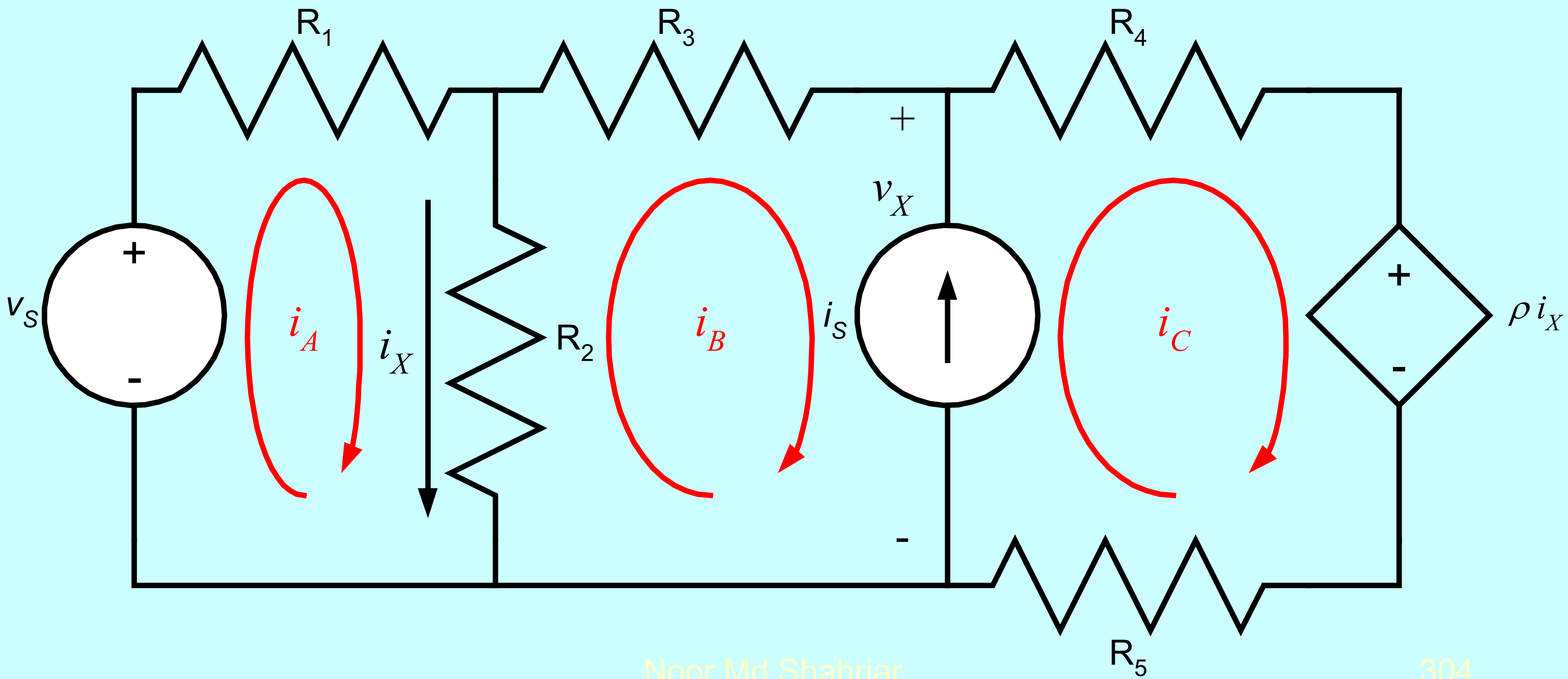


MCM – Current Source as a Part of Two Meshes – Step 3 – Part 3

Now, we can write KVL equations for meshes B and C, using v_X.

$$\text{B:}\quad (i_B - i_A)R_2 + i_B R_3 + v_X = 0, \text{ and}$$

$$\text{C:}\quad -v_X + i_C R_4 + \rho i_X + i_C R_5 = 0.$$



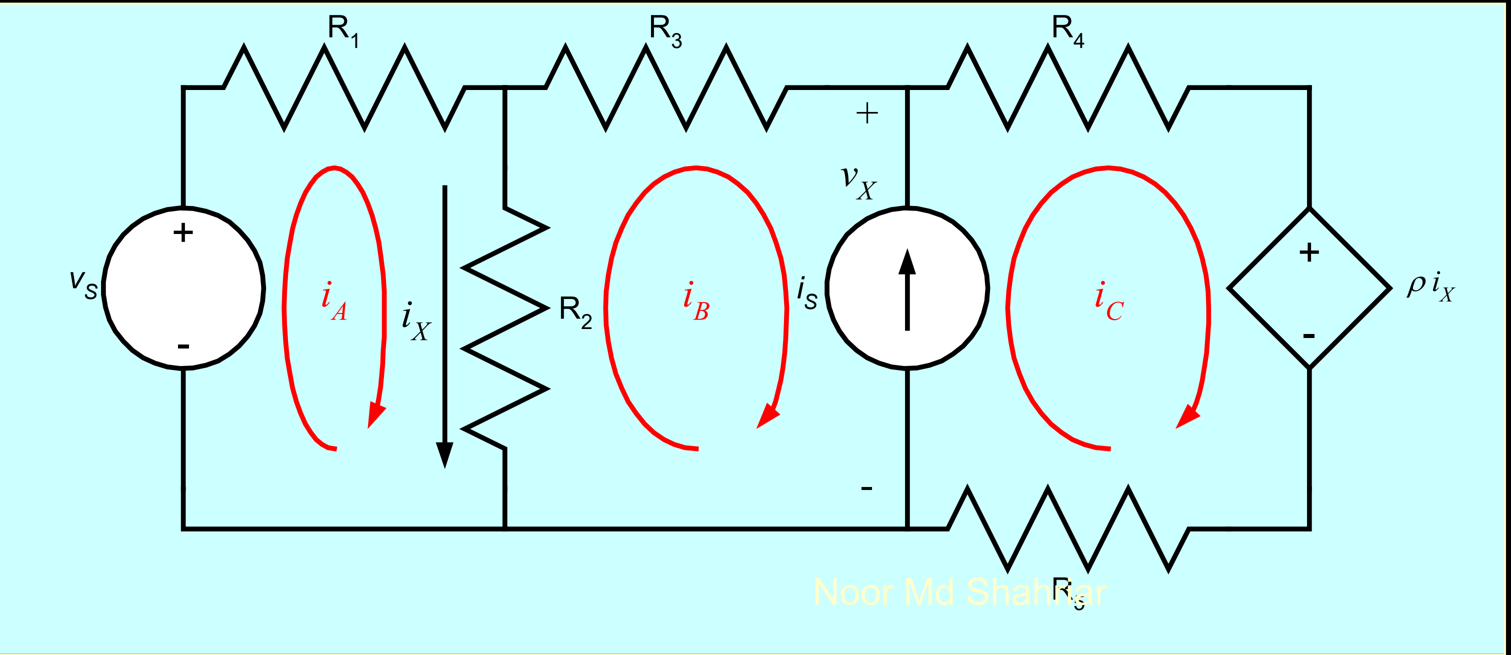
MCM – Current Source as a Part of Two Meshes – Step 3 – Part 4

Now, remember that we did not want to use the variable v_X. If we examine the equations that we have just written, we note that we can eliminate v_X by adding the two equations together. We add the B equation to the C equation, and get:

$$\text{B:}\quad (i_B - i_A)R_2 + i_B R_3 + v_X = 0$$

$$\text{+C:}\quad -v_X + i_C R_4 + \rho i_X + i_C R_5 = 0, \text{ which gives}$$

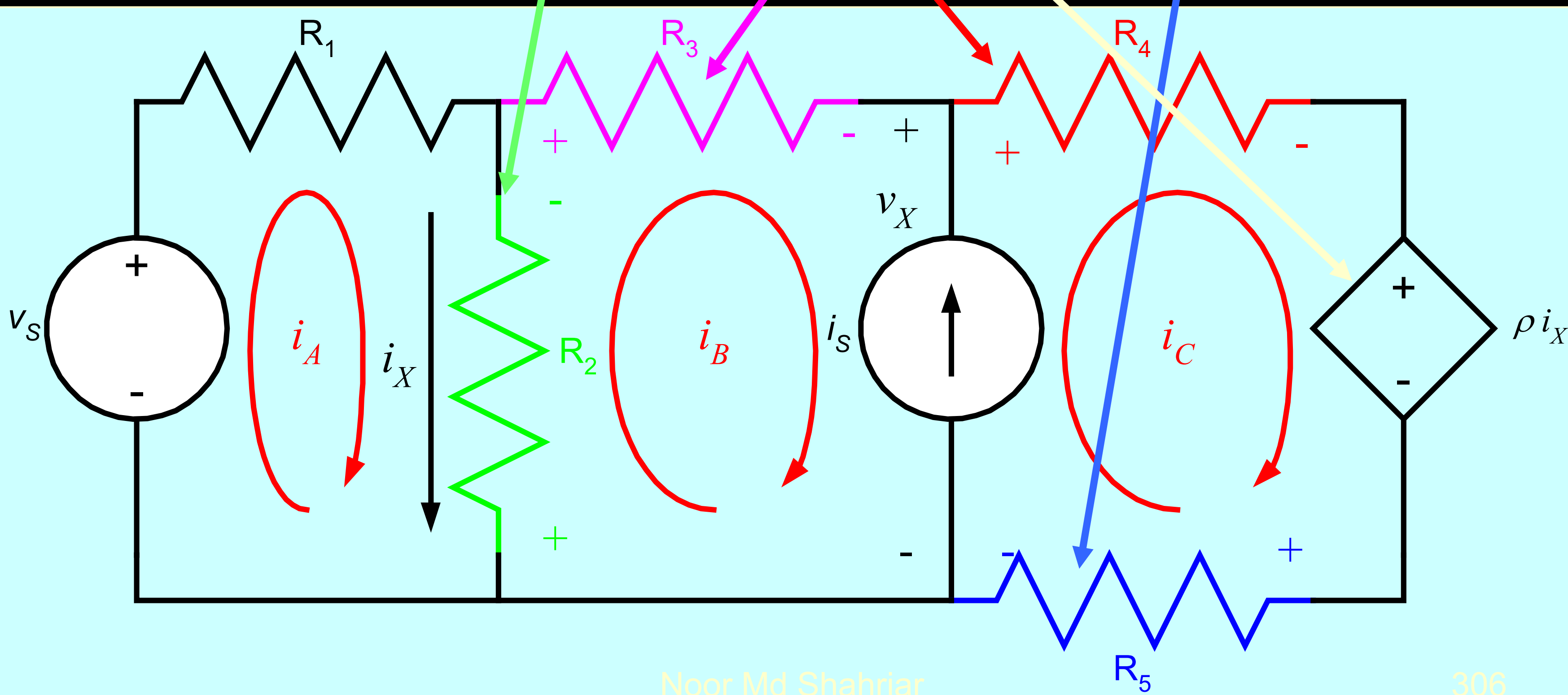
$$\text{B+C:}\quad (i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0.$$



MCM – Current Source as a Part of Two Meshes – Step 3 – Part 5

Next, we examine this new equation that we have titled B+C. If we look at the circuit, this is just KVL applied to a closed path that surrounds the current source. The correspondence between voltages and KVL terms is shown with colors.

B+C: $(i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$



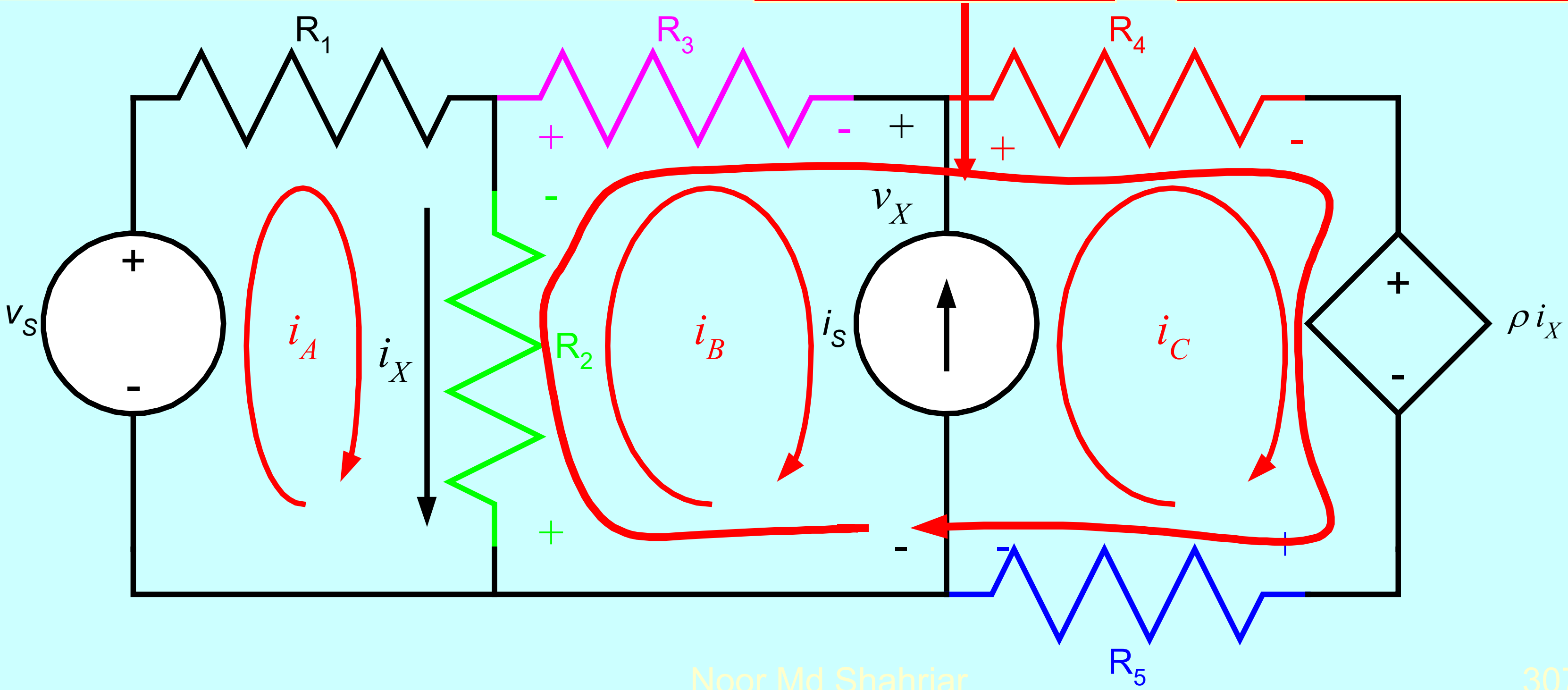
MCM – Current Source as a Part of Two Meshes – Step 3 – Part 6

The large closed path that includes the current source is called a **Supermesh**. We will call the KVL equation that we write for this closed path a **Supermesh Equation**.

B+C: $(i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$

Supermesh

Supermesh Equation



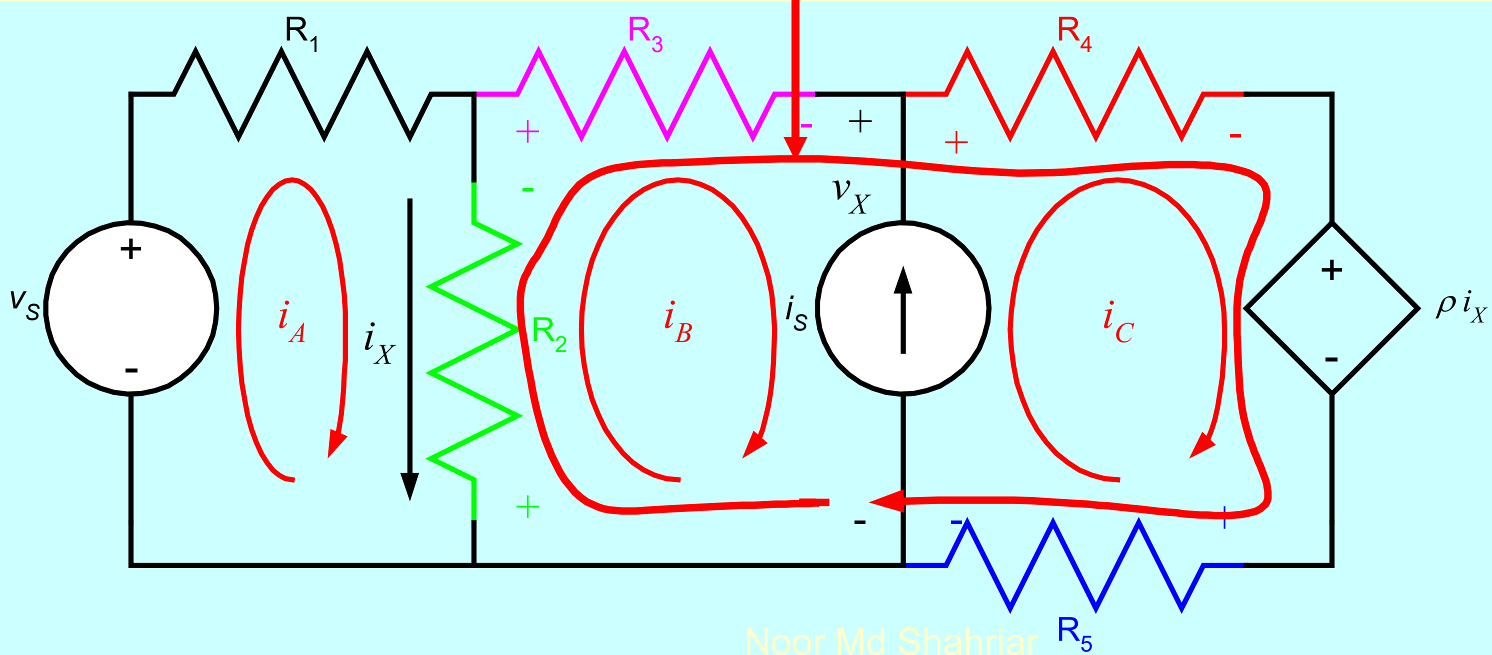
MCM – Current Source as a Part of Two Meshes – Step 3 – Part 7

The Supermesh Equation is fine, but it is not enough. With the equation for mesh A, and for variable i_X, we still only have three equations, and four unknowns. We need one more equation.

$$\text{B+C:}\quad (i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$$

Supermesh

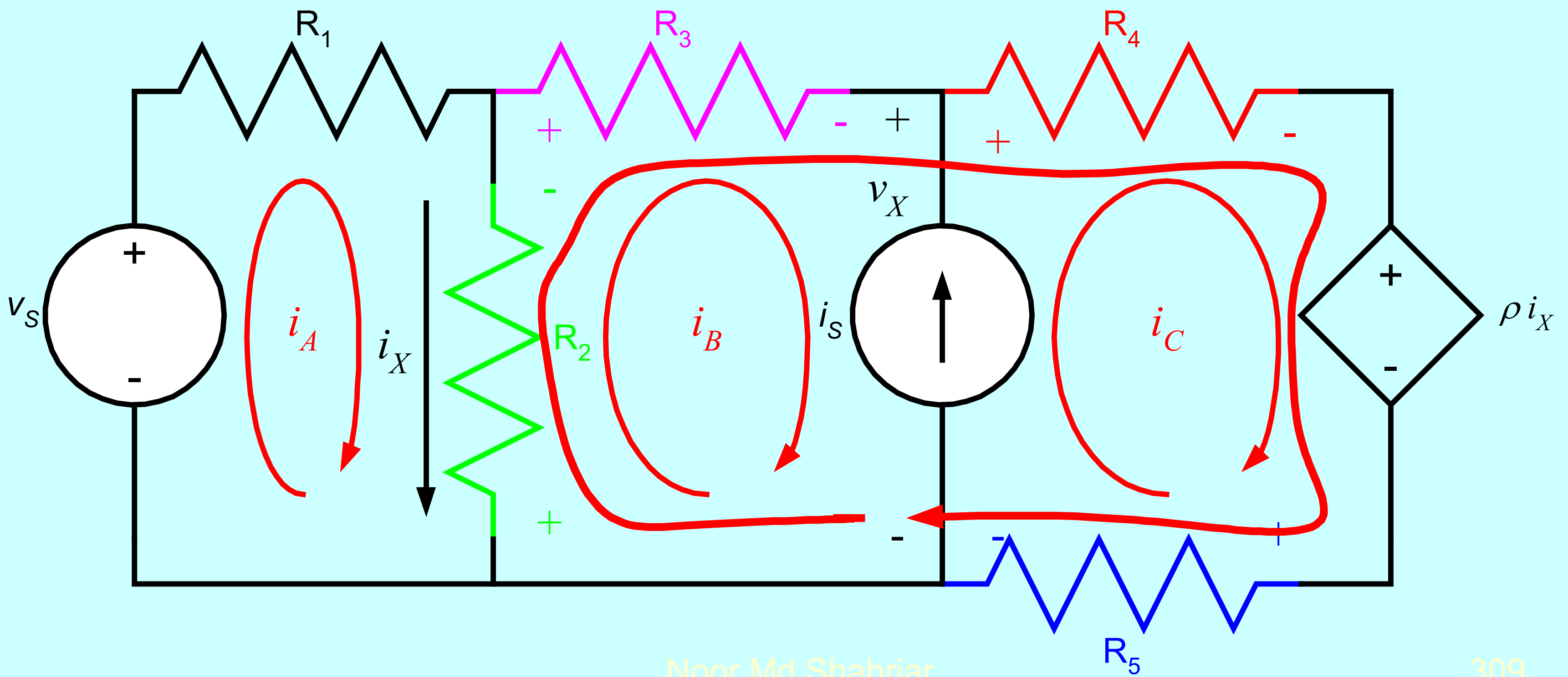
Supermesh Equation



MCM – Current Source as a Part of Two Meshes – Step 3 – Part 8

We need one more equation. We now note that we have not used the value of the current source, which we expect to influence the solution somehow. Note that the current source determines the difference between i_B and i_C.

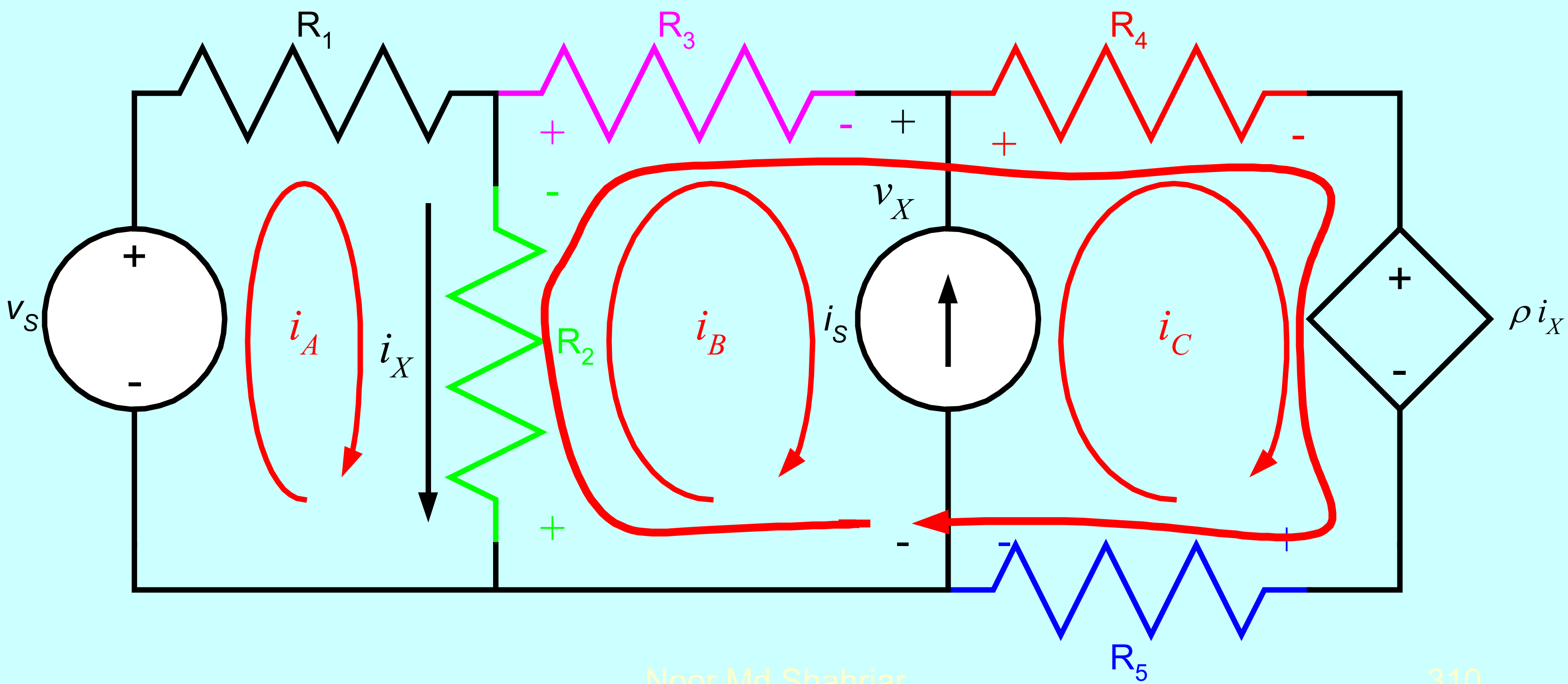
$$\text{B+C:}\quad (i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$$



MCM – Current Source as a Part of Two Meshes – Step 3 – Part 9

The current source determines the difference between i_B and i_C. We can use this to write the fourth equation we need. We write the following equation.

B+C: $i_C - i_B = i_S$.



MCM – Current Source as a Part of Two Meshes – Steps 3&4 – Part 10

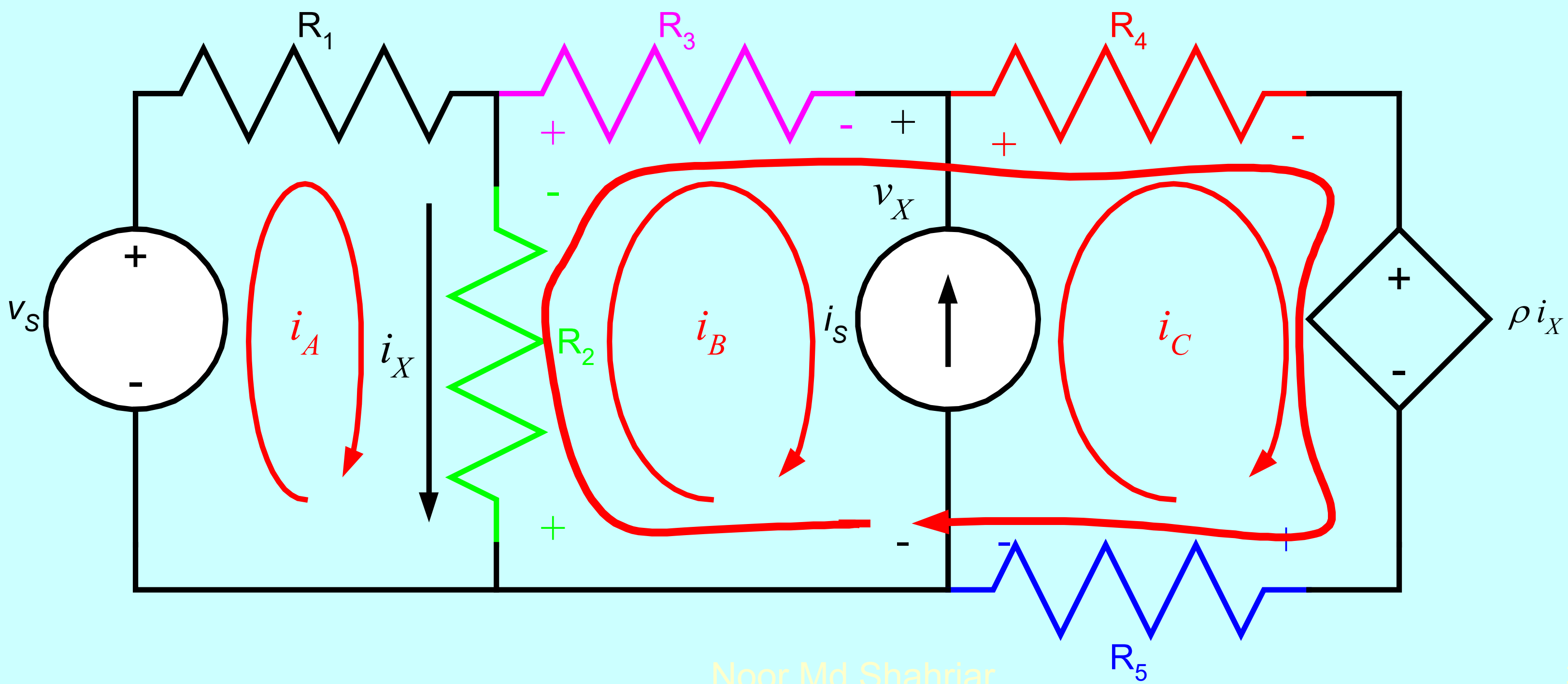
To complete the set of equations, we write the KVL equation for mesh A, and an equation for variable i_X. That gives us four equations in four unknowns, or

$$\text{A:}\quad -v_S + i_A R_1 + (i_A - i_B)R_2 = 0,$$

$$\text{B+C:}\quad (i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0,$$

$$\text{B+C:}\quad i_C - i_B = i_S,\ \text{and}$$

$$i_X:\quad i_A - i_B = i_X.$$



MCM – Current Source as a Part of Two Meshes – Steps 3&4 – Part 11

To summarize our approach then, when we have a current source as a part of two meshes, we will

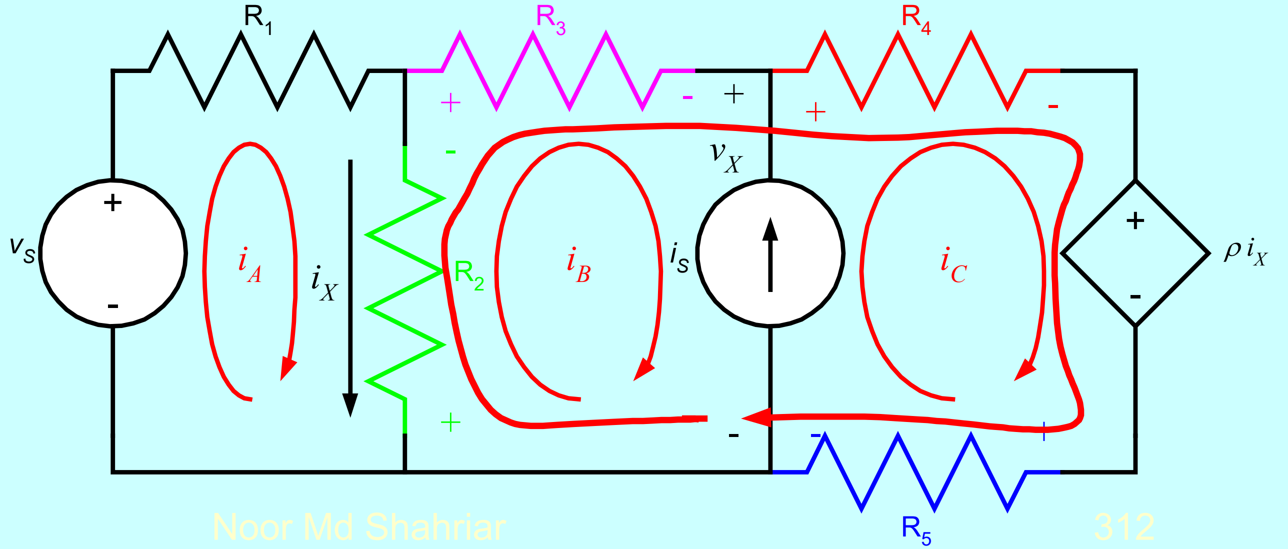
- write one equation applying KVL to a supermesh around the current source, and
- write an equation using the current source to relate the two mesh currents.

A: $-v_S + i_A R_1 + (i_A - i_B)R_2 = 0$

B+C: $(i_B - i_A)R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$

B+C: $i_C - i_B = i_S$

i_X: $i_A - i_B = i_X$



MCM – Current Source as a Part of Two Meshes – Steps 3&4 – Part 12

We write:

- one equation applying KVL to a supermesh around the current source, and
- one equation using the current source to relate the two mesh currents.

Supermesh Equation

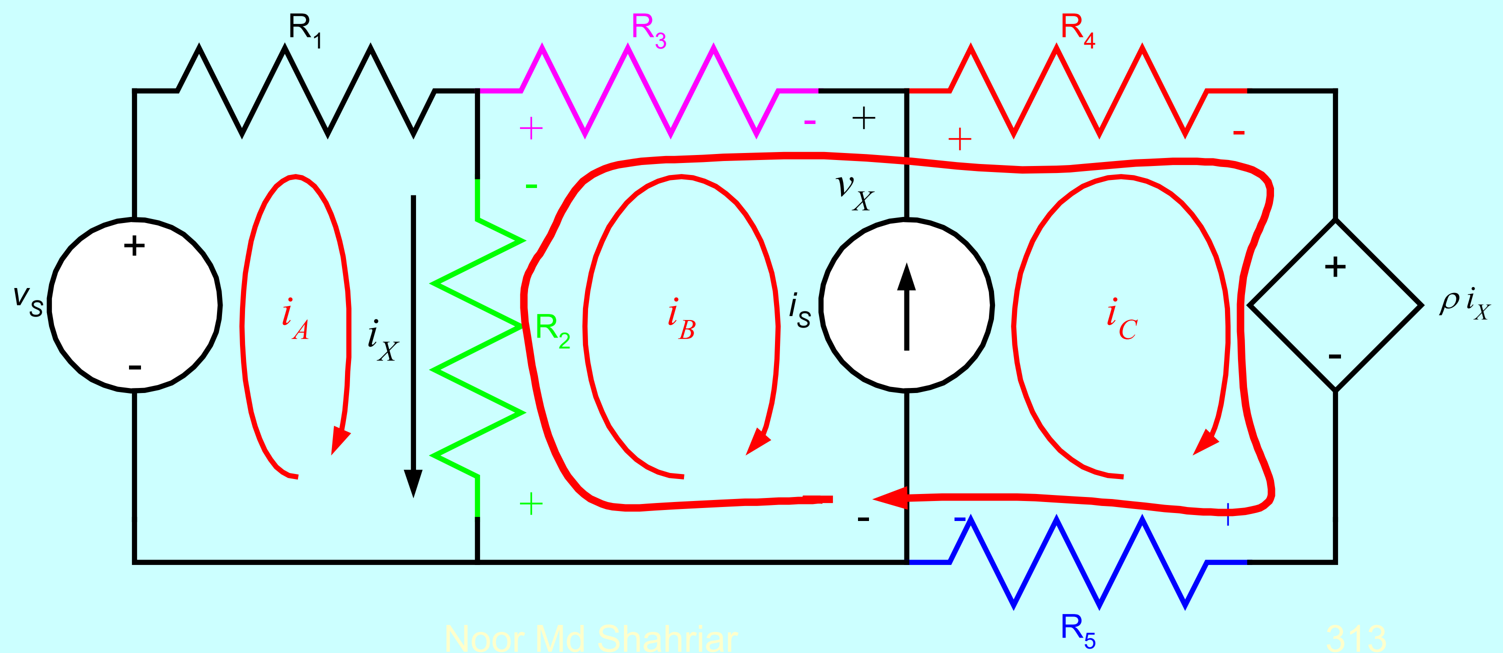
Constraint Equation

A: $-v_S + i_A R_1 + (i_A - i_B) R_2 = 0$

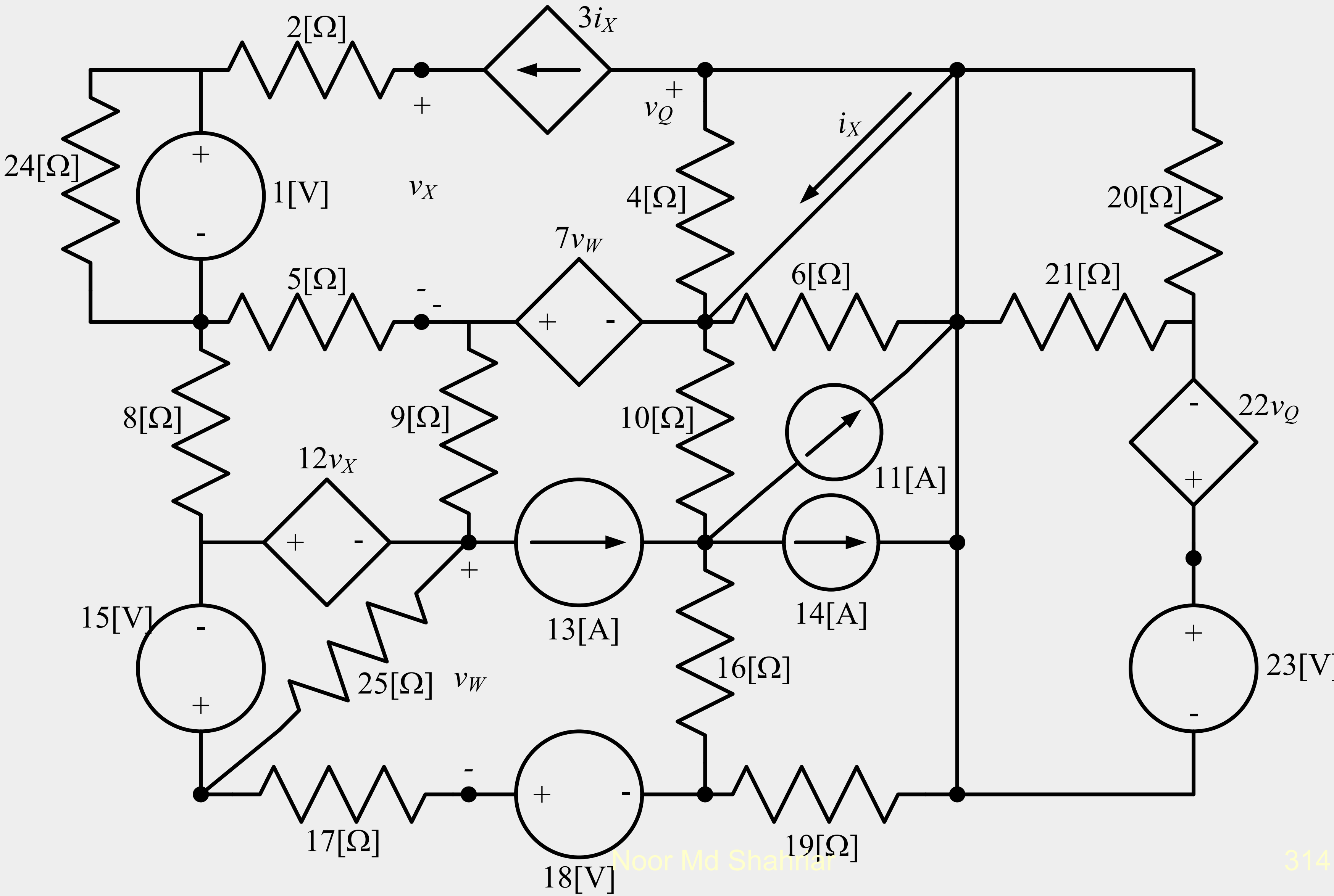
B+C: $(i_B - i_A) R_2 + i_B R_3 + i_C R_4 + \rho i_X + i_C R_5 = 0$

B+C: $i_C - i_B = i_S$

i_X: $i_A - i_B = i_X$



Example Problem: Use the mesh-current method to write a set of equations that could be used to solve the circuit below. Do not attempt to simplify the circuit. Do not attempt to solve the equations.

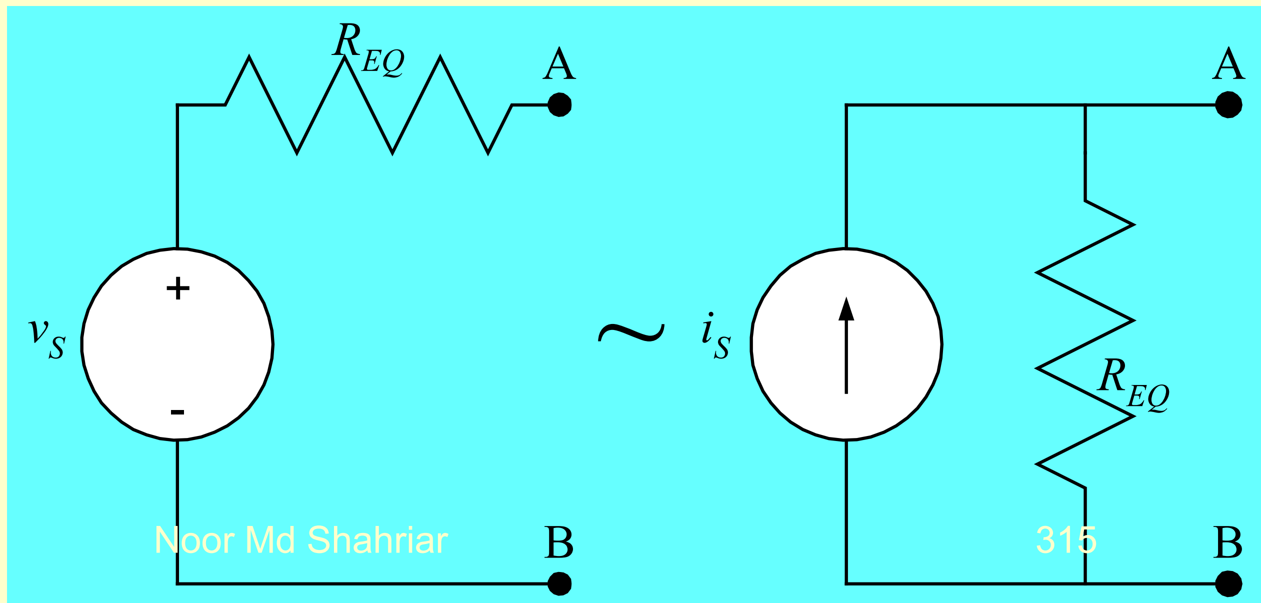


Source Transformations Defined

The equivalent circuits called the source transformation can be defined as follows:

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

$$v_S = i_S R_{EQ}.$$

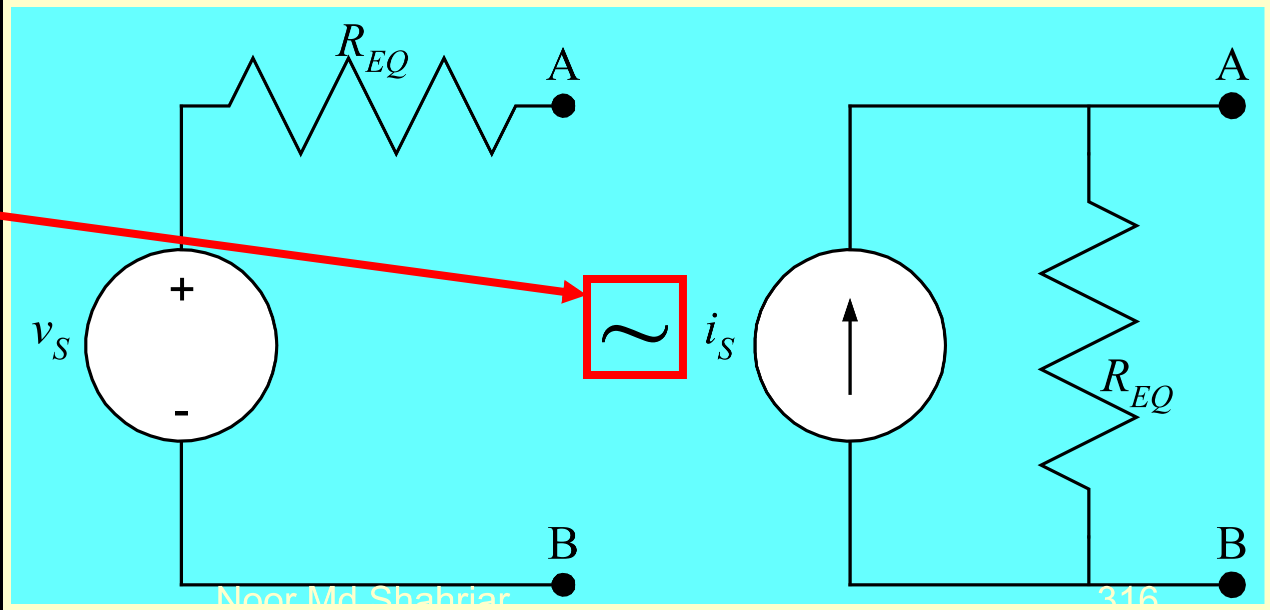


Notation

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

$$v_S = i_S R_{EQ}.$$

We have used the symbol "~" to indicate equivalence here. Some textbooks use a double-sided arrow (⇔ or ↔), or even a single-sided arrow (⇒ or →), to indicate this same thing.

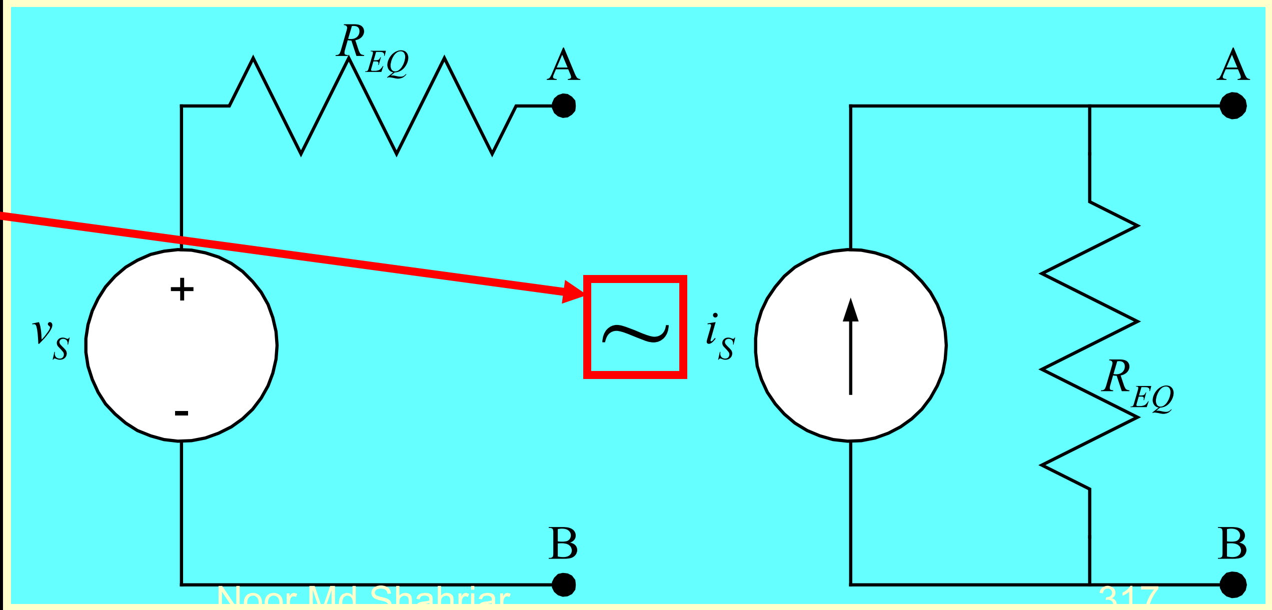


Note 1

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

$$v_S = i_S R_{EQ}.$$

This equivalence can go in either direction. That is, we can replace the circuit on the right with the one on the left, or the other way around. Neither one is simpler; we just prefer one or the other in some situations.

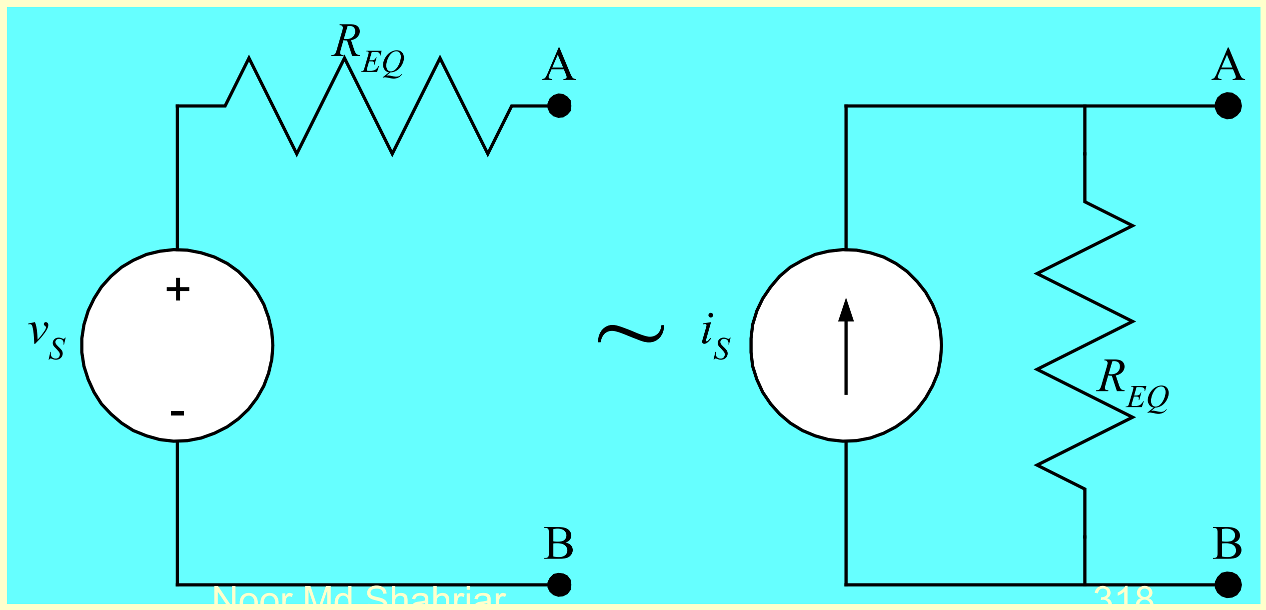


Note 2

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

This equation is not really Ohm's Law. It looks like Ohm's Law, and has the same form. However, it should be noted that Ohm's Law relates voltage and current for a resistor. This relates the values of sources and resistances in two different equivalent circuits. However, if you wish to remember this by relating it to Ohm's Law, that is fine.

$$v_S = i_S R_{EQ}.$$

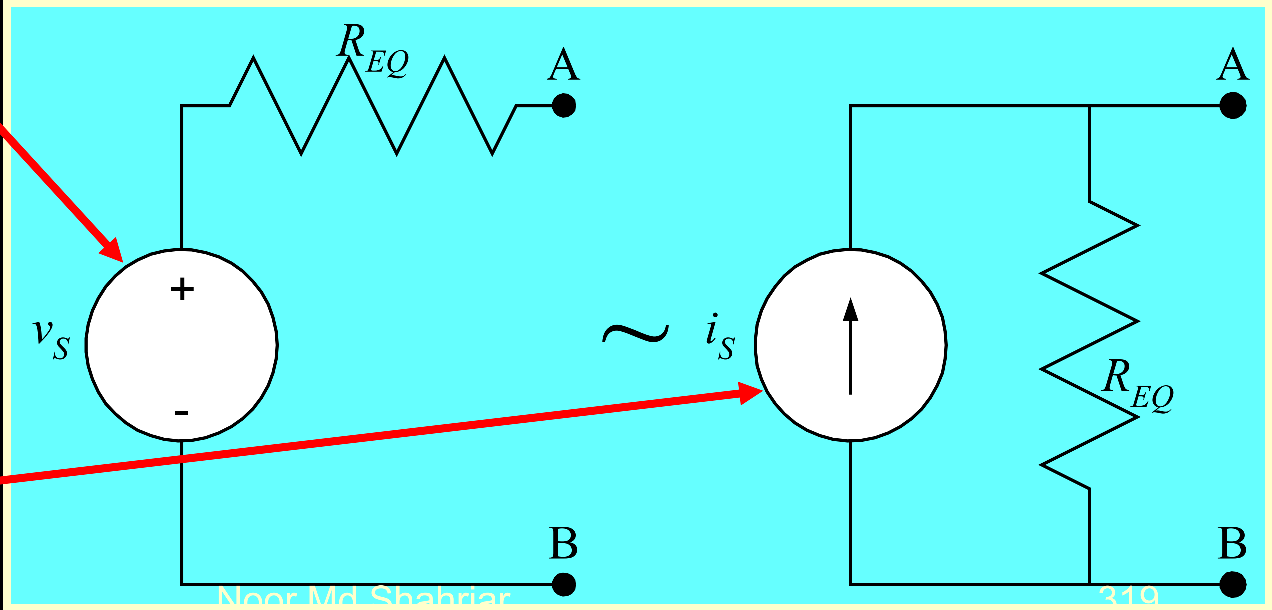


Note 3

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

$$v_S = i_S R_{EQ}.$$

The polarities of the sources with respect to the terminals is important. If the reference polarity for the voltage source is as given here (voltage drop from A to B), then the reference polarity for the current source must be as given here (current flow from B to A). This is one good reason for naming the terminals of these equivalents.

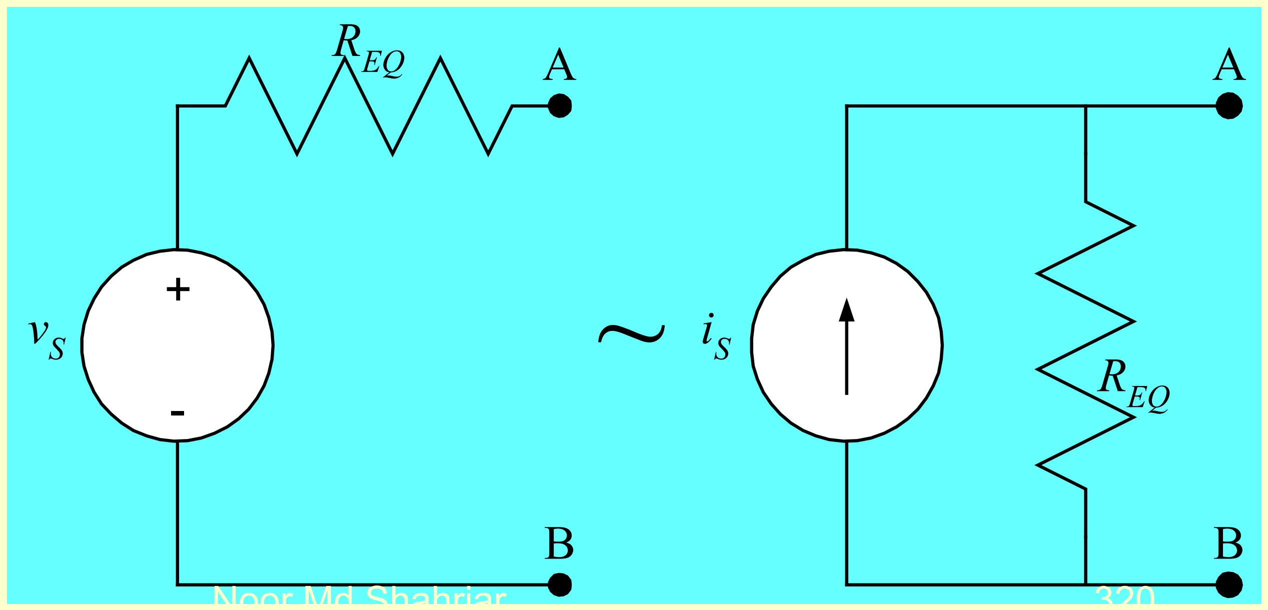


Note 4

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

$$v_S = i_S R_{EQ}.$$

As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalent circuits. For example, when these two equivalent circuits are connected to an open circuit, in one the resistor dissipates power, and in the other it does not.



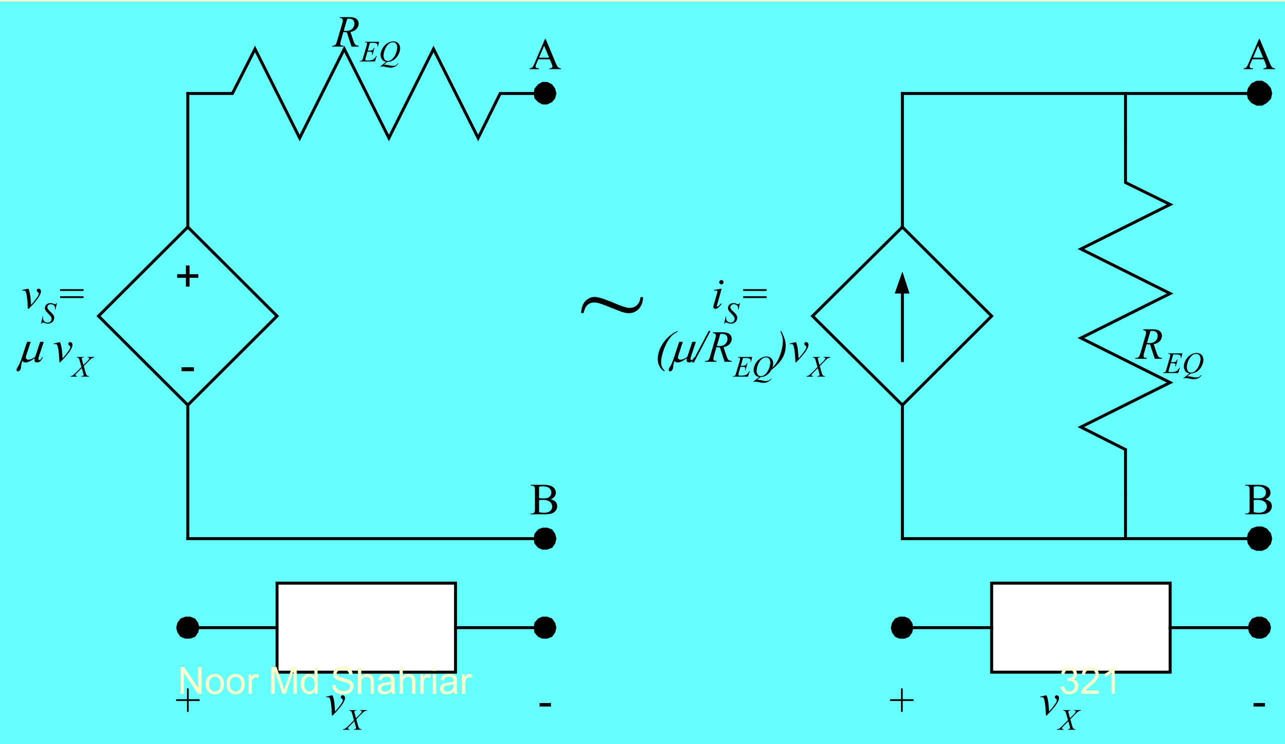
Note 5

Go back to Overview slide.

A portion of a circuit where we have a voltage source in series with a resistance is equivalent to current source in parallel with a resistance. The resistances for these two equivalents are equal. These two cases are equivalent as long as the resistances are equal and if the voltage source and current source are related by

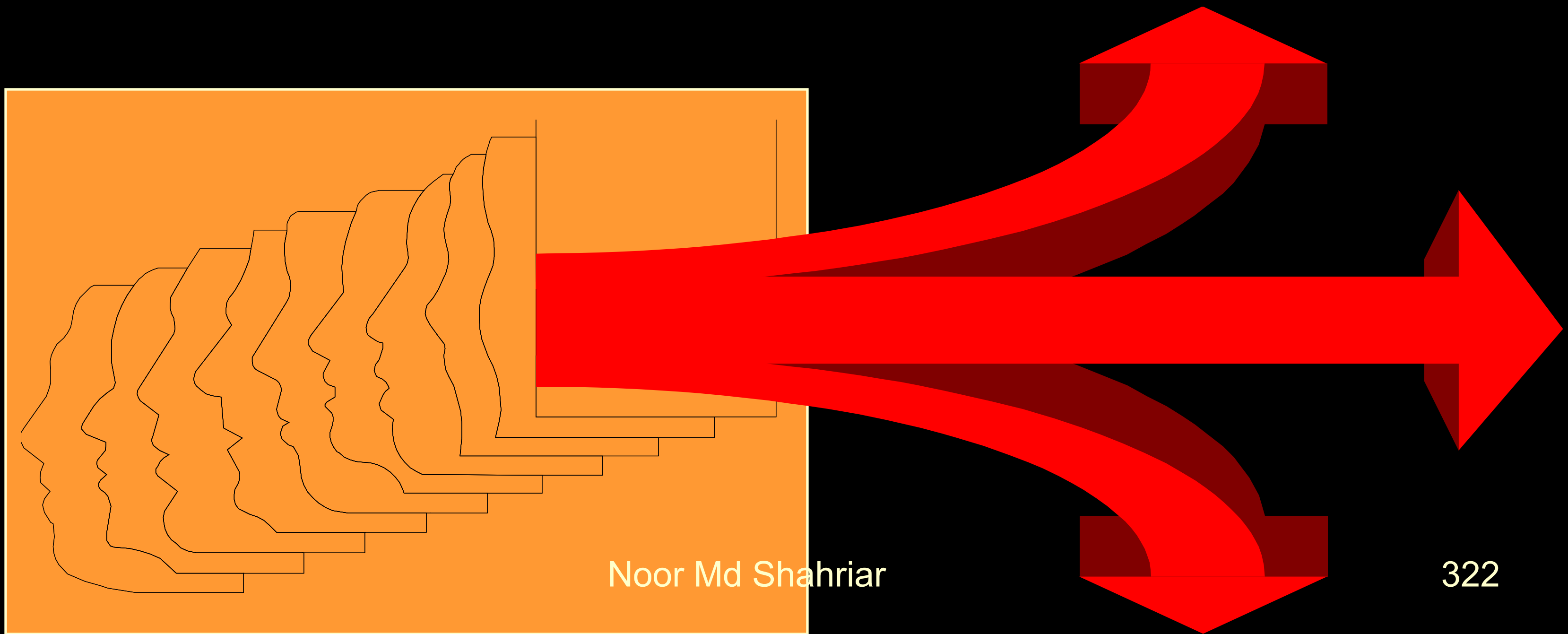
$$v_S = i_S R_{EQ}.$$

These equivalent circuits hold for dependent sources as well as independent sources. The key is that the variable, which the dependent sources depend on, must remain intact. That is, the voltage or current that the dependent sources use must be outside of the circuit being replaced.



Other Useful Transformations

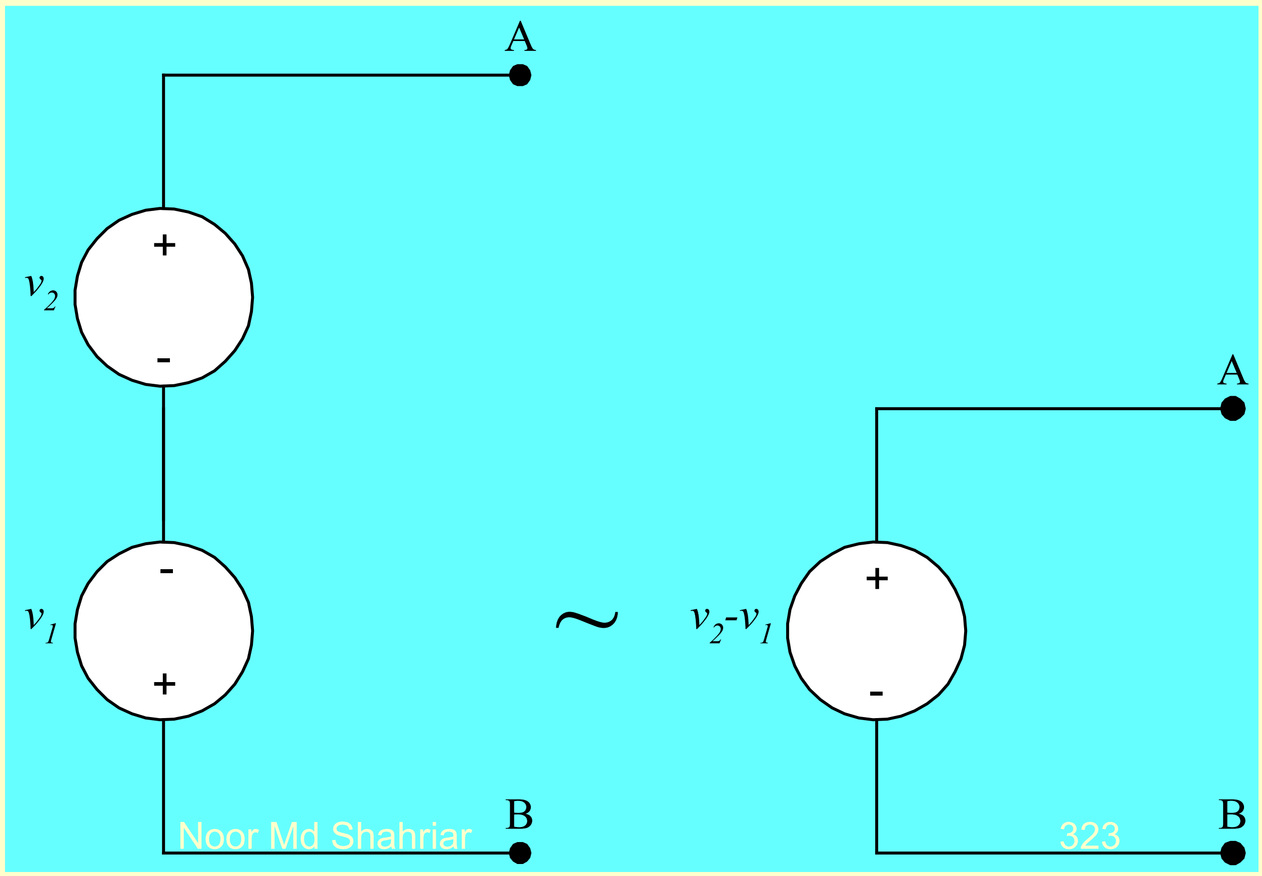
There are some other useful transformations, relating to sources, that can be defined at this point. These transformations do not have a common name, and in a sense they derive from the definitions of ideal voltage sources and ideal current sources. Still, since they are equivalent circuits relating sources, that have much the same form as source transformations, they are listed in the slides that follow.



Other Useful Transformations – 1

As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalents.

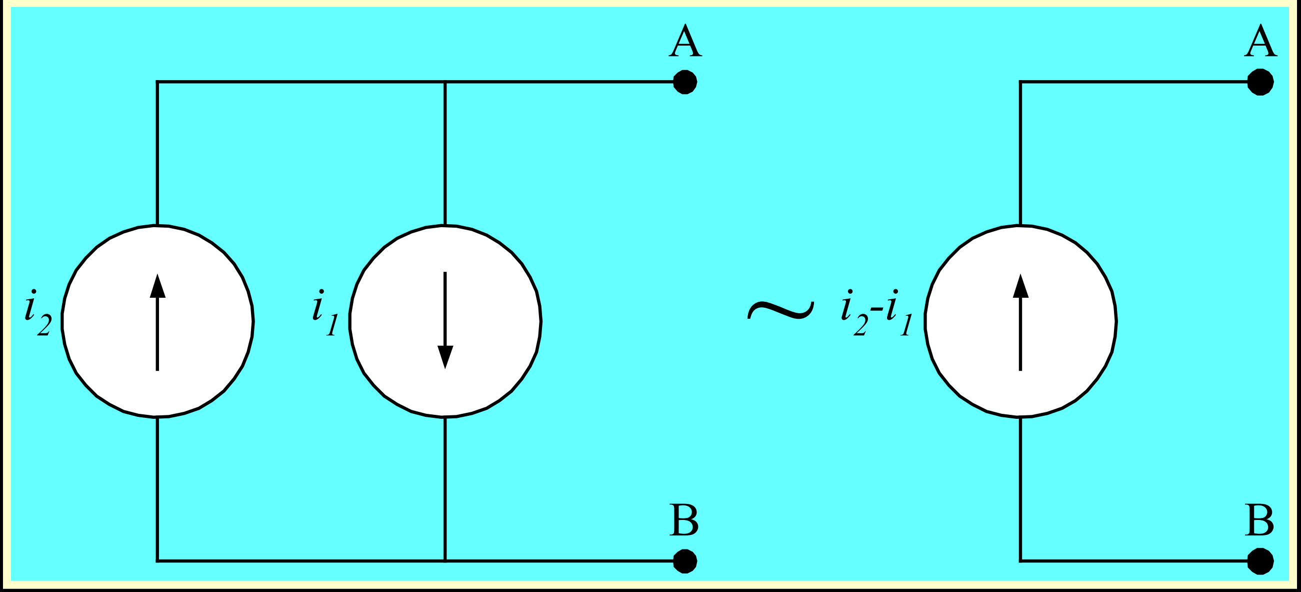
Voltage sources in series can be replaced by a single voltage source, where the value of the equivalent source is equal to the algebraic sum of the voltage sources it is replacing. An example is shown here with two sources with random polarities.



Other Useful Transformations – 2

Current sources in parallel can be replaced by a single current source, where the value of the equivalent source is equal to the algebraic sum of the current sources it is replacing. An example is shown here with two sources with random polarities.

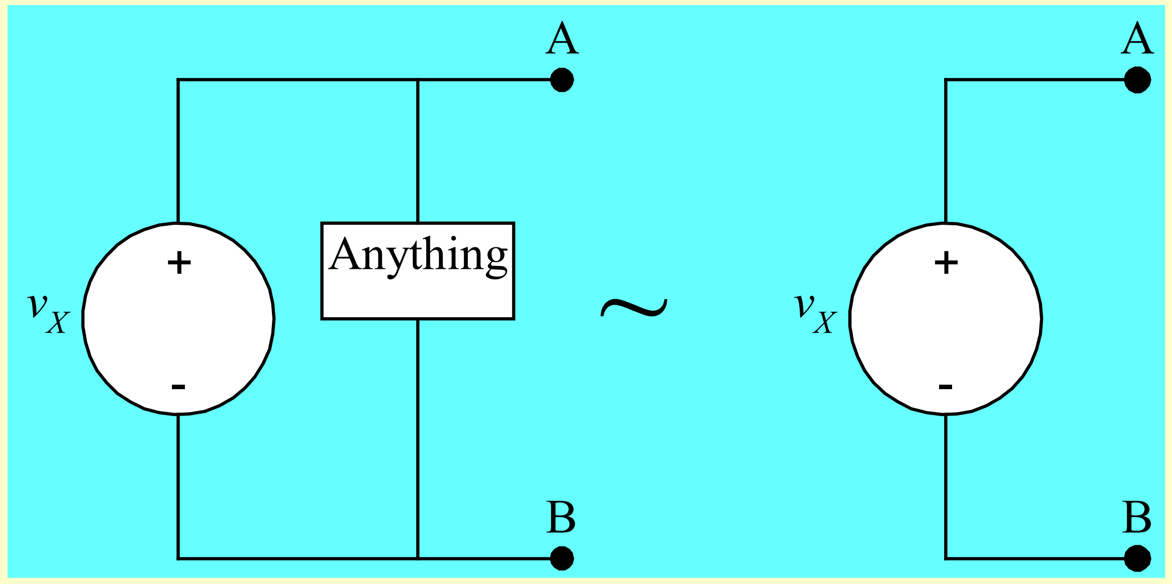
As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalents.



Other Useful Transformations – 3

A voltage source in parallel with anything can be replaced by that voltage source. The "anything" can be a resistor, a current source, or any other combination of elements. If the "anything" is a voltage source, the two voltage sources must be equal for KVL to hold.

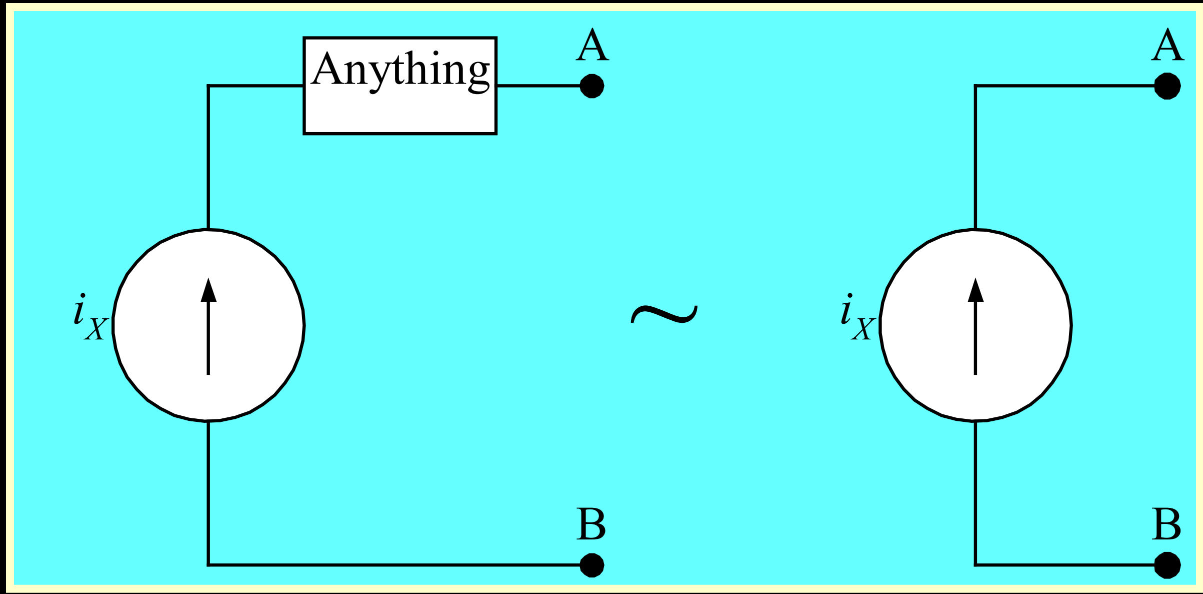
As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalents.



Other Useful Transformations – 4

A current source in series with anything can be replaced by that current source. The "anything" can be a resistor, a voltage source, or any other combination of elements. If the "anything" is a current source, the two current sources must be equal for KCL to hold.

As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalents.



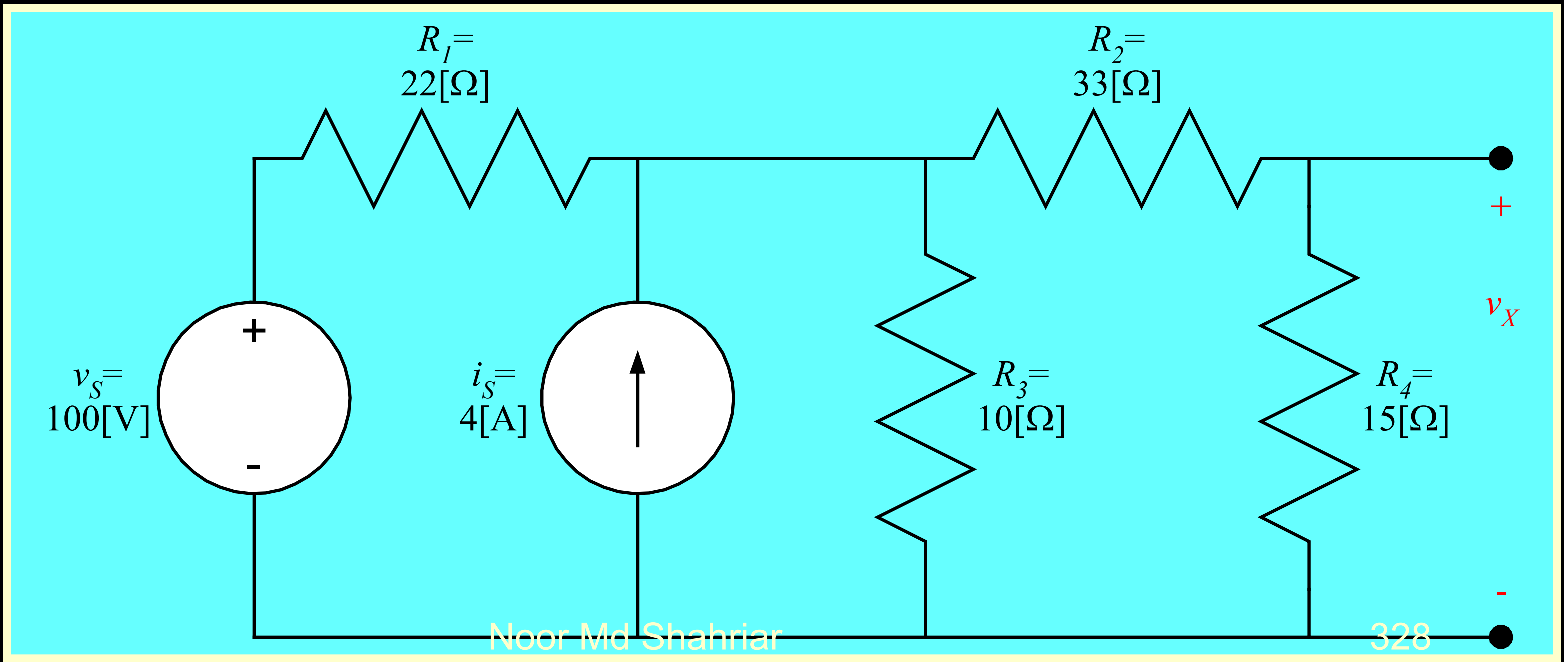
Go back to Overview slide.

Notes

1. These equivalent circuits can go in either direction. That is, we can replace the circuit on the right with the one on the left, or the other way around.
2. The polarities of the sources with respect to the terminals are important. This is one good reason for naming the terminals of these equivalents.
3. As with all equivalent circuits, these are equivalent only with respect to the things connected to the equivalent circuits.
4. These equivalent circuits hold for dependent sources as well as independent sources. The key is that the variable, which the dependent sources depend on, must remain intact. That is, the voltage or current that the dependent sources use must be outside of the circuit being replaced.

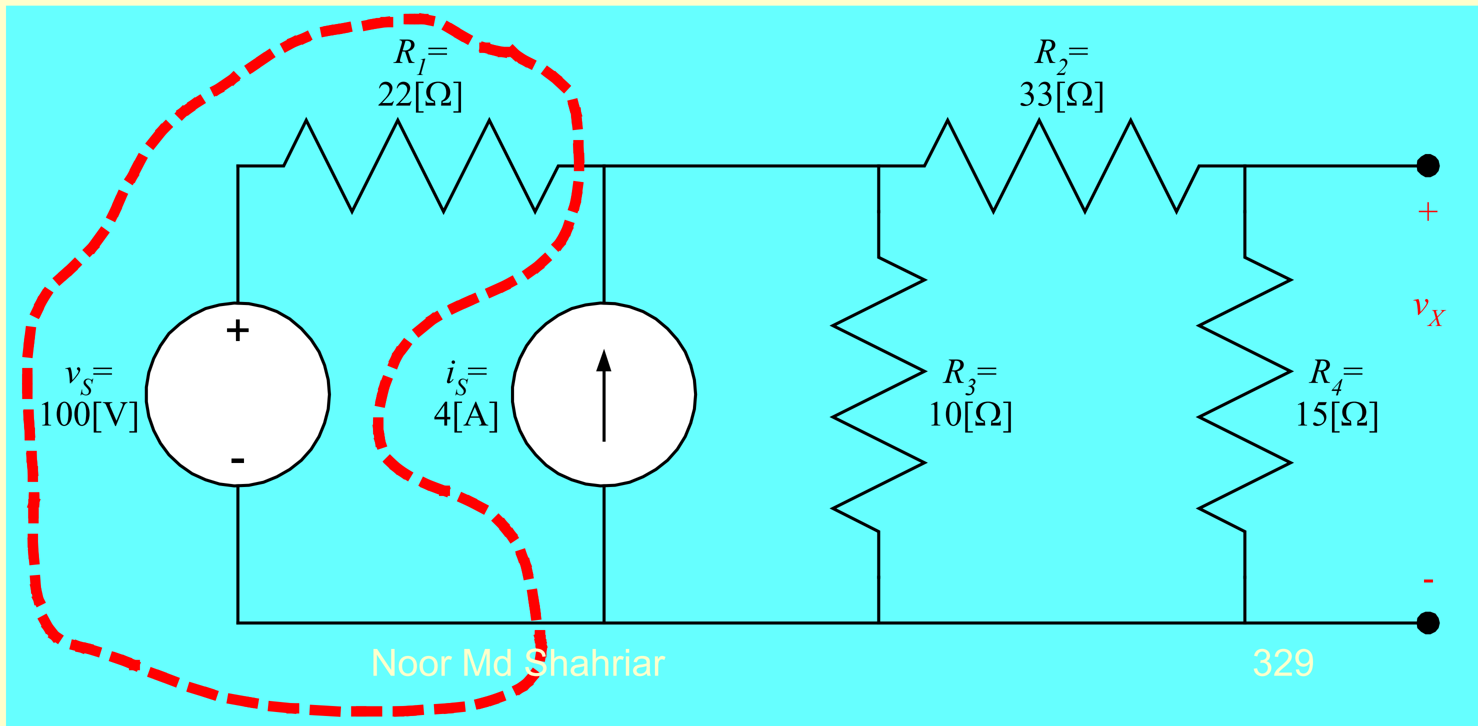
Example Problem

We wish to solve for the voltage v_X in the circuit given below. While we could certainly solve this by writing a series of KVL and KCL equations, we are going to solve it instead by using a series of equivalent circuits and simplify the circuit down step by step.



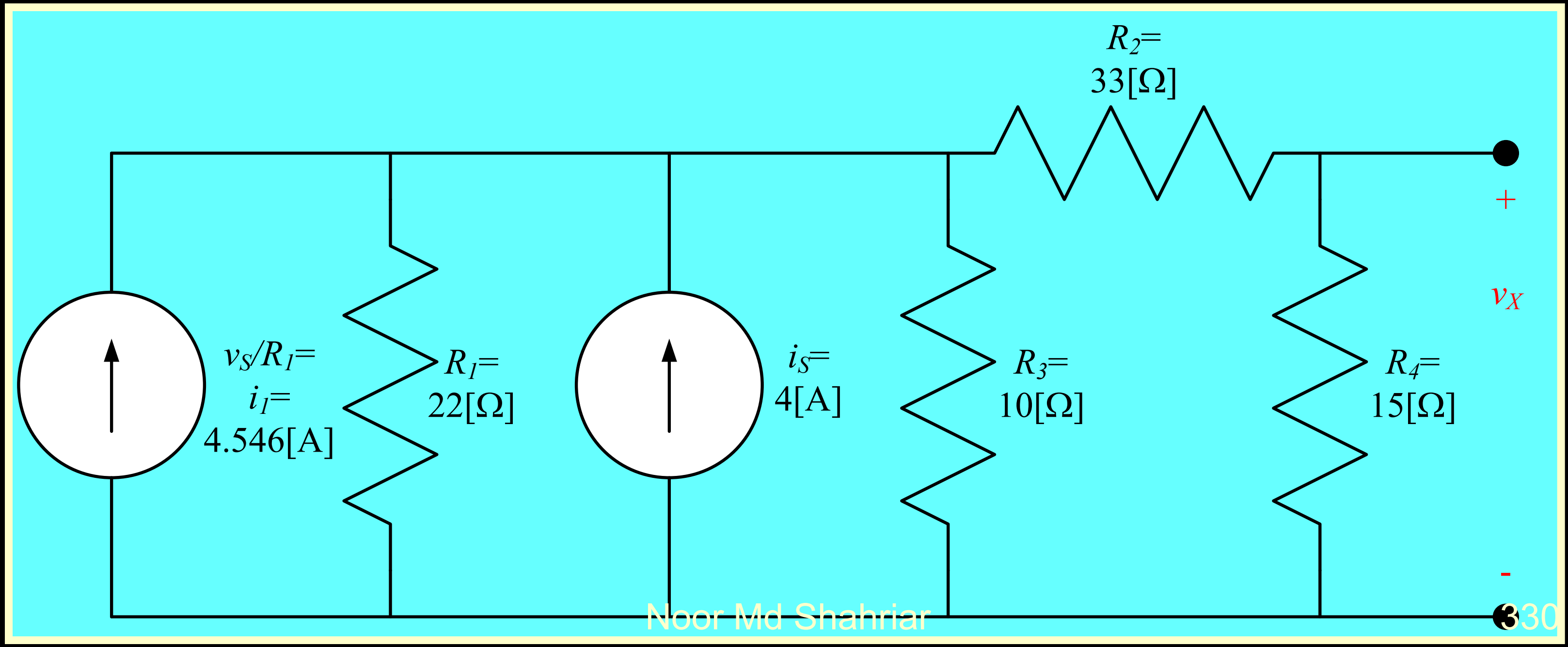
Example Problem – Step 1

We wish to solve for the voltage v_X in the circuit given below. We note that we have a voltage source, v_S, in series with a resistor, R_1. We can replace them with a current source in parallel with a resistor. When we do, we will have current sources in parallel and resistors in parallel, which we can simplify further. So, let's take the first step.



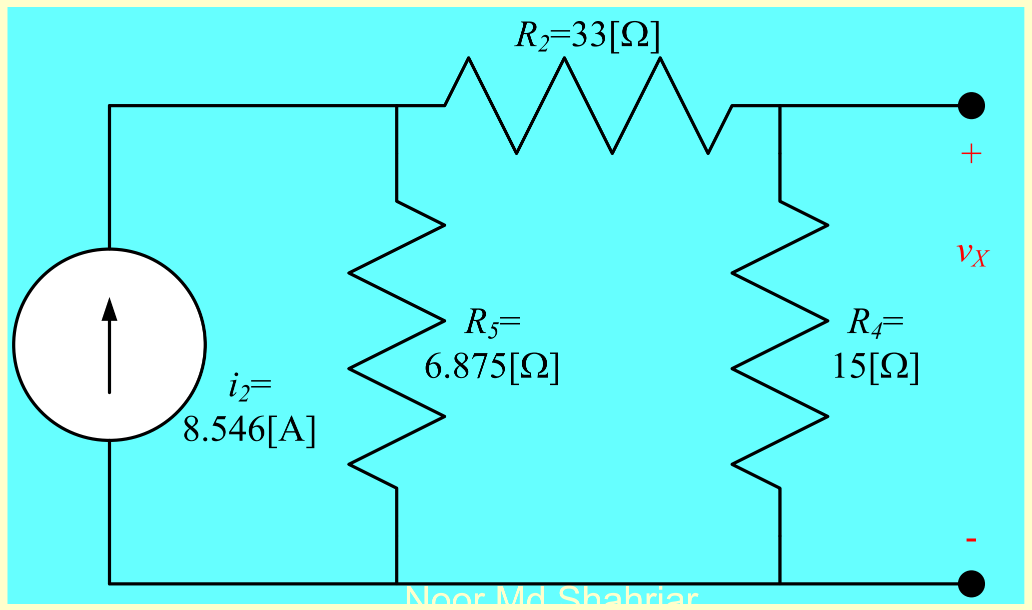
Example Problem – Step 2

We want to replace the voltage source in series with a resistor, with a current source in parallel with a resistor. Here, we have made this replacement. Note that we now have two current sources in parallel, and two resistors in parallel. Since the voltage we are looking for is outside these combinations, we can replace them with their equivalents. That is our next step.



Example Problem – Step 3

We have replaced the parallel current sources and parallel resistors with their equivalents. Now, we can note that we have a current source in parallel with a resistor. We could replace this with a voltage source in series with a resistor, and then we could simplify the circuit further. Let's do this.



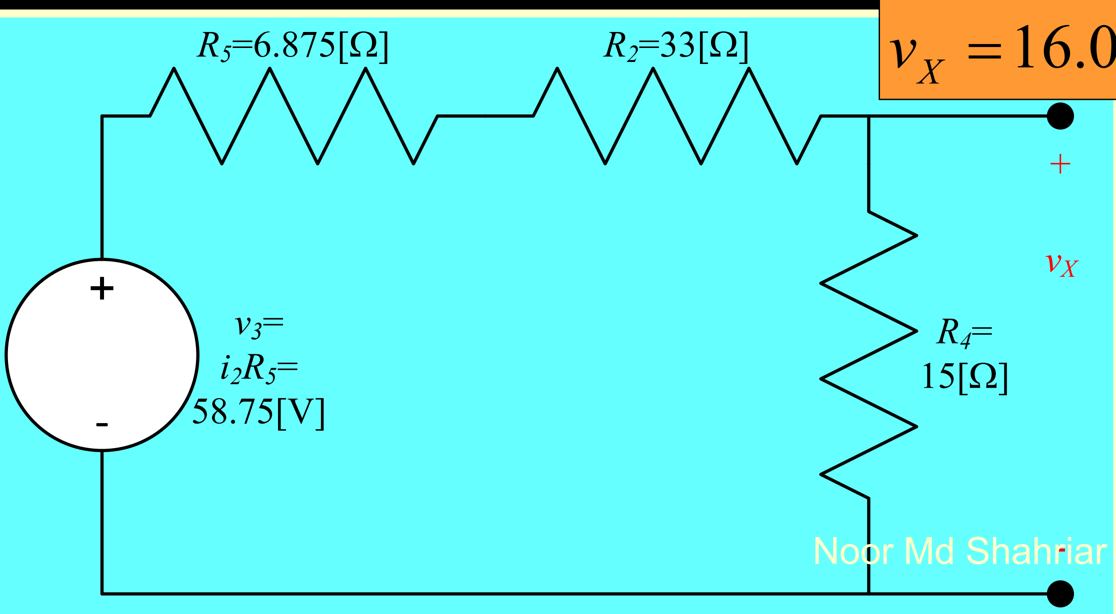
Example Problem – Step 4

We have replaced the current source in parallel with a resistor with a voltage source in series with a resistor. At this point, we have three resistors in series, and we want the voltage across one of them. This means that we use the voltage divider rule, and write,

$$v_X = v_3 \frac{R_4}{R_4 + R_5 + R_2} =$$

$$v_X = 58.75[\text{V}] \frac{15[\Omega]}{15[\Omega] + 6.875[\Omega] + 33[\Omega]}$$

$$v_X = 16.06[\text{V}].$$



Noor Md Shahriar

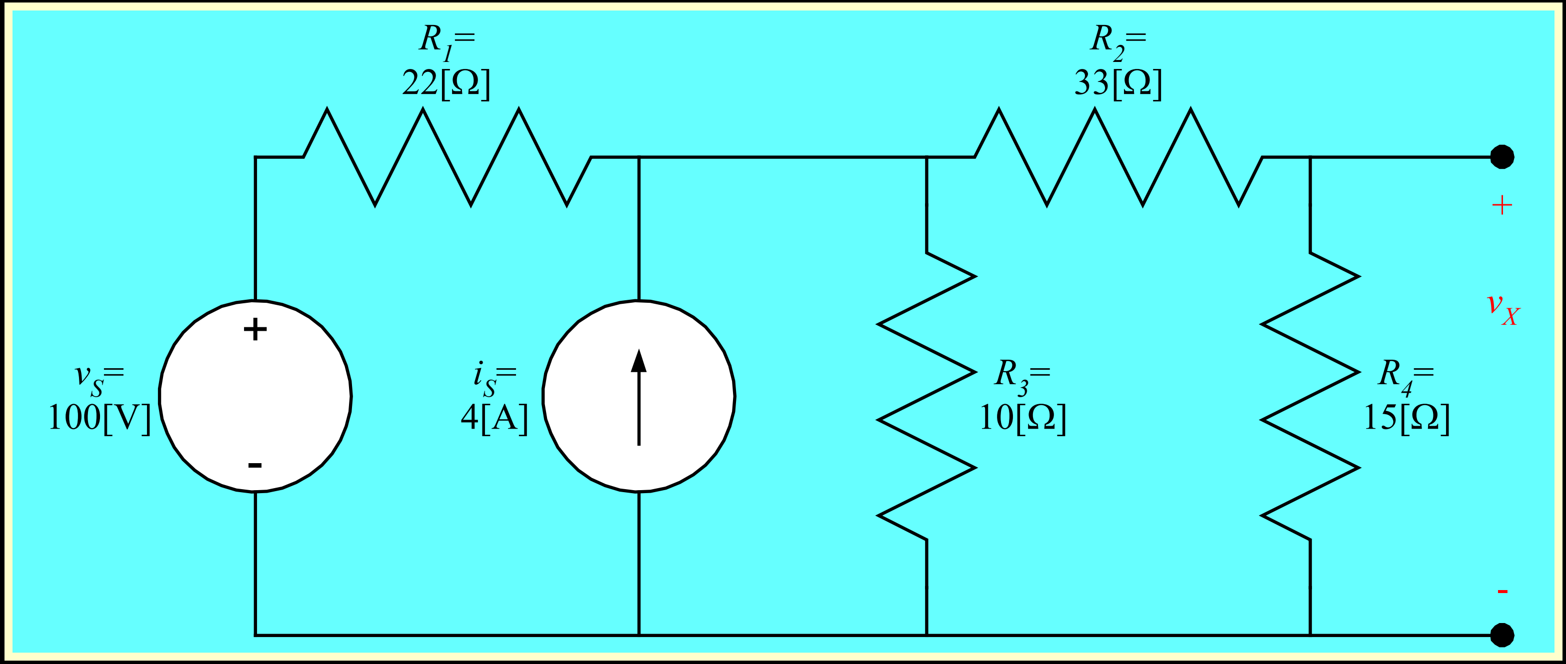
Go to Comments slide.

Go back to Overview slide.

Example Problem – Solution

The solution for this problem then is

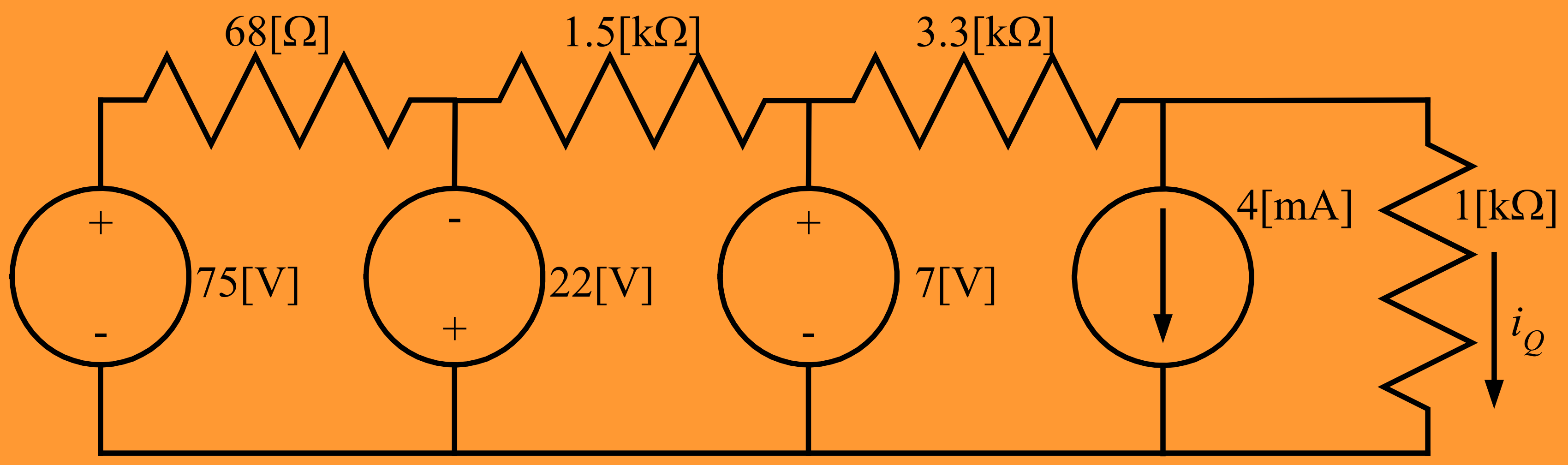
$$v_X = 16.06[\mathrm{V}].$$



Go to Comments slide.

Go back to Overview slide.

Example Problem



Find i_Q.

Week -11

Page- (336-366)

Thévenin's Theorem

Thévenin's Theorem Defined

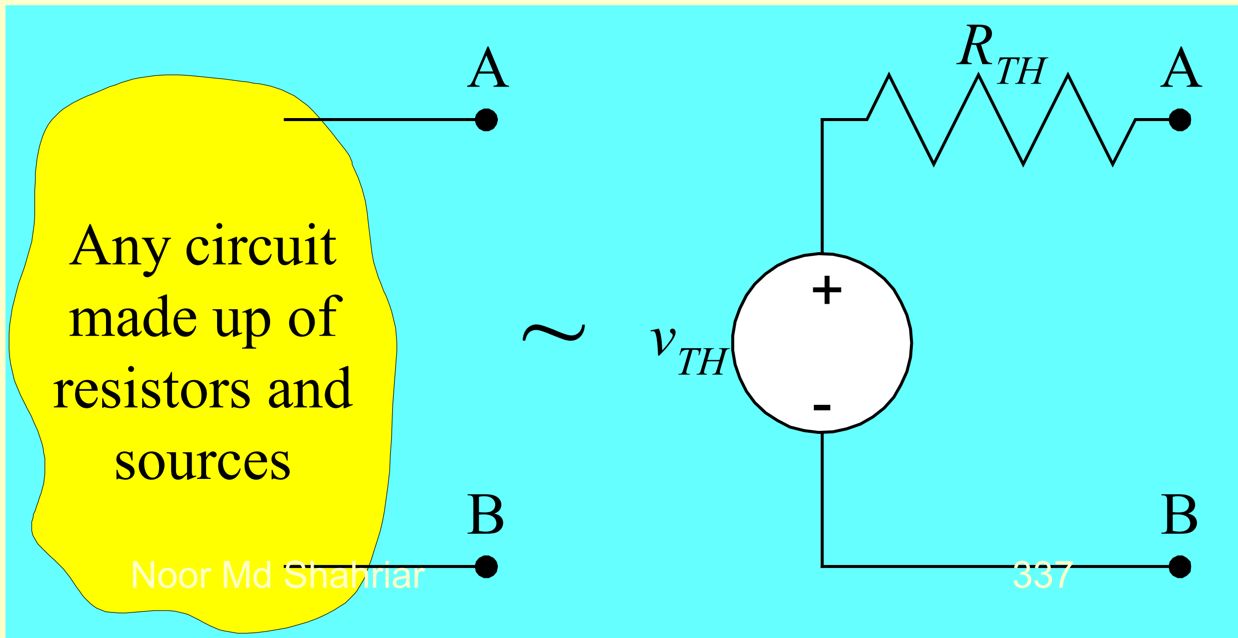
Thévenin's Theorem is another equivalent circuit. Thévenin's Theorem can be stated as follows:

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

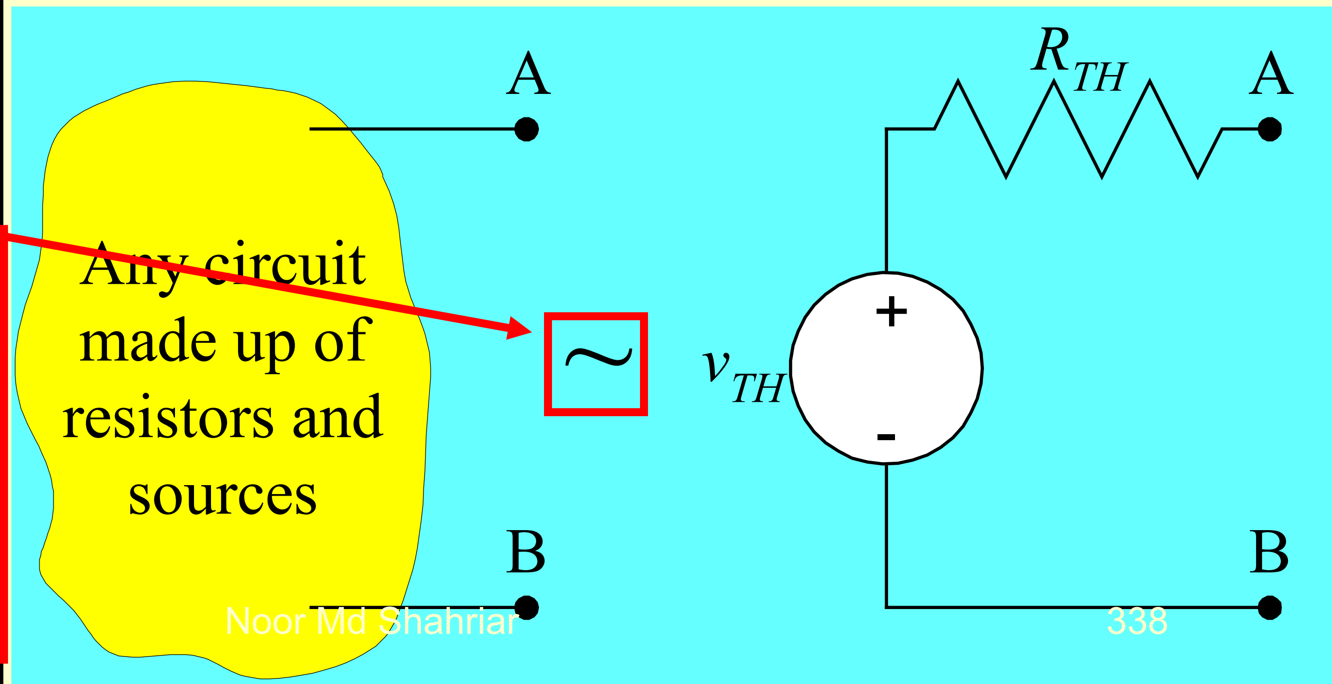


Notation

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

$$v_{TH} = \text{open-circuit voltage, and}$$
$$R_{TH} = \text{equivalent resistance.}$$



We have used the symbol "~" to indicate equivalence here. Some textbooks use a double-sided arrow (⇔ or ↔), or even a single-sided arrow (⇒ or →), to indicate this same thing.

Note 1

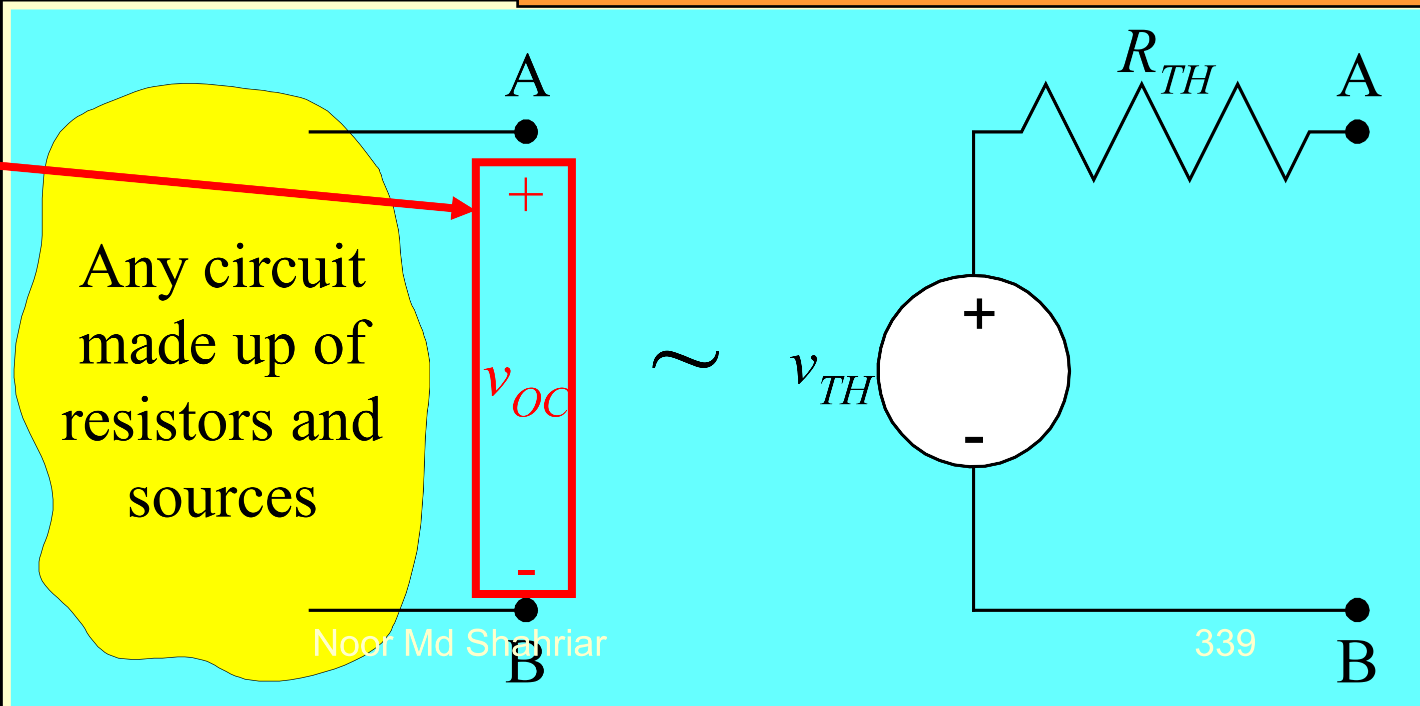
Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

We have introduced a term called the open-circuit voltage. This is the voltage for the circuit that we are finding the equivalent of, with nothing connected to the circuit. Connecting nothing means an open circuit. This voltage is shown here.



Note 2

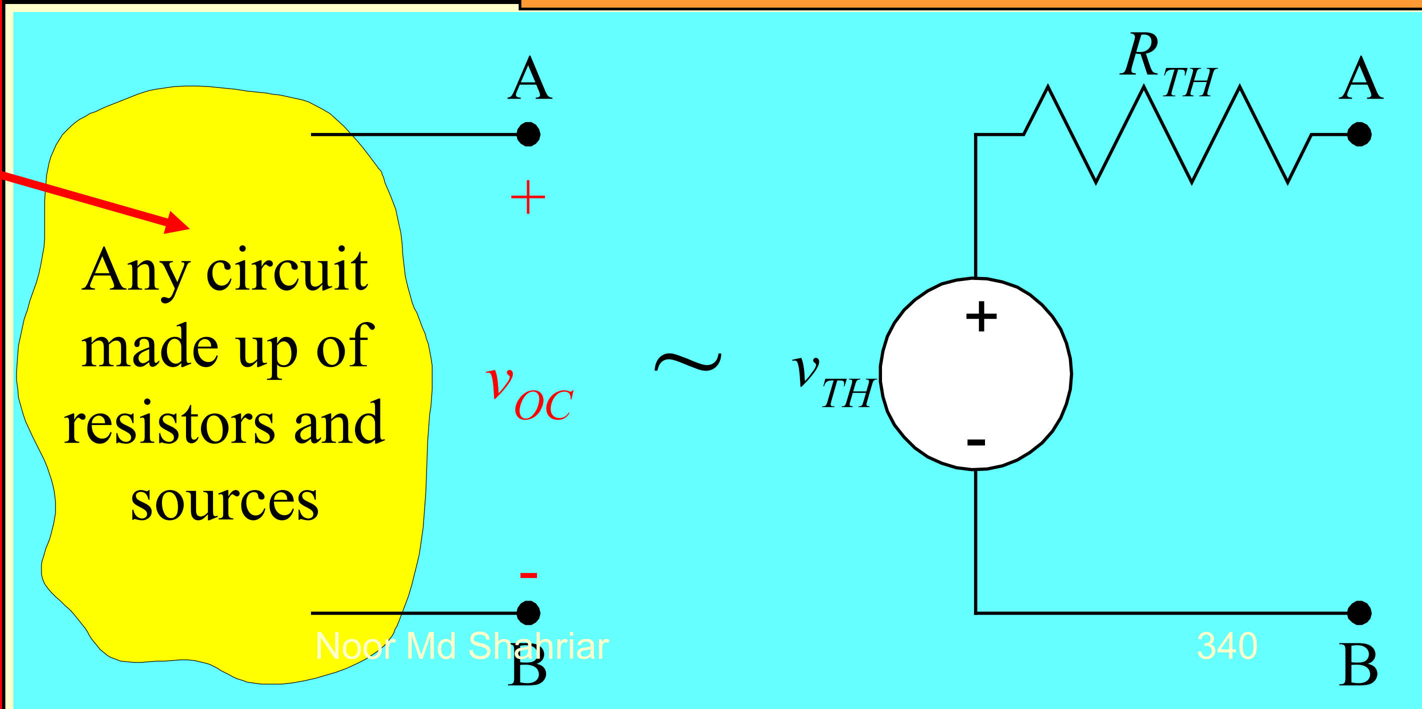
Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

We have introduced a term called the equivalent resistance. This is the resistance for the circuit that we are finding the equivalent of, with the independent sources set equal to zero. Any dependent sources are left in place.



Note 3

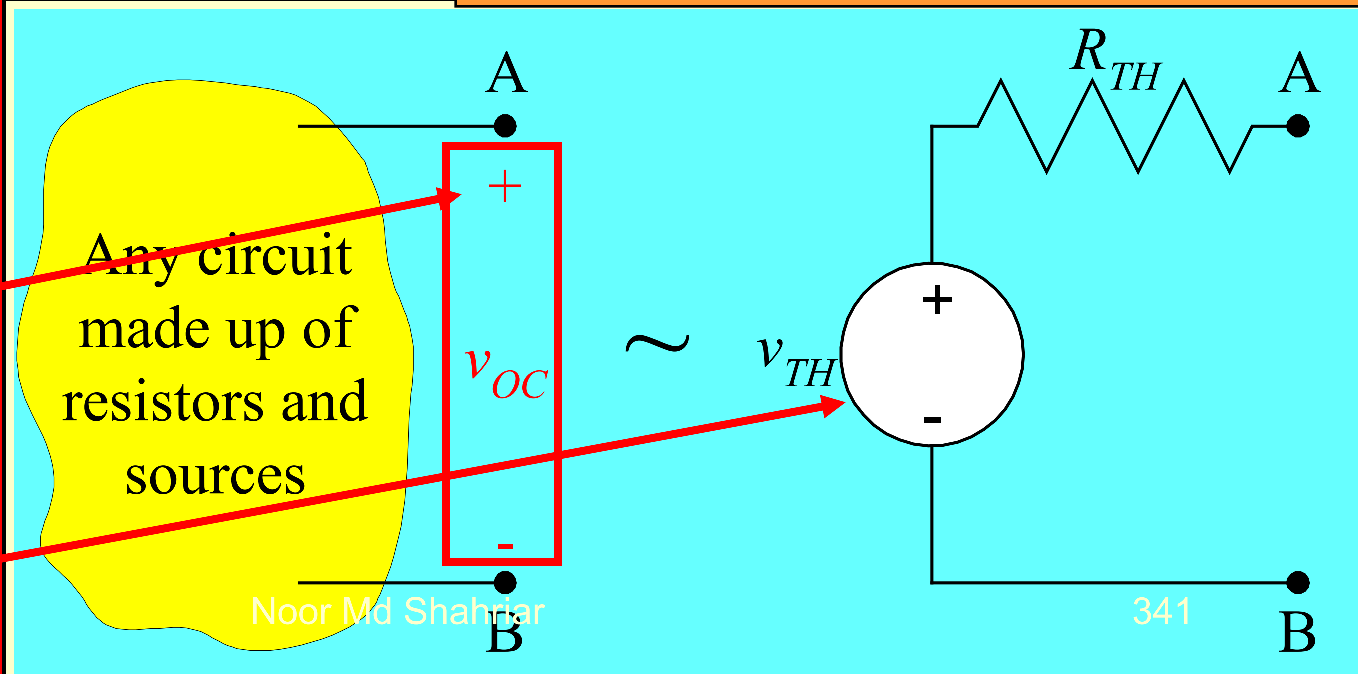
Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

The polarities of the source with respect to the terminals is important. If the reference polarity for the open-circuit voltage is as given here (voltage drop from A to B), then the reference polarity for the voltage source must be as given here (voltage drop from A to B).



Note 4

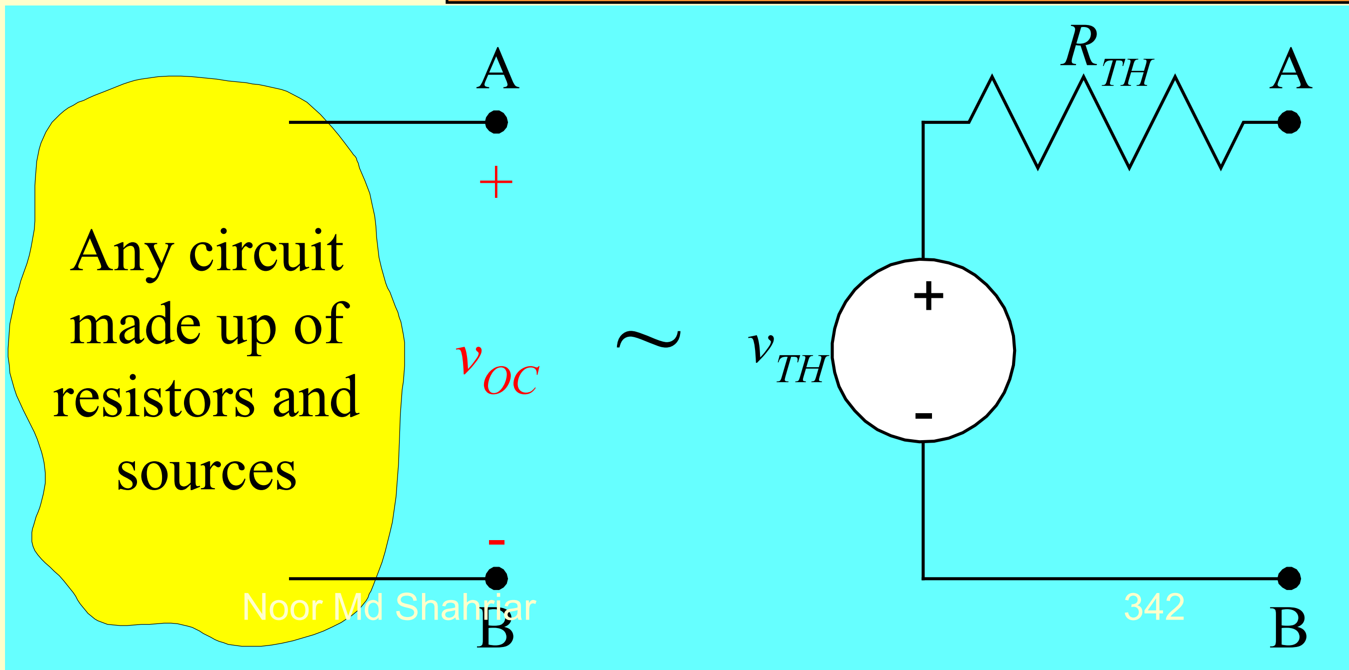
Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalent circuits.



Note 5

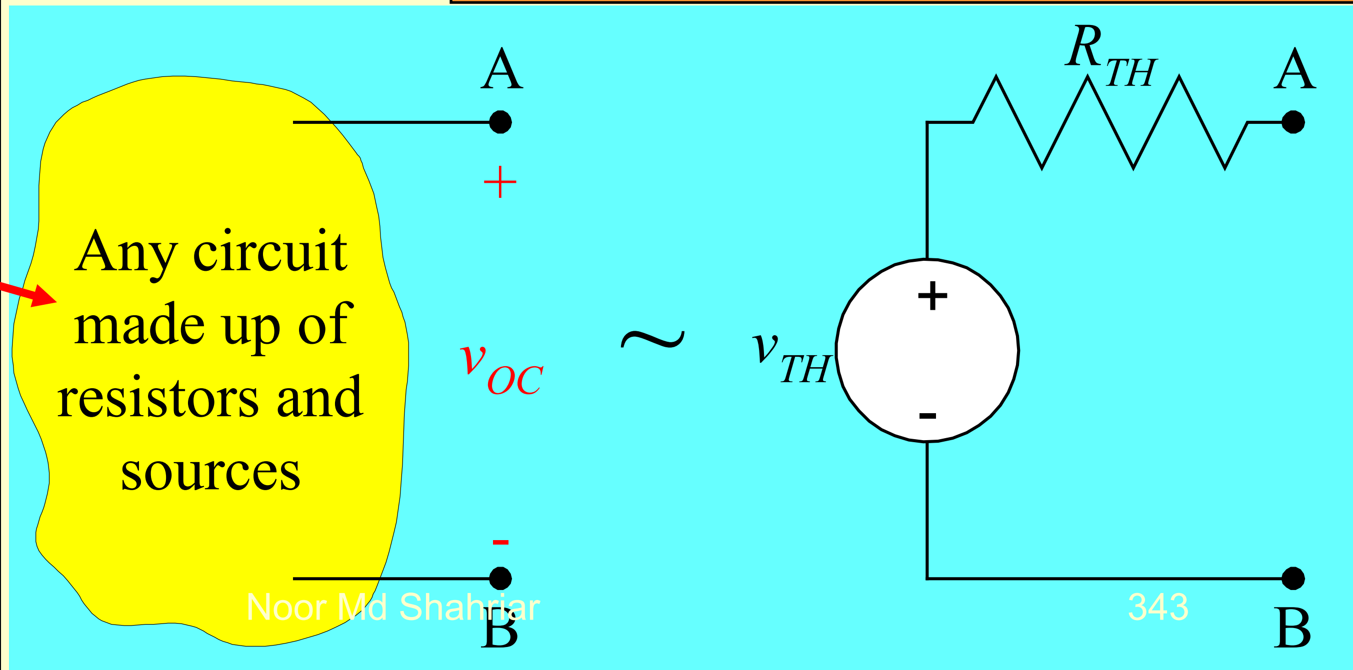
Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{TH} = open-circuit voltage, and

R_{TH} = equivalent resistance.

When we have dependent sources in the circuit shown here, it will make some calculations more difficult, but does not change the validity of the theorem.



Short-Circuit Current – 1

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

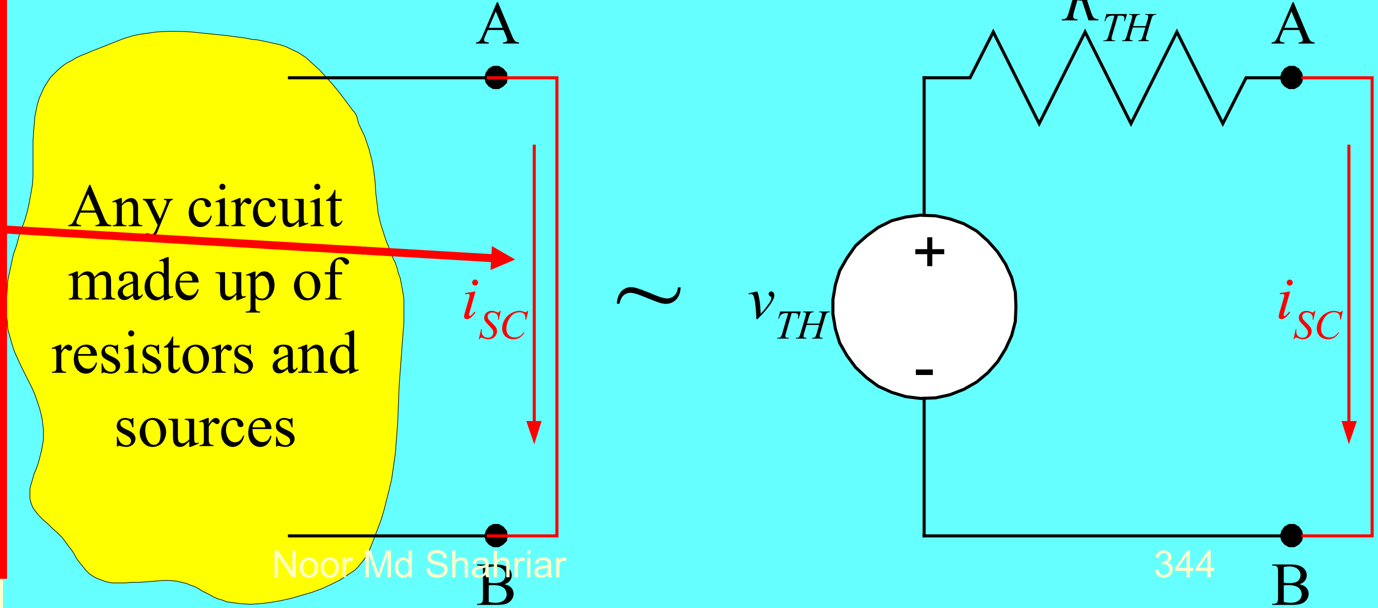
The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{OC} = open-circuit voltage,

i_{SC} = short-circuit current, and

R_{EQ} = equivalent resistance.

A useful concept is the concept of short-circuit current. This is the current that flows through a wire, or short circuit, connected to the terminals of the circuit. This current is shown here as i_{SC}.



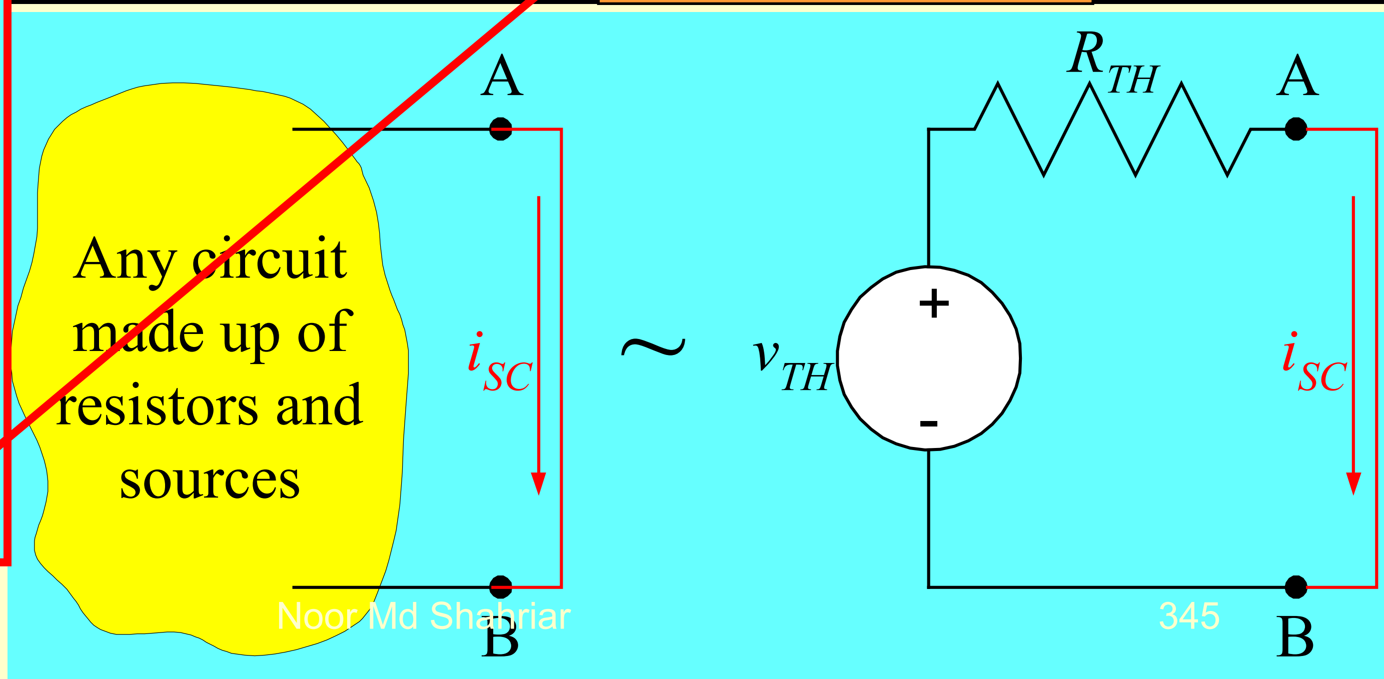
Short-Circuit Current – 2

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

$$v_{OC} = i_{SC} R_{EQ}.$$

When we look at the circuit on the right, we can see that the short-circuit current is equal to v_{TH}/R_{TH}, which is also v_{OC}/R_{EQ}. Thus, we obtain the important expression for i_{SC}, shown here.



Go back to Overview slide.

Extra note

We have shown that for the Thévenin equivalent, the open-circuit voltage is equal to the short-circuit current times the equivalent resistance. This is fundamental and important. However, it is not Ohm's Law.

This equation is not really Ohm's Law. It looks like Ohm's Law, and has the same form. However, it should be noted that Ohm's Law relates voltage and current for a resistor. This relates the values of voltages, currents and resistances in two different connections to an equivalent circuit. However, if you wish to remember this by relating it to Ohm's Law, that is fine.

$$v_{OC} = i_{SC} R_{EQ}.$$

Remember that

$v_{OC} = v_{TH},$

and

$R_{EQ} = R_{TH}.$

Finding the Thévenin Equivalent

We have shown that for the Thévenin equivalent, the open-circuit voltage is equal to the short-circuit current times the equivalent resistance. In general we can find the Thévenin equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, R_{EQ}.

$$v_{OC} = i_{SC} R_{EQ}.$$

Once we find any two, we can find the third by using this equation.

Remember that $v_{OC} = v_{TH}$, and $R_{EQ} = R_{TH}$.

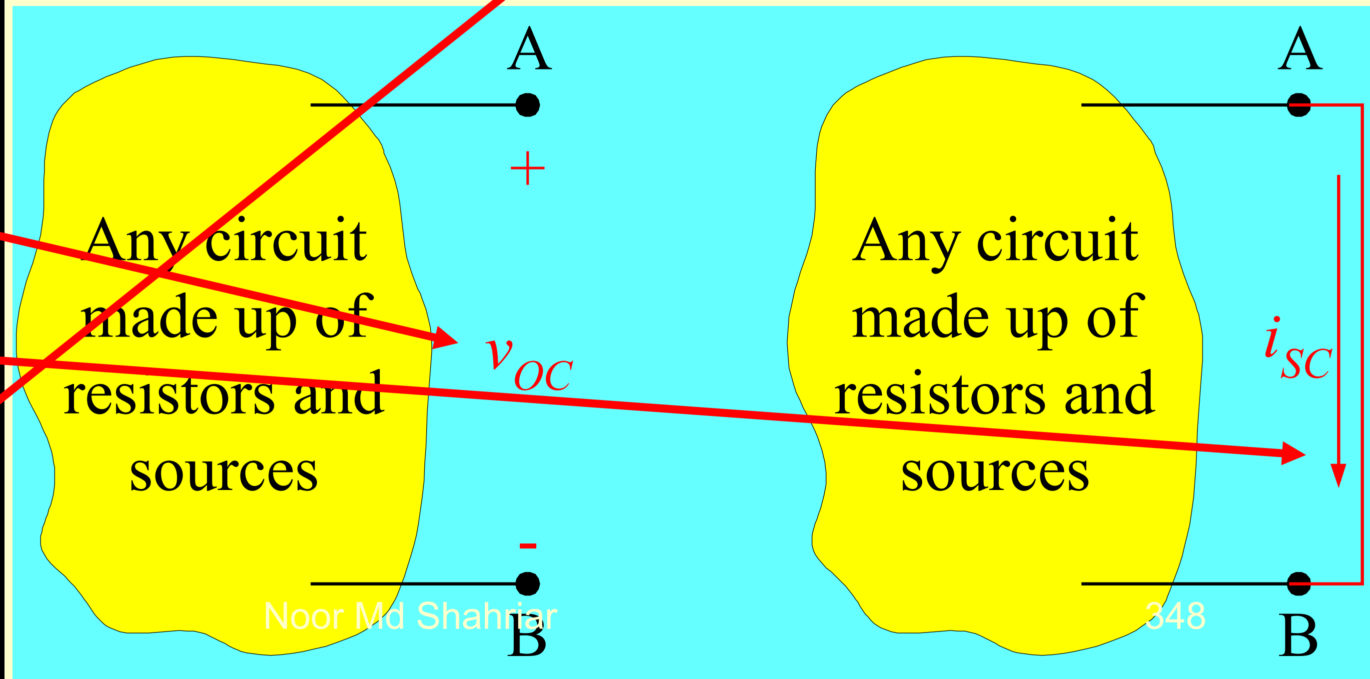
Finding the Thévenin Equivalent – Note 1

We can find the Thévenin equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, $v_{OC} = v_{TH}$
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, $R_{EQ} = R_{TH}$.

$$v_{OC} = i_{SC} R_{EQ}.$$

One more time, the reference polarities of our voltages and currents matter. If we pick v_{OC} at A with respect to B, then we need to pick i_{SC} going from A to B. If not, we need to change the sign in this equation.



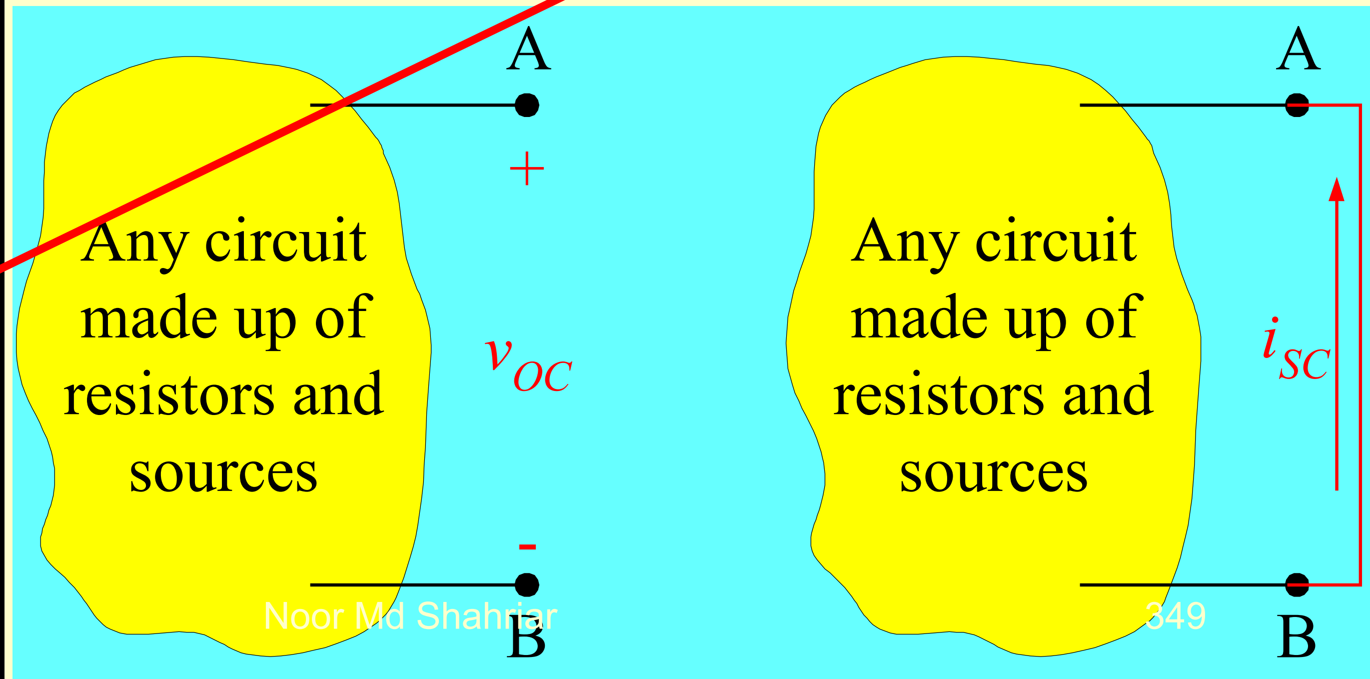
Finding the Thévenin Equivalent – Note 2

We can find the Thévenin equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, $v_{OC} = v_{T}$
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, $R_{EQ} = R_{TH}$.

$$v_{OC} = -i_{SC} R_{EQ}.$$

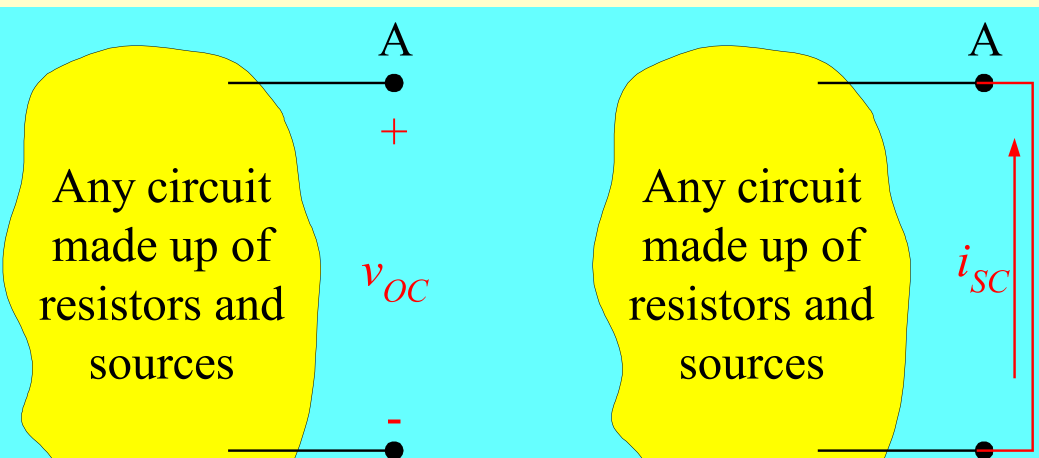
As an example, if we pick v_{OC} and i_{SC} with the reference polarities given here, we need to change the sign in the equation as shown. This is a consequence of the sign in Ohm's Law. For a further explanation, see the next slide.



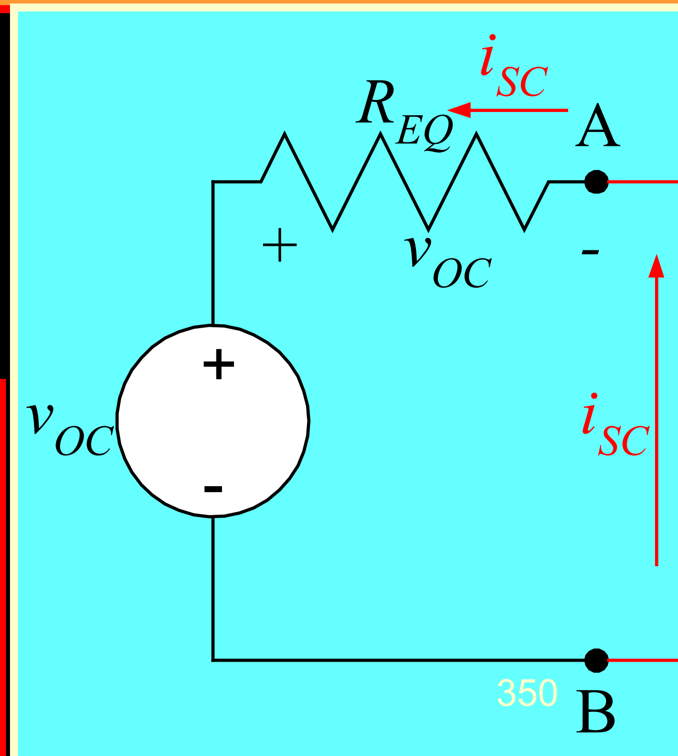
Finding the Thévenin Equivalent – Note 3

We can find the Thévenin equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, $v_{OC} = v_{TH}$,
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, $R_{EQ} = R_{TH}$.



$$v_{OC} = -i_{SC} R_{EQ}.$$

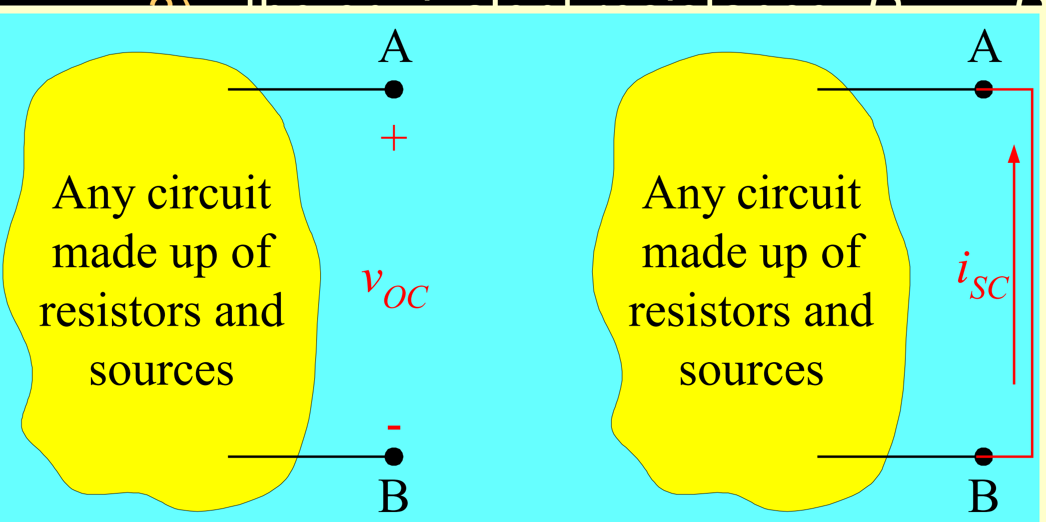


As an example, if we pick v_{OC} and i_{SC} with the reference polarities given here, we need to change the sign in the equation as shown. This is a consequence of Ohm's Law, which for resistor R_{EQ} requires a minus sign, since the voltage and current are in the active sign convention.

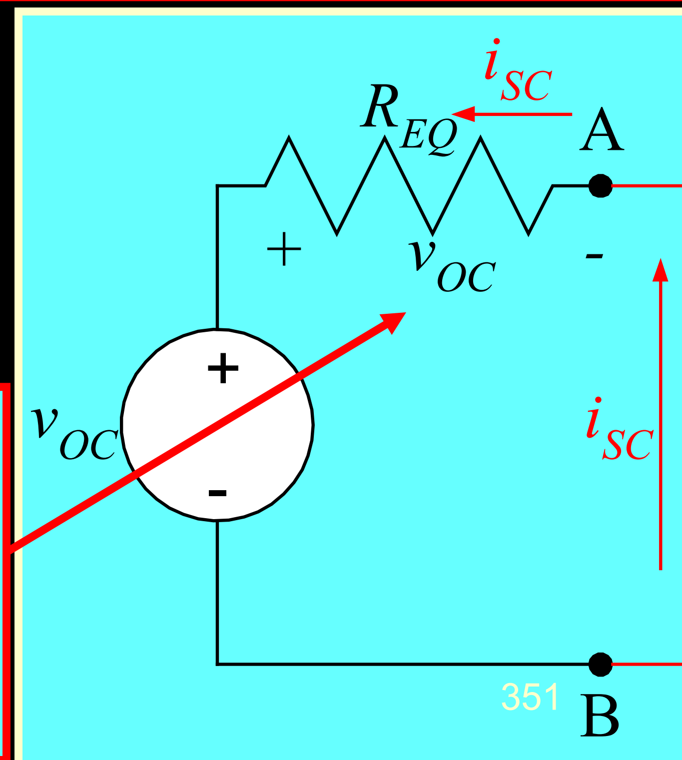
Finding the Thévenin Equivalent – Note 4

We can find the Thévenin equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, $v_{OC} = v_{TH}$,
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, $R_{EQ} = R_{TH}$.



$$v_{OC} = -i_{SC} R_{EQ}.$$



Be very careful here! We have labeled the voltage across the resistance R_{EQ} as v_{OC}. This is true only for this special case. This v_{OC} is not the voltage at A with respect to B in this circuit. In this circuit, that voltage is zero due to the short. Due to the short, the voltage across R_{EQ} is v_{OC}.

Go back to Overview slide.

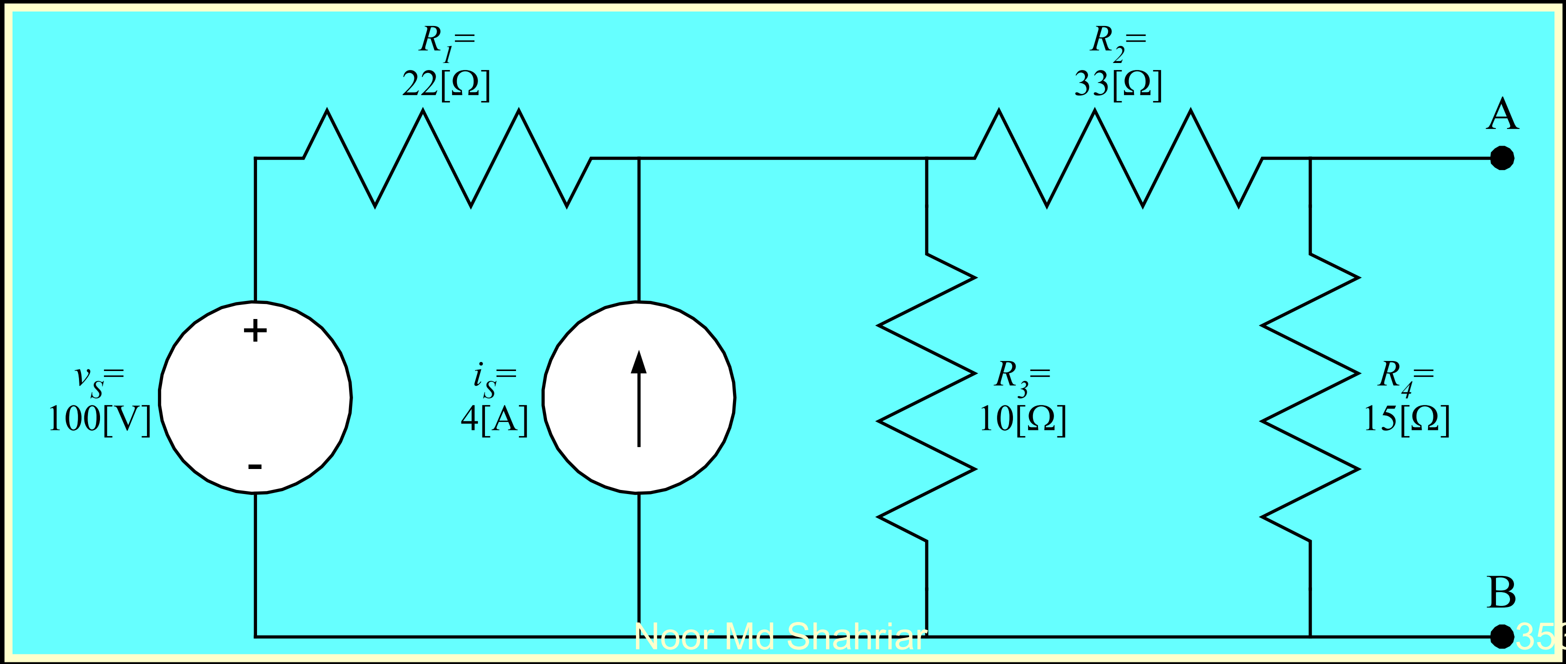
Notes

1. We can find the Thévenin equivalent of any circuit made up of voltage sources, current sources, and resistors. The sources can be any combination of dependent and independent sources.
2. We can find the values of the Thévenin equivalent by finding the open-circuit voltage and short-circuit current. The reference polarities of these quantities are important.
3. To find the equivalent resistance, we need to set the independent sources equal to zero. However, the dependent sources will remain. This requires some care. We will discuss finding the equivalent resistance with dependent sources in the fourth part of this module.
4. As with all equivalent circuits, the Thévenin equivalent is equivalent only with respect to the things connected to it.

Example Problem

We wish to find the Thévenin equivalent of the circuit below, as seen from terminals A and B.

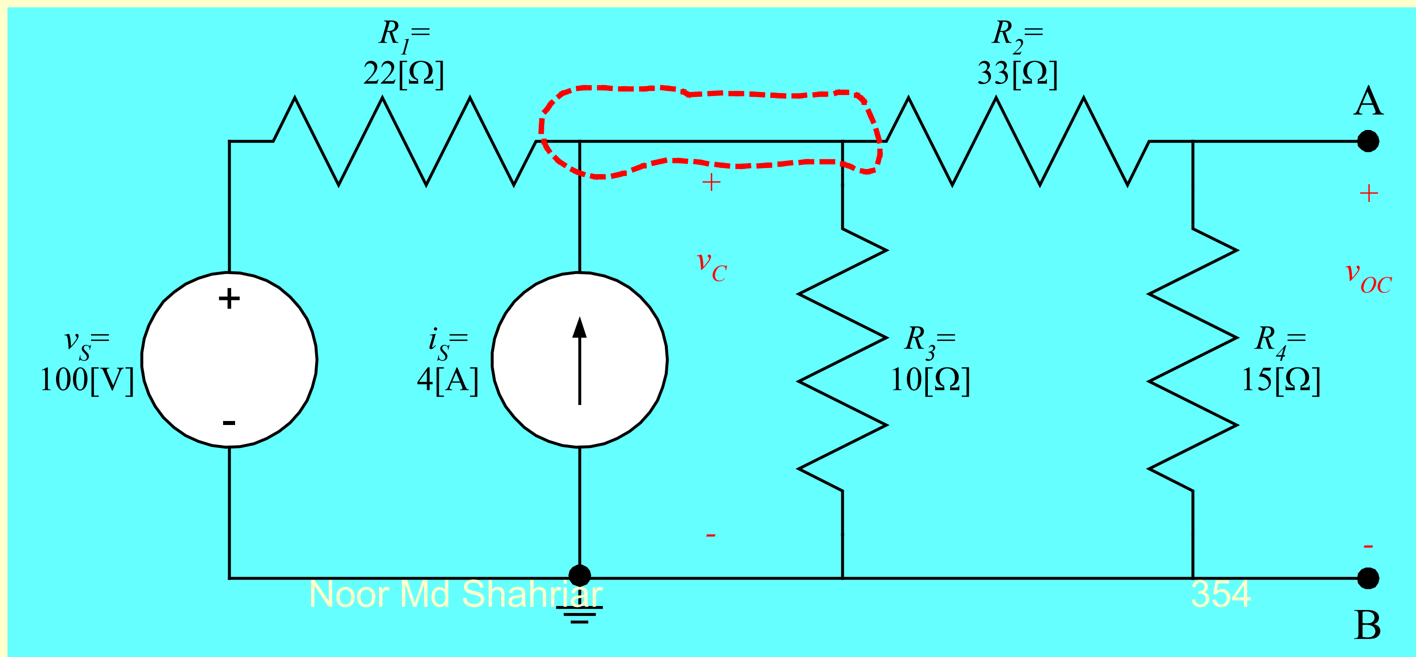
Note that there is an unstated assumption here; we assume that we will later connect something to these two terminals. Having found the Thévenin equivalent, we will be able to solve that circuit more easily by using that equivalent. Note also that we solved this same circuit in the last part of this module; we can compare our answer here to what we got then.



Example Problem – Step 1

We wish to find the open-circuit voltage v_{OC} with the polarity defined in the circuit given below. We have also defined the node voltage v_C, which we will use to find v_{OC}.

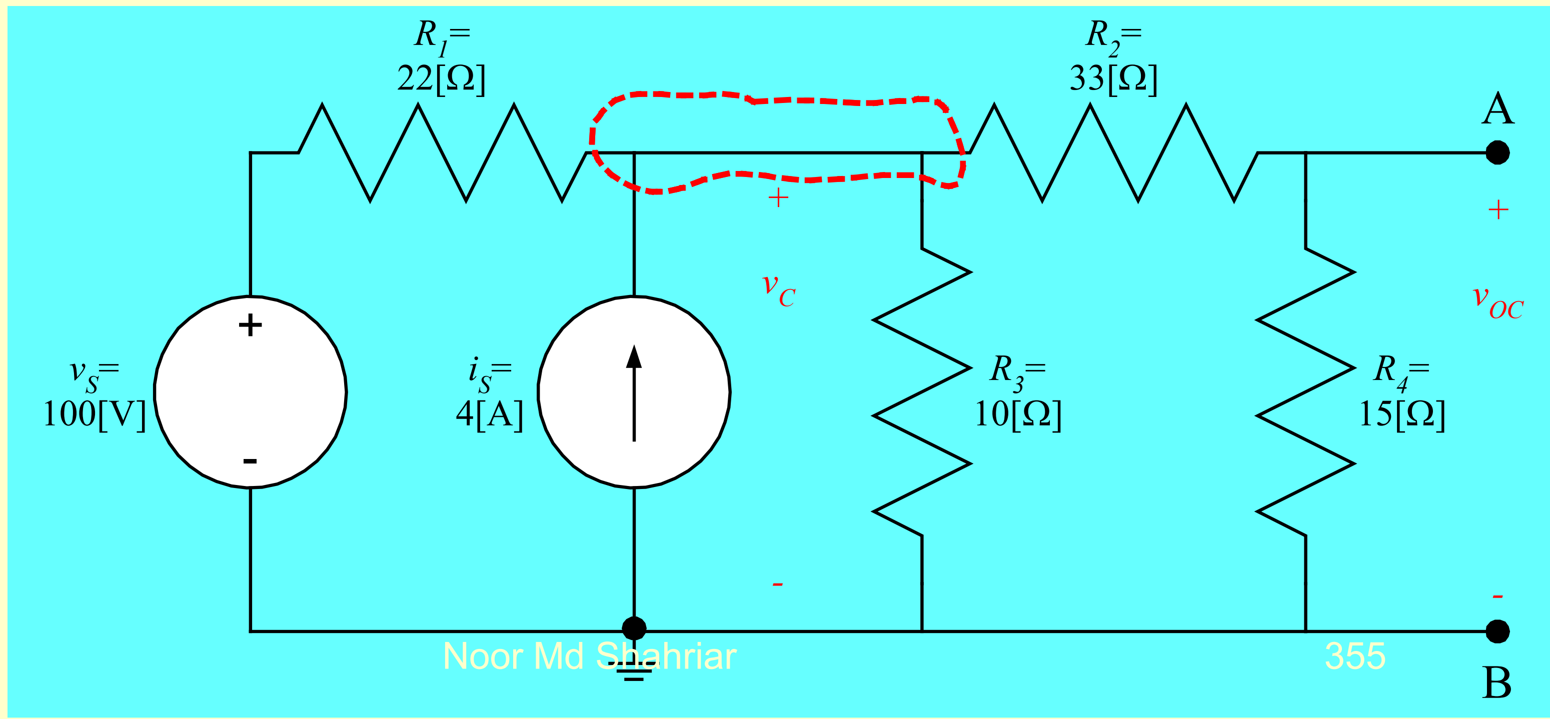
In general, remember, we need to find two out of three of the quantities v_{OC}, i_{SC}, and R_{EQ}. In this problem we will find two, and then find the third just as a check. In general, finding the third quantity is not required.



Example Problem – Step 2

We wish to find the node voltage v_C, which we will use to find v_{OC}. Writing KCL at the node encircled with a dashed red line, we have

$$\frac{v_C}{R_2 + R_4} + \frac{v_C}{R_3} - i_S + \frac{v_C - v_S}{R_1} = 0.$$



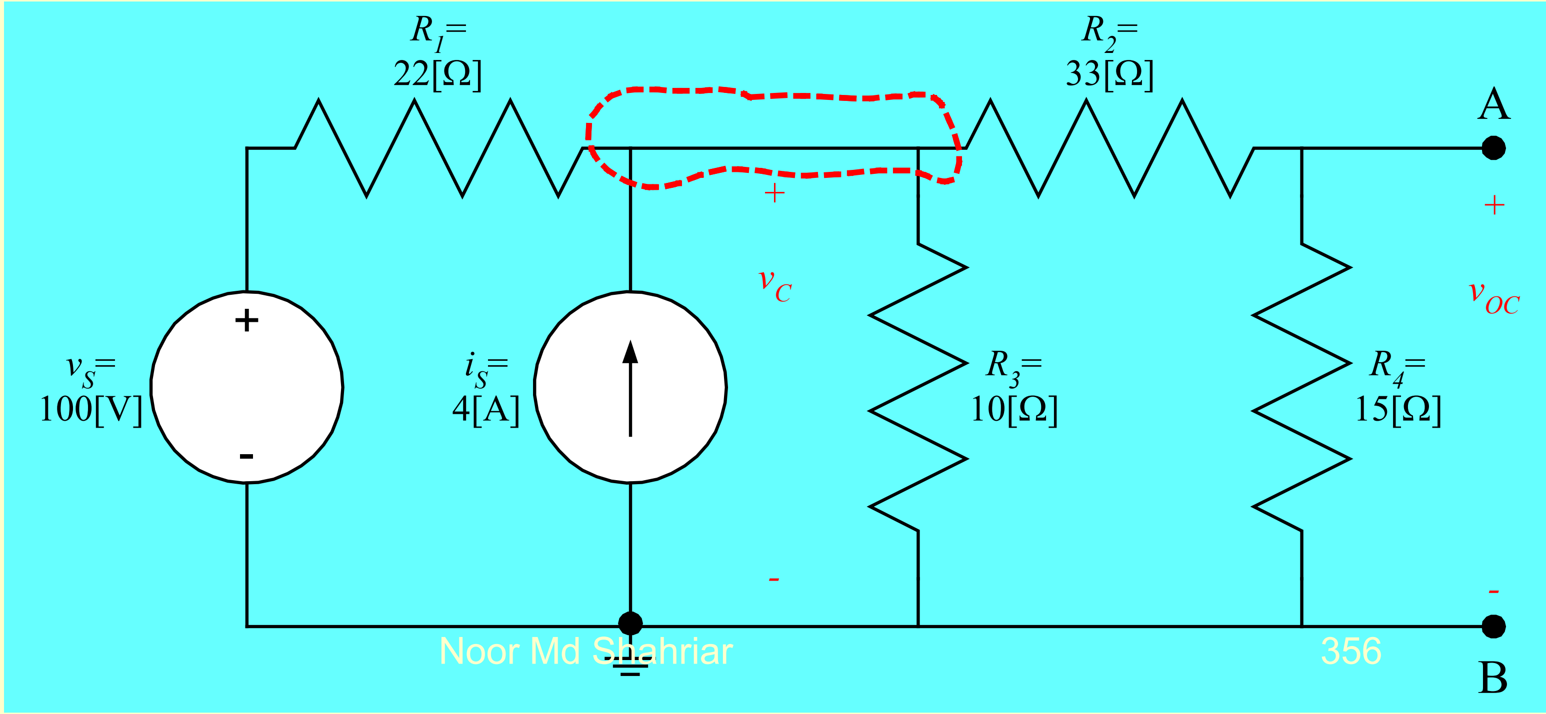
Example Problem – Step 3

Substituting in values, we have

$$\frac{v_C}{48[\Omega]} + \frac{v_C}{10[\Omega]} - 4[\text{A}] + \frac{v_C - 100[\text{V}]}{22[\Omega]} = 0.$$ Solving, we get

$$0.1663[\text{S}]v_C = 4[\text{A}] + 4.545[\text{A}],$$ or

$$v_C = 51.4[\text{V}].$$



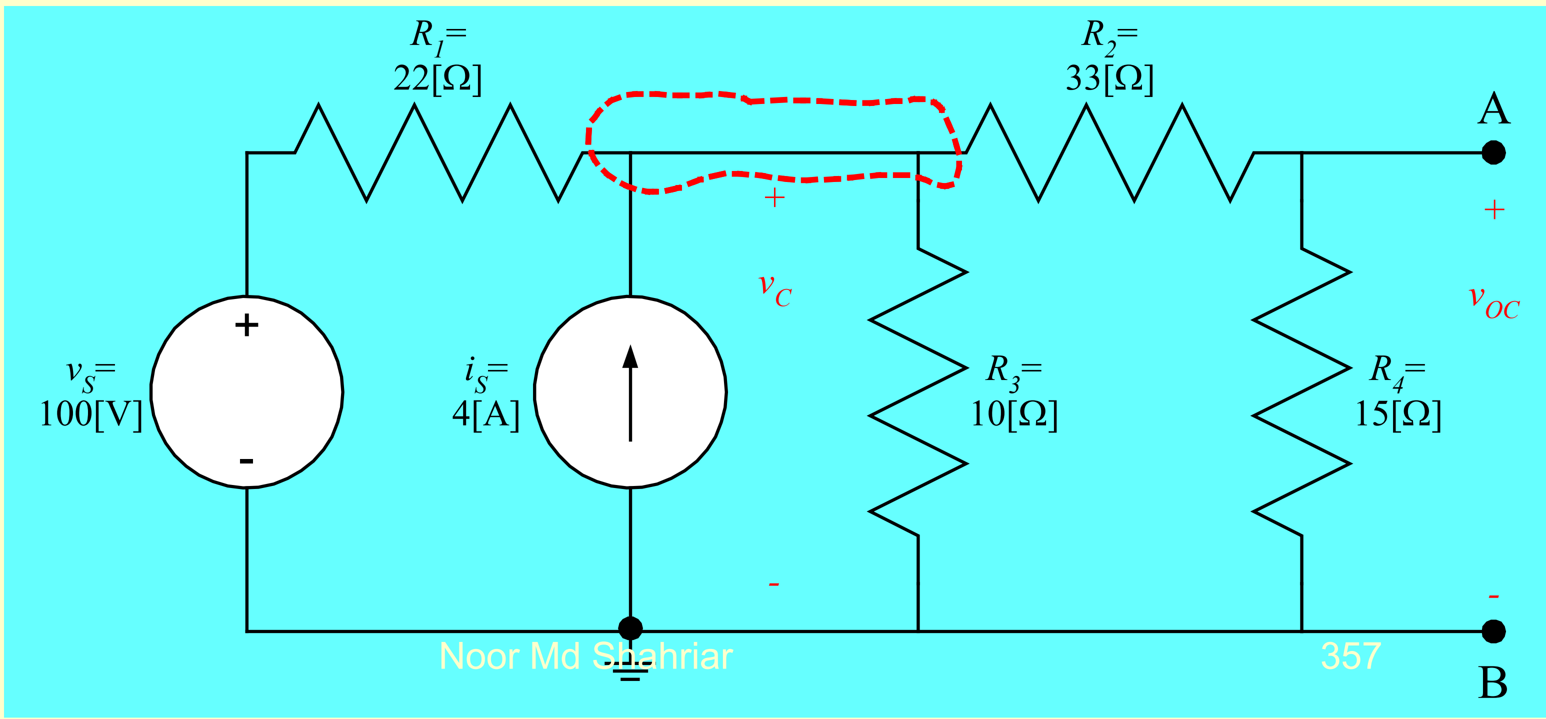
Example Problem – Step 4

Then, using VDR, we can find

$$v_{OC} = v_C \frac{15[\Omega]}{15[\Omega]+33[\Omega]}.$$ Solving, we get

$$v_{OC} = 16[V].$$

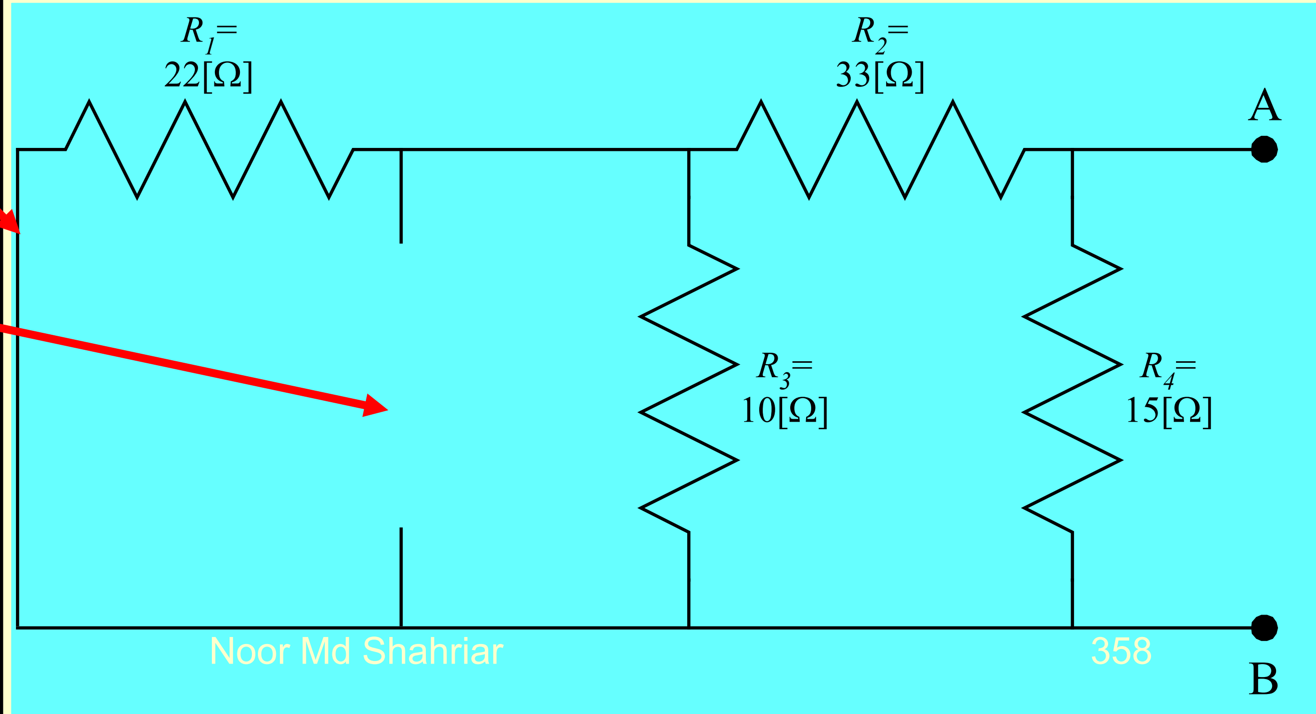
Note that when we solved this problem before, we got this same voltage.



Example Problem – Step 5

Next, we will find the equivalent resistance, R_{EQ}. The first step in this solution is to set the independent sources equal to zero. We then have the circuit below.

Note that the voltage source becomes a short circuit, and the current source becomes an open circuit. These represent zero-valued sources.

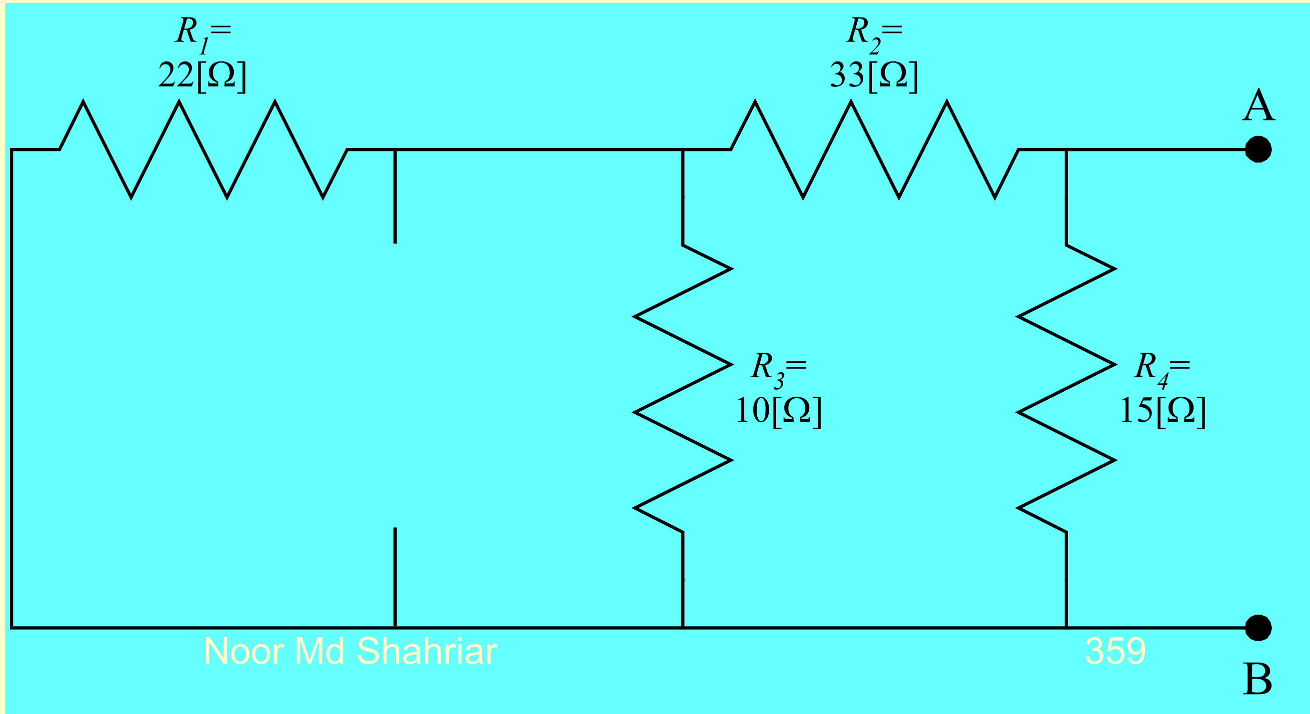


Example Problem – Step 6

To find the equivalent resistance, R_{EQ}, we simply combine resistances in parallel and in series. The resistance between terminals A and B, which we are calling R_{EQ}, is found be recognizing that R_1 and R_3 are in parallel. That parallel combination is in series with R_2. That series combination is in parallel with R_4. We have

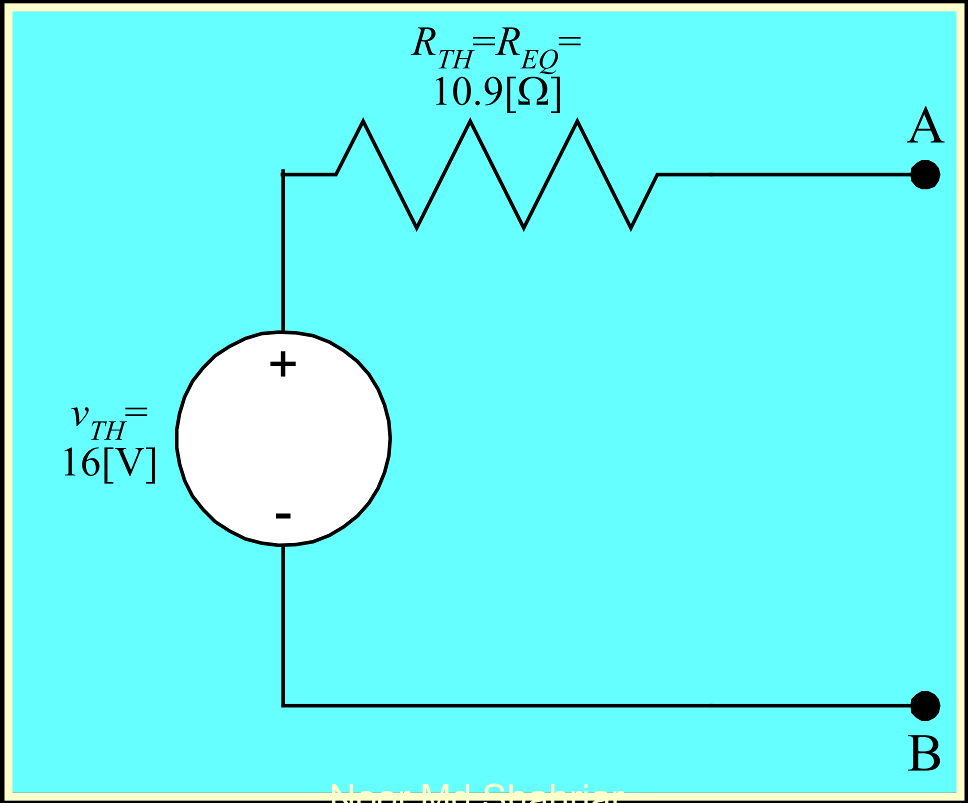
$$R_{EQ} = \{(R_1 \,||\, R_3) + R_2\} \,||\, R_4 = \{(22[\Omega] \,||\, 10[\Omega]) + 33[\Omega]\} \,||\, 15[\Omega].$$ Solving, we get

$$R_{EQ} = 10.9[\Omega].$$



Example Problem – Step 7 (Solution)

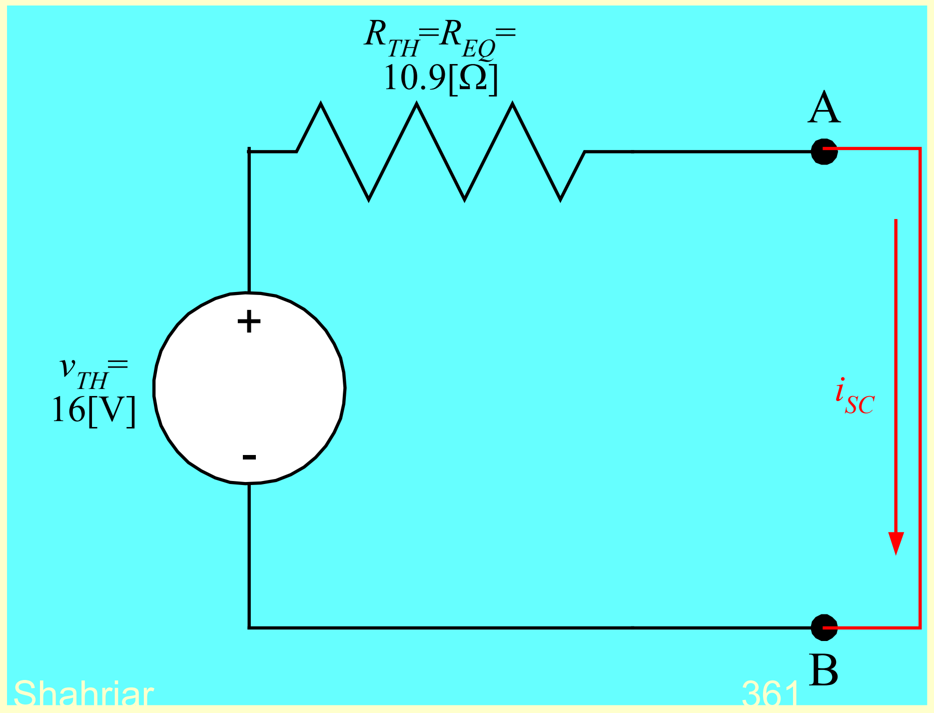
To complete this problem, we would typically redraw the circuit, showing the complete Thévenin's equivalent, along with terminals A and B. This has been done here. This shows the proper polarity for the voltage source.



Example Problem – Step 8 (Check)

Let's check this solution, by finding the short-circuit current in the original circuit, and compare it to the short-circuit current in the Thévenin's equivalent. We will start with the Thévenin's equivalent shown here. We have

$$i_{SC} = \frac{v_{TH}}{R_{EQ}} = \frac{16[\text{V}]}{10.9[\Omega]} = 1.5[\text{A}].$$



Example Problem – Step 9 (Check)

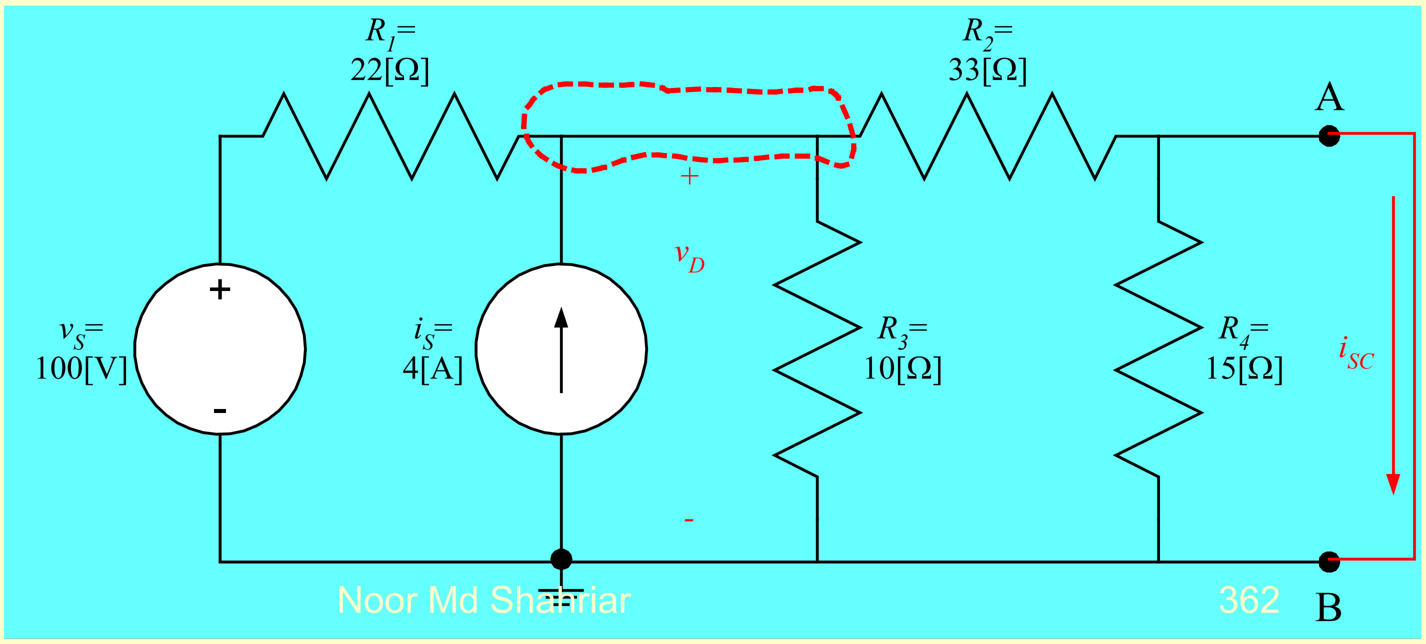
Let's find the short-circuit current in the original circuit. We have

$$\frac{v_D}{33[\Omega]} + \frac{v_D}{10[\Omega]} - 4[A] + \frac{v_D - 100[V]}{22[\Omega]} = 0.$$ Solving, we get

$$0.1758[S] v_D = 4[A] + 4.545[A],$$ or

$$v_D = 48.6[V].$$

Note that resistor R_4 is neglected, since it has no voltage across it, and therefore no current through it.

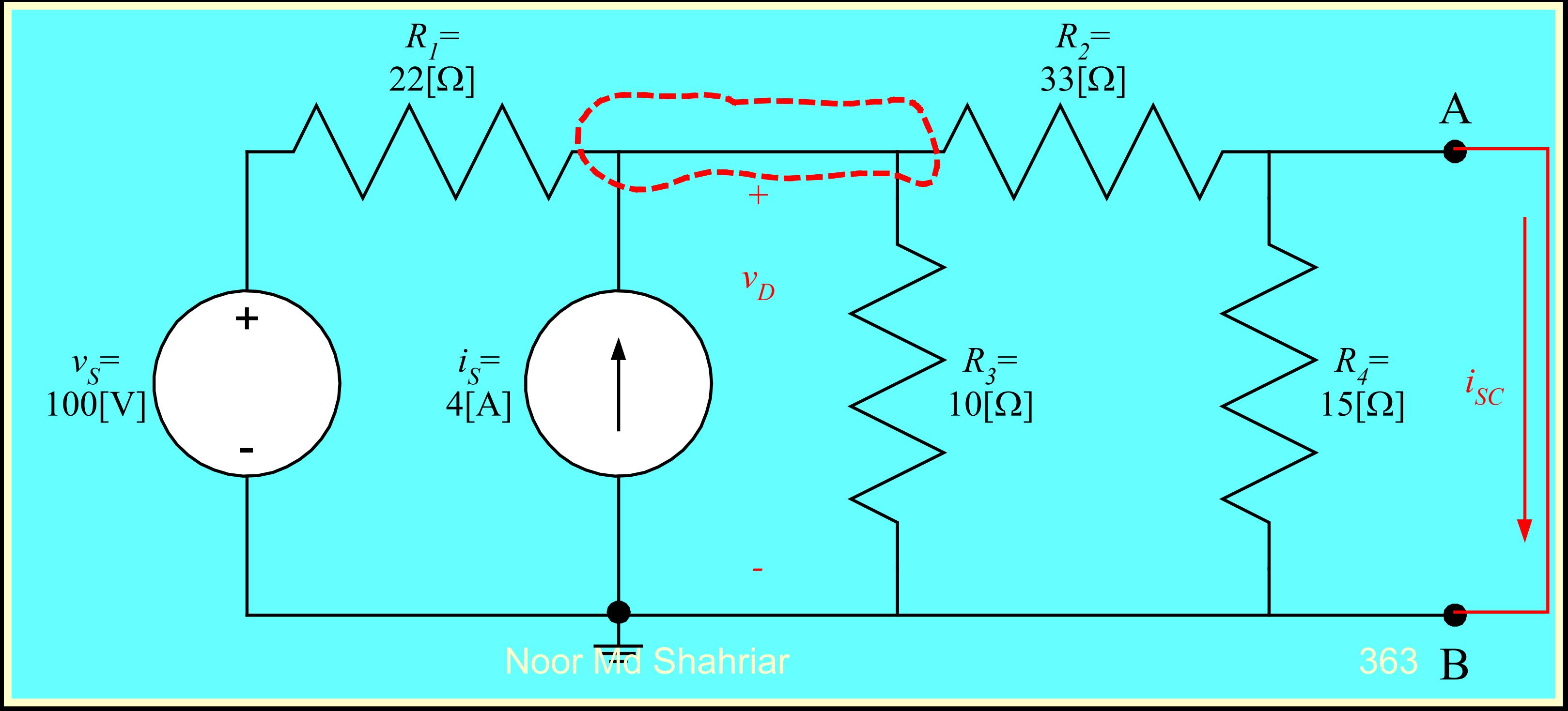


Example Problem – Step 10 (Check)

With this result, we can find the short-circuit current in the original circuit.

$$i_{SC} = \frac{v_D}{33[\Omega]} = \frac{48.6[V]}{33[\Omega]} = 1.5[A].$$

This is the same result that we found using the Thévenin's equivalent earlier.

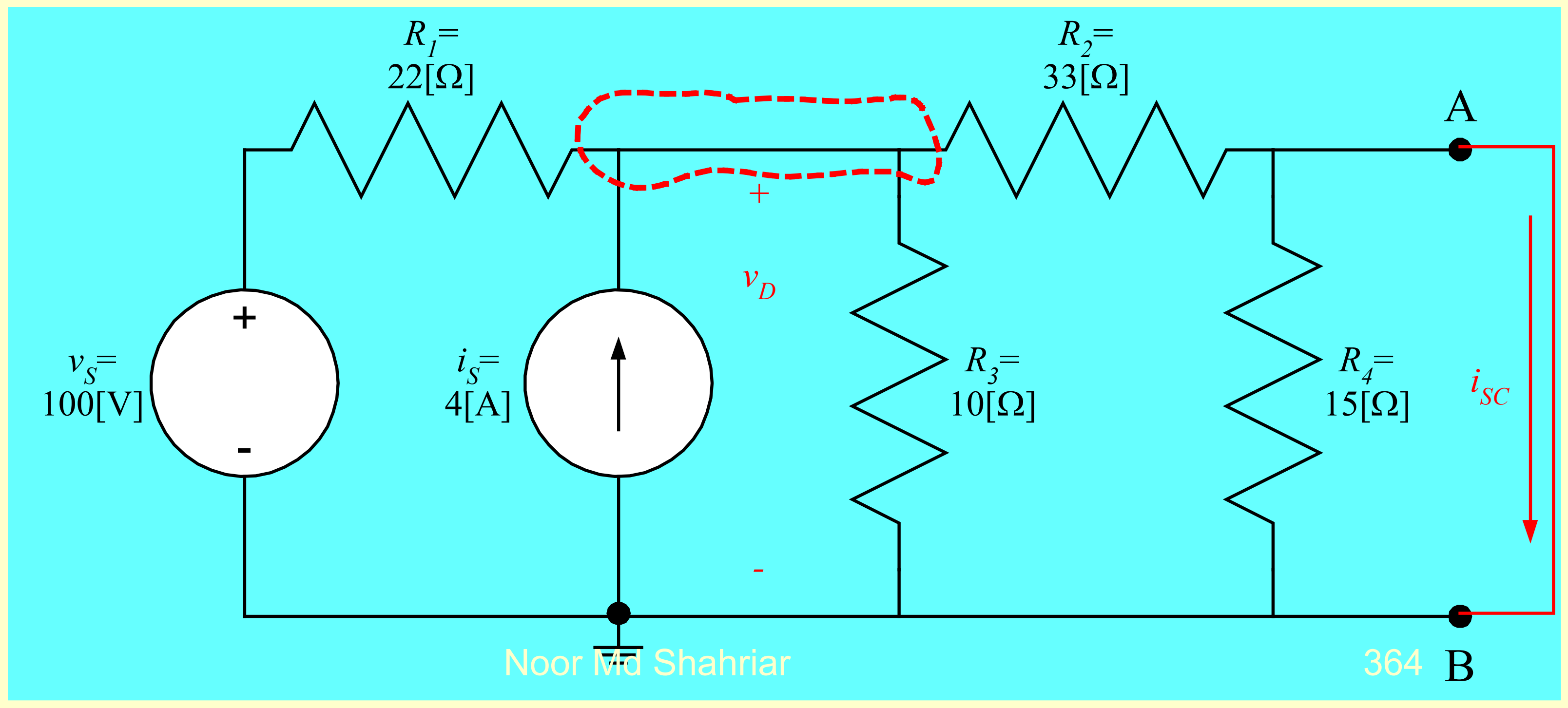


Go back to Overview slide.

Example Problem – Step 11 (Check)

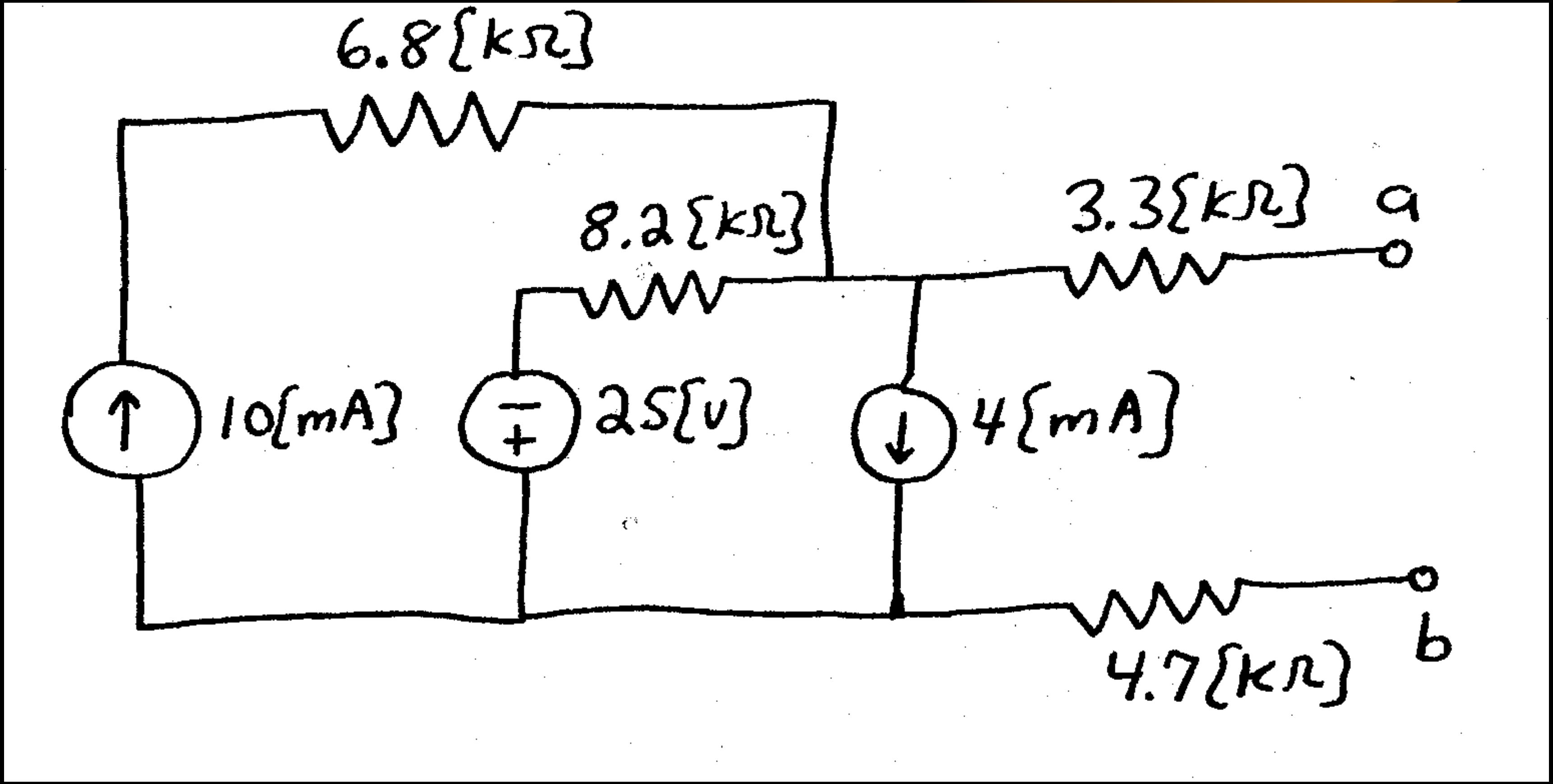
This is important. This shows that we could indeed have found any two of three of the quantities: open-circuit voltage, short-circuit current, and equivalent resistance.

$$i_{SC} = \frac{v_{OC}}{R_{EQ}} = \frac{16[\mathrm{V}]}{10.9[\Omega]} = 1.5[A].$$



Sample Problem #1

Find the Thévenin equivalent of the circuit shown, with respect to terminals a and b. Draw the equivalent, labeling terminals a and b.



Soln: v_{TH} = 24.2[V], R_{TH} = 16.2[kΩ]

Week -12

Page- (367-432)

Norton’s Theorem

Norton's Theorem Defined

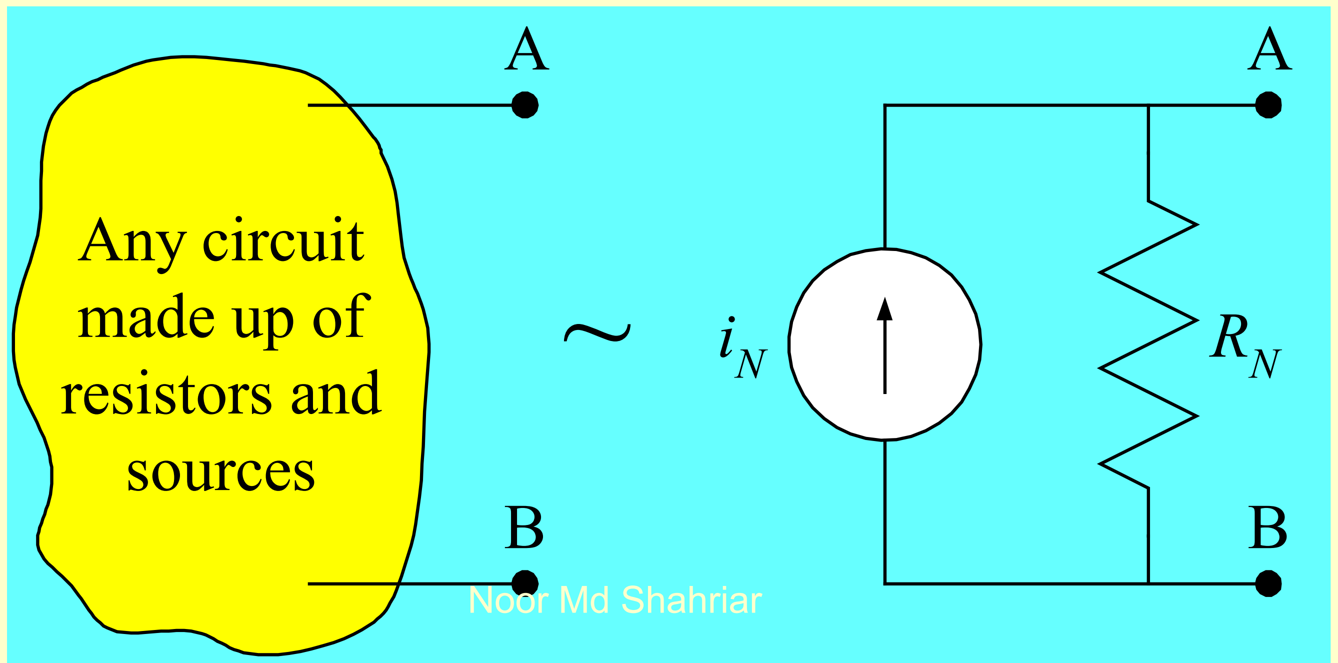
Norton's Theorem is another equivalent circuit. Norton's Theorem can be stated as follows:

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a current source in parallel with a resistance.

The current source is equal to the short-circuit current for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

i_N = short-circuit current, and

R_N = equivalent resistance.



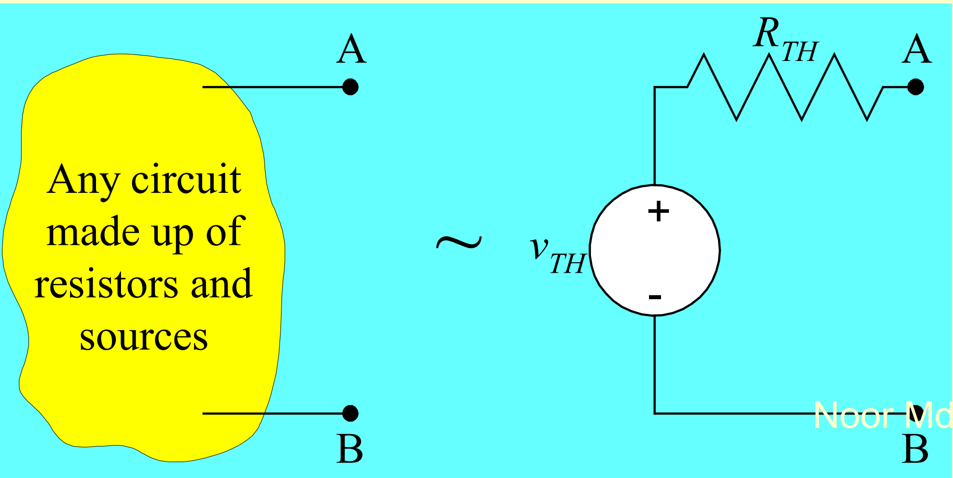
Note 1

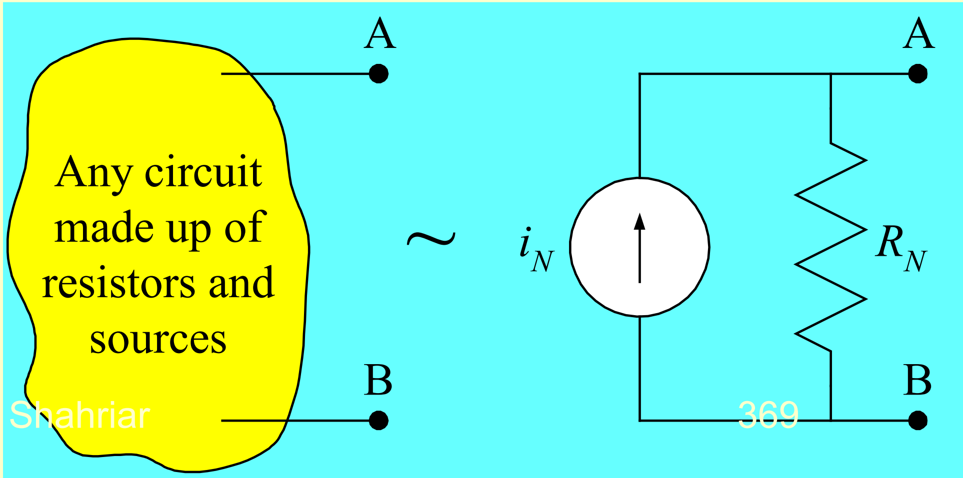
It is probably obvious to you, if you studied the last two parts of this module, that if Thévenin's Theorem is valid, then Norton's Theorem is valid, because Norton's Theorem is simply a source transformation of Thévenin's Theorem. Note that the resistance value is the same in both cases, that is, $R_{TH} = R_N = R_{EQ}$.

v_{OC} = open-circuit voltage,

i_{SC} = short-circuit current, and

R_{EQ} = equivalent resistance.





Note 2

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a current source in parallel with a resistance.

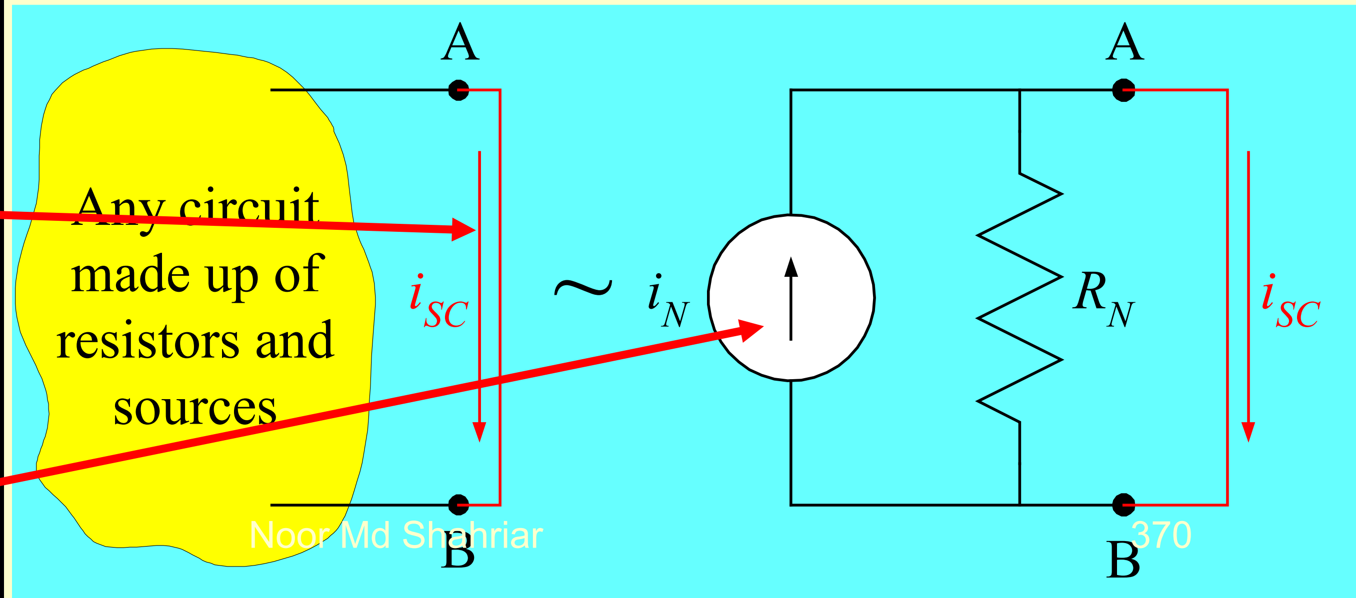
The current source is equal to the short-circuit current for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{OC} = open-circuit voltage,

i_{SC} = short-circuit current, and

R_{EQ} = equivalent resistance.

The polarity of the current source with respect to the terminals is important. If the reference polarity for the short-circuit current is as given here (flowing from A to B), then the reference polarity for the current source must be as given here (current from B to A).



Note 3

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a current source in parallel with a resistance.

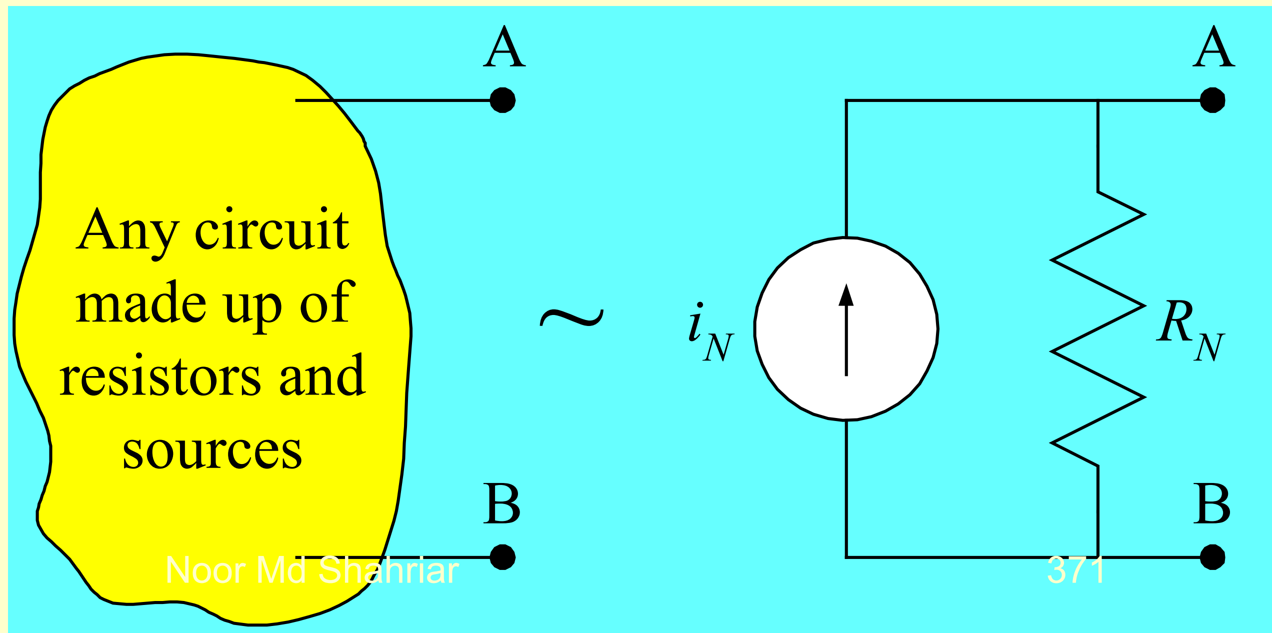
The current source is equal to the short-circuit current for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{OC} = open-circuit voltage,

i_{SC} = short-circuit current, and

R_{EQ} = equivalent resistance.

As with all equivalent circuits, these two are equivalent only with respect to the things connected to the equivalent circuits.



Note 4

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a current source in parallel with a resistance.

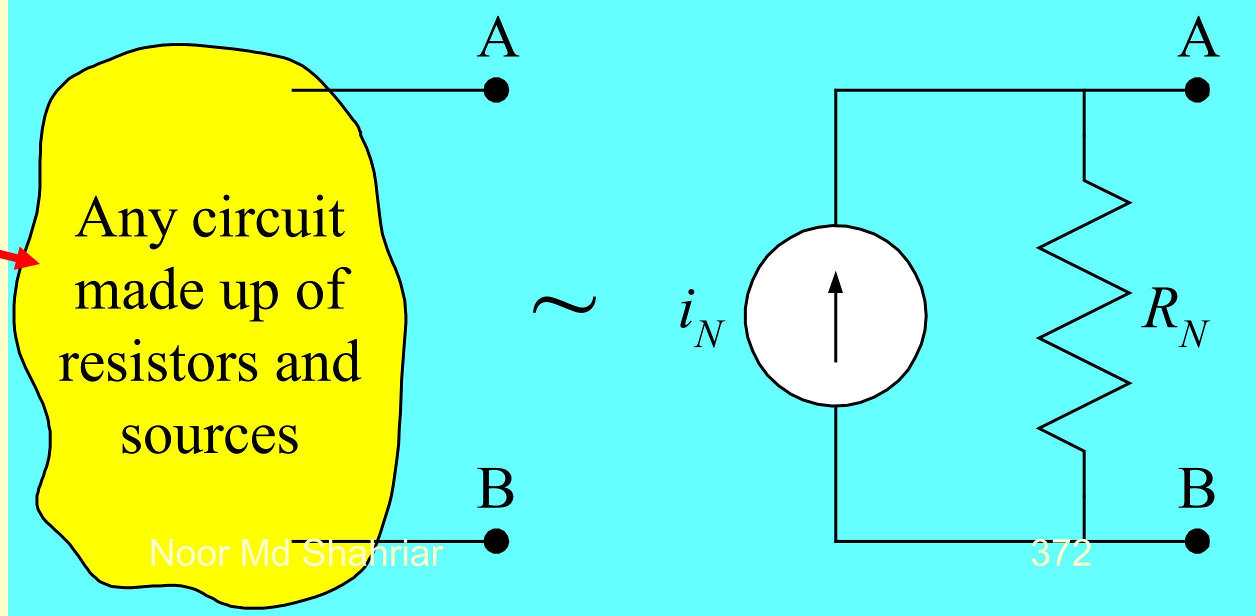
The current source is equal to the short-circuit current for the two-terminal circuit, and the resistance is equal to the equivalent resistance of the circuit.

v_{OC} = open-circuit voltage,

i_{SC} = short-circuit current, and

R_{EQ} = equivalent resistance.

When we have dependent sources in the circuit shown here, it will make some calculations more difficult, but does not change the validity of the theorem.

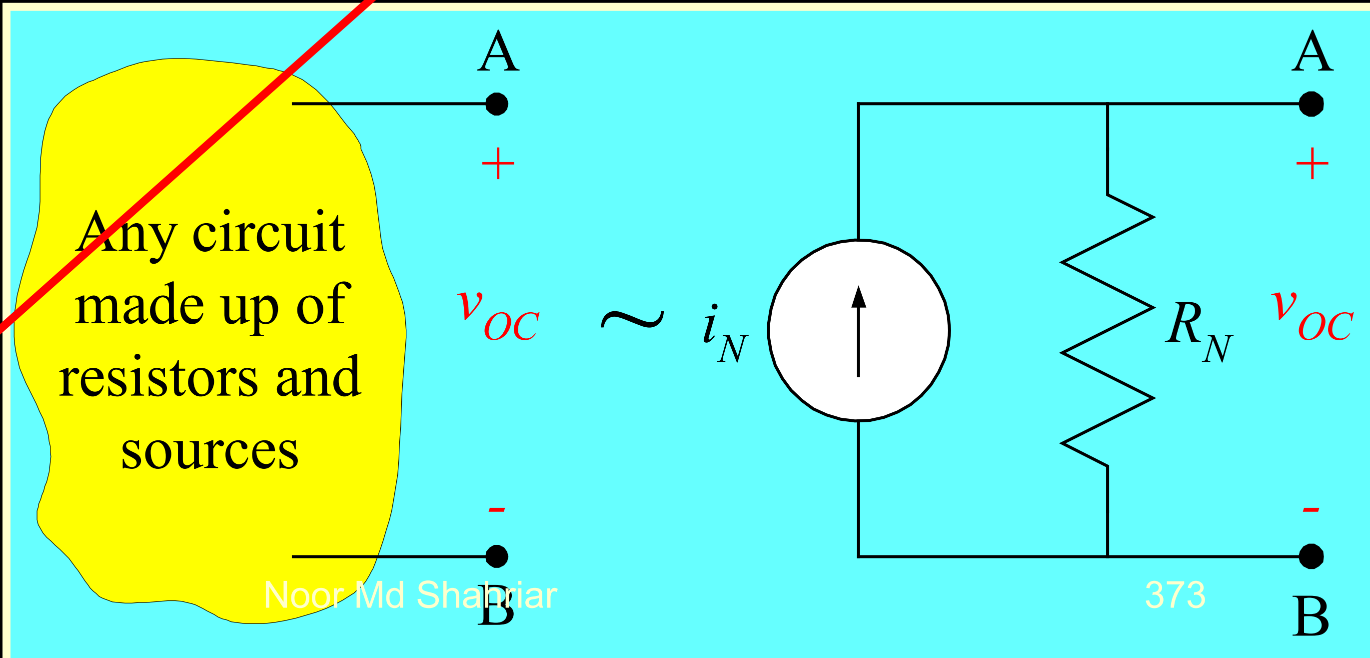


Short-Circuit Current and Open-Circuit Voltage

The open-circuit voltage that results from the Norton equivalent is equal to the product of the Norton current source and the Norton resistance. This leads to the same equation that we used previously, for the Thévenin equivalent,

$$v_{OC} = i_{SC} R_{EQ}.$$

When we look at the circuit on the right, we can see that the open-circuit voltage is equal to $i_N R_N$, which is also $i_{SC} R_{EQ}$. Thus, we obtain the important expression for v_{OC}, shown here.



Go back to Overview slide.

Extra note

We have shown that for the Norton equivalent, the open-circuit voltage is equal to the short-circuit current times the equivalent resistance. This is fundamental and important. However, it is not Ohm's Law.

This equation is not really Ohm's Law. It looks like Ohm's Law, and has the same form. However, it should be noted that Ohm's Law relates voltage and current for a resistor. This relates the values of voltages, currents and resistances in two different connections to an equivalent circuit. However, if you wish to remember this by relating it to Ohm's Law, that is fine.

$$v_{OC} = i_{SC} R_{EQ}.$$

Remember that

$i_{SC} = i_N$,

and

$R_{EQ} = R_N$.

Finding the Norton Equivalent

We have shown that for the Norton equivalent, the open-circuit voltage is equal to the short-circuit current times the equivalent resistance. In general we can find the Norton equivalent of a circuit by finding
any two of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, i_{SC}, and
3) the equivalent resistance, R_{EQ}.

Once we find any two, we can find the third by using this equation,

$$v_{OC} = i_{SC} R_{EQ}.$$

Remember that
$i_{SC} = i_N$,
and
$R_{EQ} = R_N$.

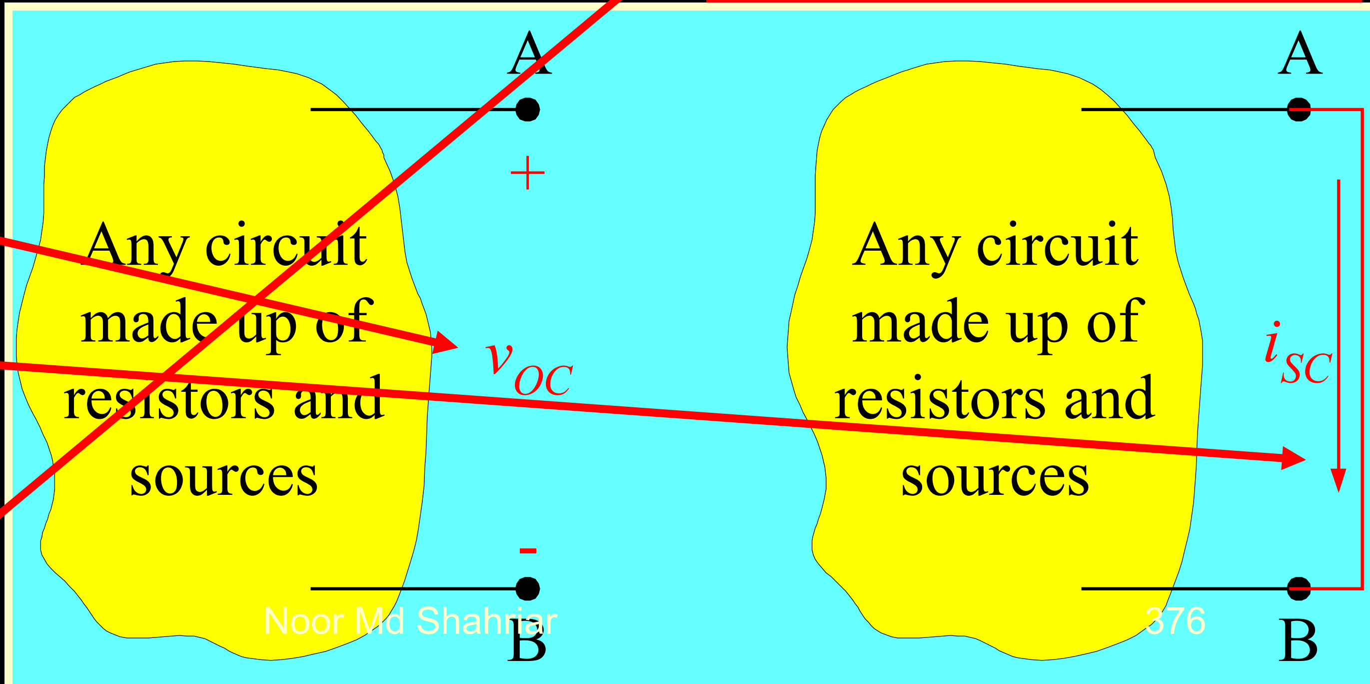
Finding the Norton Equivalent – Note 1

We can find the Norton equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, $i_{SC} = i_N$, and
3) the equivalent resistance, $R_{EQ} = R_N$.

$$v_{OC} = i_{SC} R_{EQ}.$$

One more time, the reference polarities of our voltages and currents matter. If we pick v_{OC} at A with respect to B, then we need to pick i_{SC} going from A to B. If not, we need to change the sign in this equation.



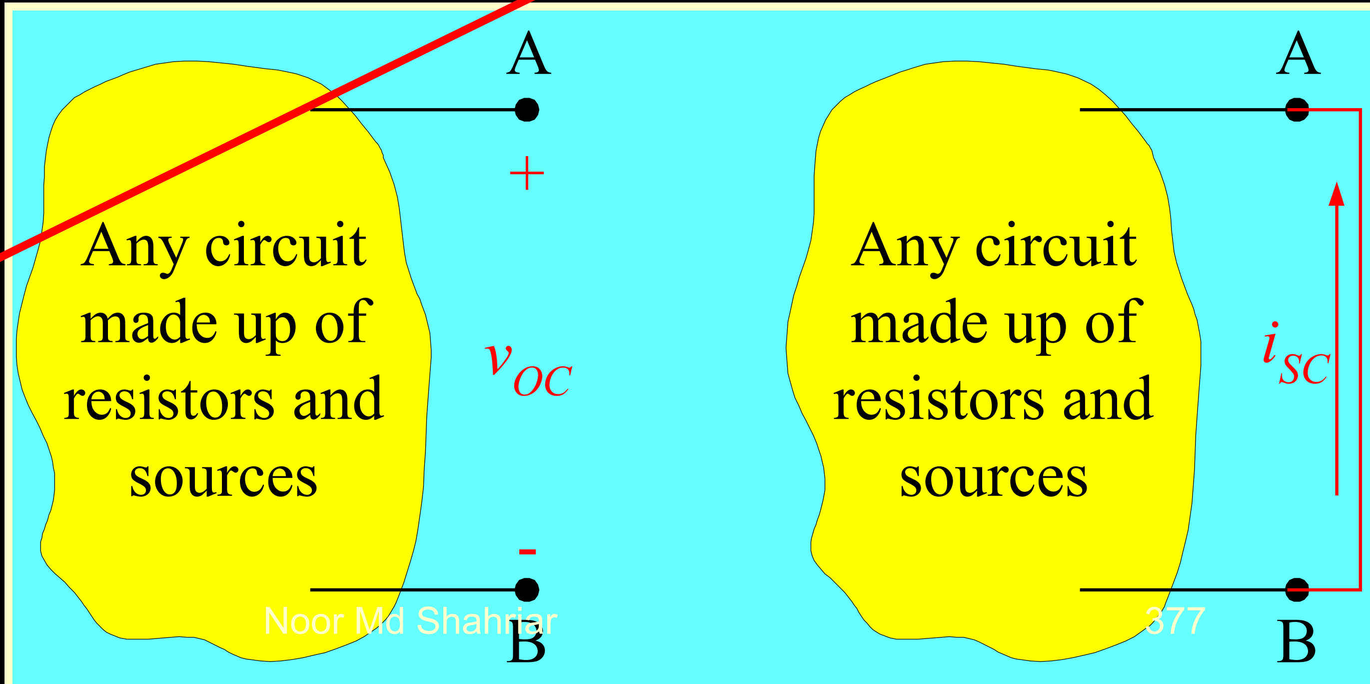
Finding the Norton Equivalent – Note 2

We can find the Norton equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, $i_{SC} = i_N$, and
3) the equivalent resistance, $R_{EQ} = R_N$.

$$v_{OC} = -i_{SC} R_{EQ}.$$

As an example, if we pick v_{OC} and i_{SC} with the reference polarities given here, we need to change the sign in the equation as shown. This is a consequence of the sign in Ohm's Law. For a further explanation, see the next slide.

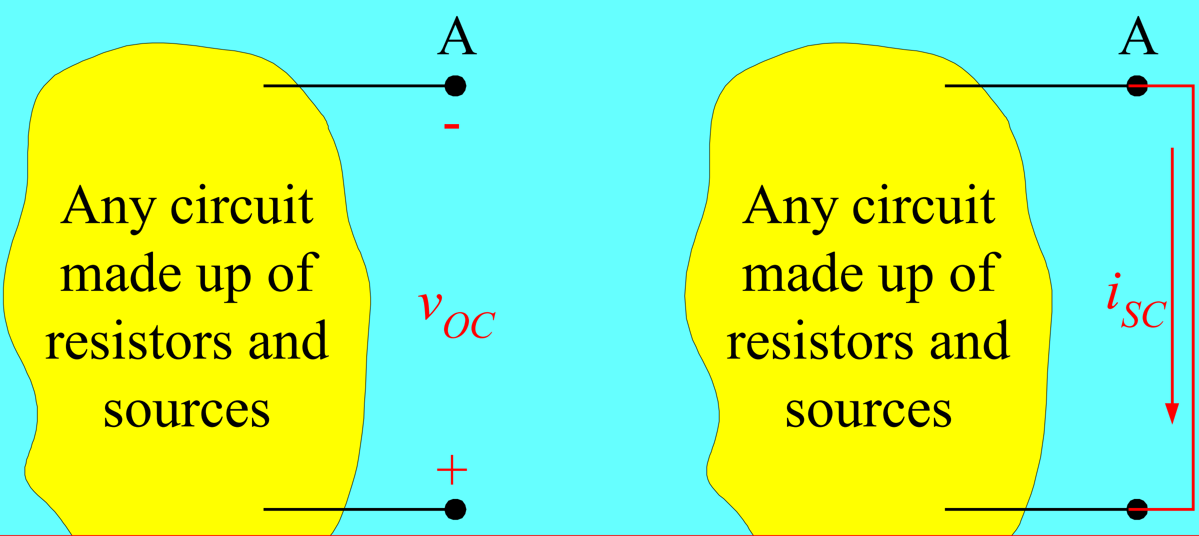


Finding the Norton Equivalent – Note 3

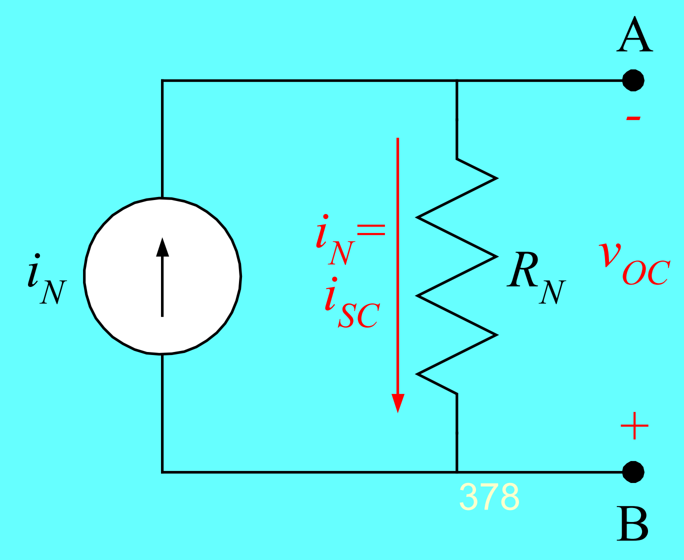
We can find the Norton equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, $i_{SC} = i_N$, and
3) the equivalent resistance, $R_{EQ} = R_N$.

$$v_{OC} = -i_{SC} R_{EQ}.$$



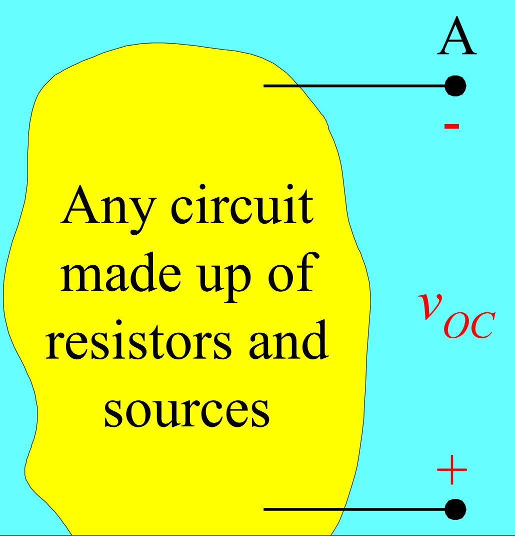
As an example, if we pick v_{OC} and i_{SC} with the reference polarities given here, we need to change the sign in the equation as shown. This is a consequence of Ohm’s Law, which for resistor R_N requires a minus sign, since the voltage and current are in the active sign relationship for R_N.

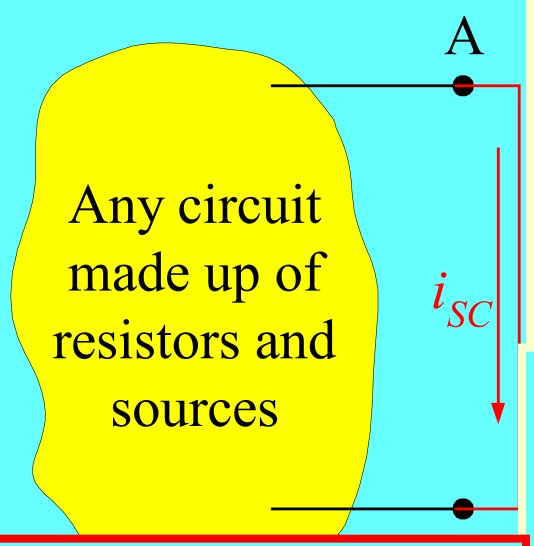


Finding the Norton Equivalent – Note 4

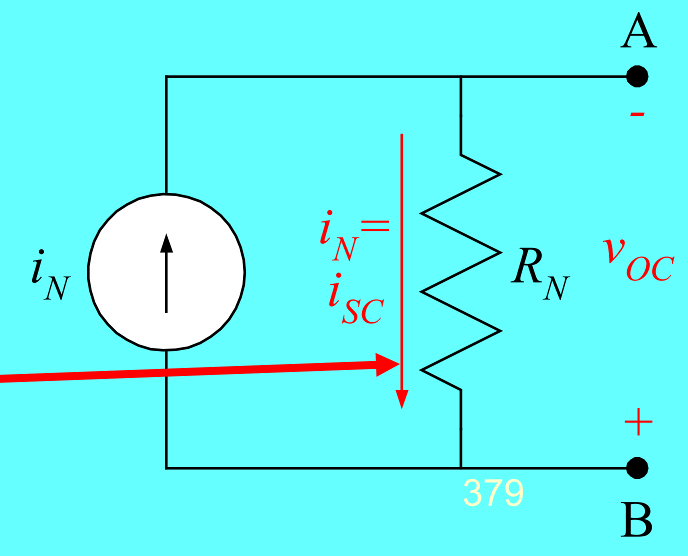
We can find the Norton equivalent of a circuit by finding ***any two*** of the following three things:

1) the open circuit voltage, v_{OC},
2) the short-circuit current, $i_{SC} = i_N$, and
3) the equivalent resistance, $R_{EQ} = R_N$.





$$v_{OC} = -i_{SC} R_{EQ}.$$



Be very careful here! We have labeled the current through R_N as i_{SC}. This is true only for this special case. This i_{SC} is not the current through the open circuit. The current through an open circuit is always zero. The current i_{SC} only goes through R_N because of the open circuit.

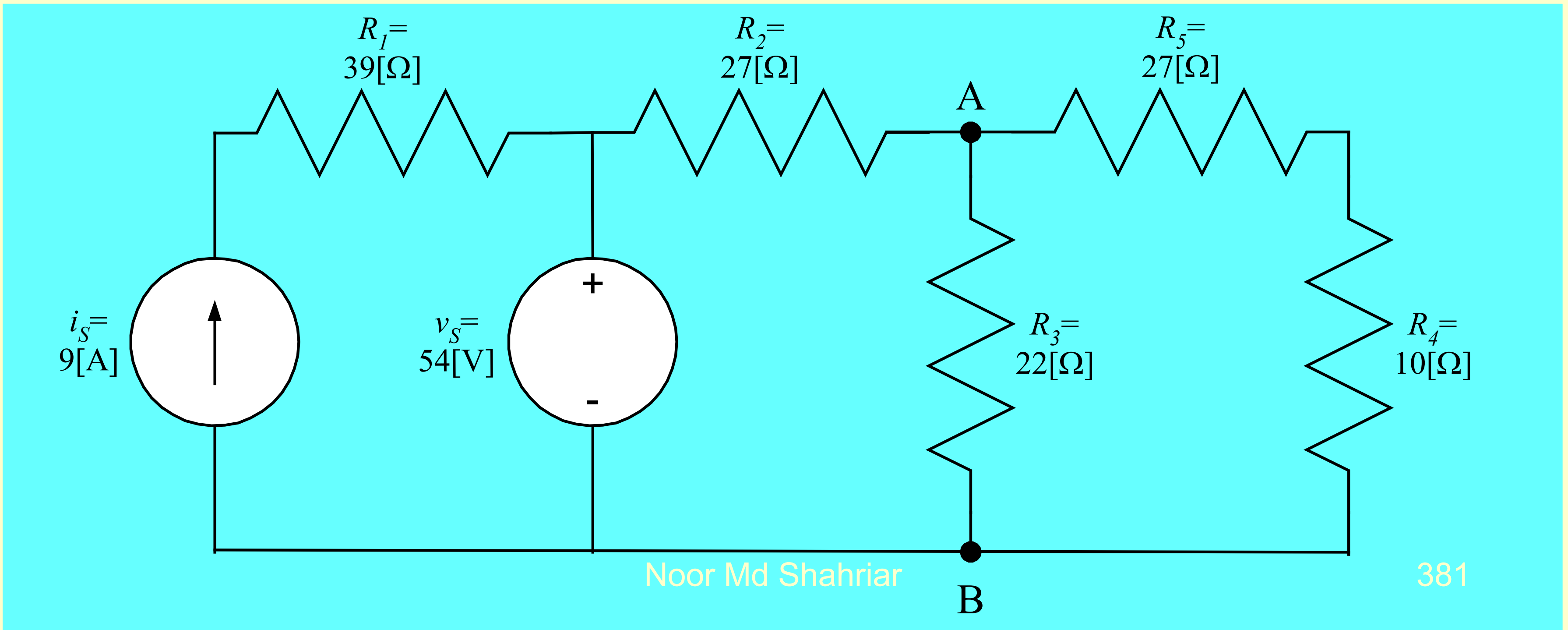
Notes

1. We can find the Norton equivalent of any circuit made up of voltage sources, current sources, and resistors. The sources can be any combination of dependent and independent sources.
2. We can find the values of the Norton equivalent by finding the open-circuit voltage and short-circuit current. The reference polarities of these quantities are important.
3. To find the equivalent resistance, we need to set the independent sources equal to zero. However, the dependent sources will remain. This requires some care. We will discuss finding the equivalent resistance with dependent sources in the fourth part of the module.
4. As with all equivalent circuits, the Norton equivalent is equivalent only with respect to the things connected to it.

Example Problem

We wish to find the Norton equivalent of the circuit below, as seen from terminals A and B.

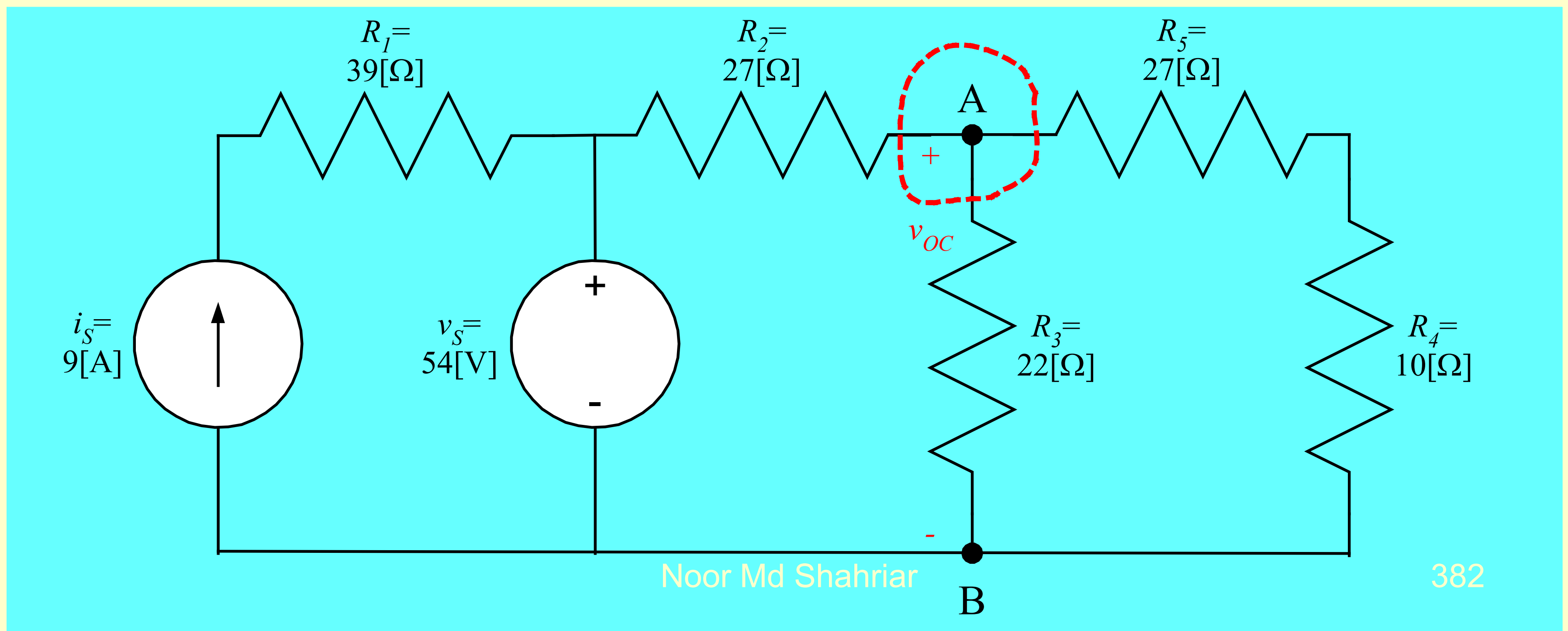
Note that there is an unstated assumption here; we assume that we will later connect something to these two terminals. Having found the Norton equivalent, we will be able to solve that circuit more easily by using that equivalent.



Example Problem – Step 1

We wish to find the open-circuit voltage v_{OC} with the polarity defined in the circuit given below.

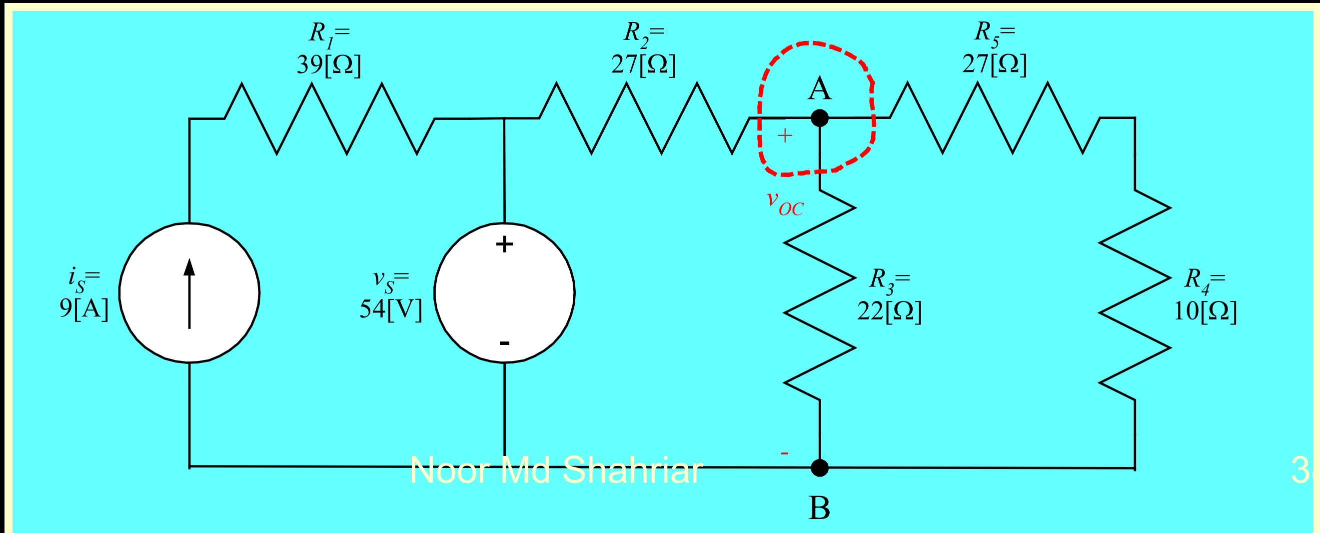
In general, remember, we need to find two out of three of the quantities v_{OC}, i_{SC}, and R_{EQ}. In this problem we will find two, and then find the third just as a check. In general, finding the third quantity is not required.



Example Problem – Step 1 (Note)

We wish to find the open-circuit voltage v_{OC} with the polarity defined in the circuit given below.

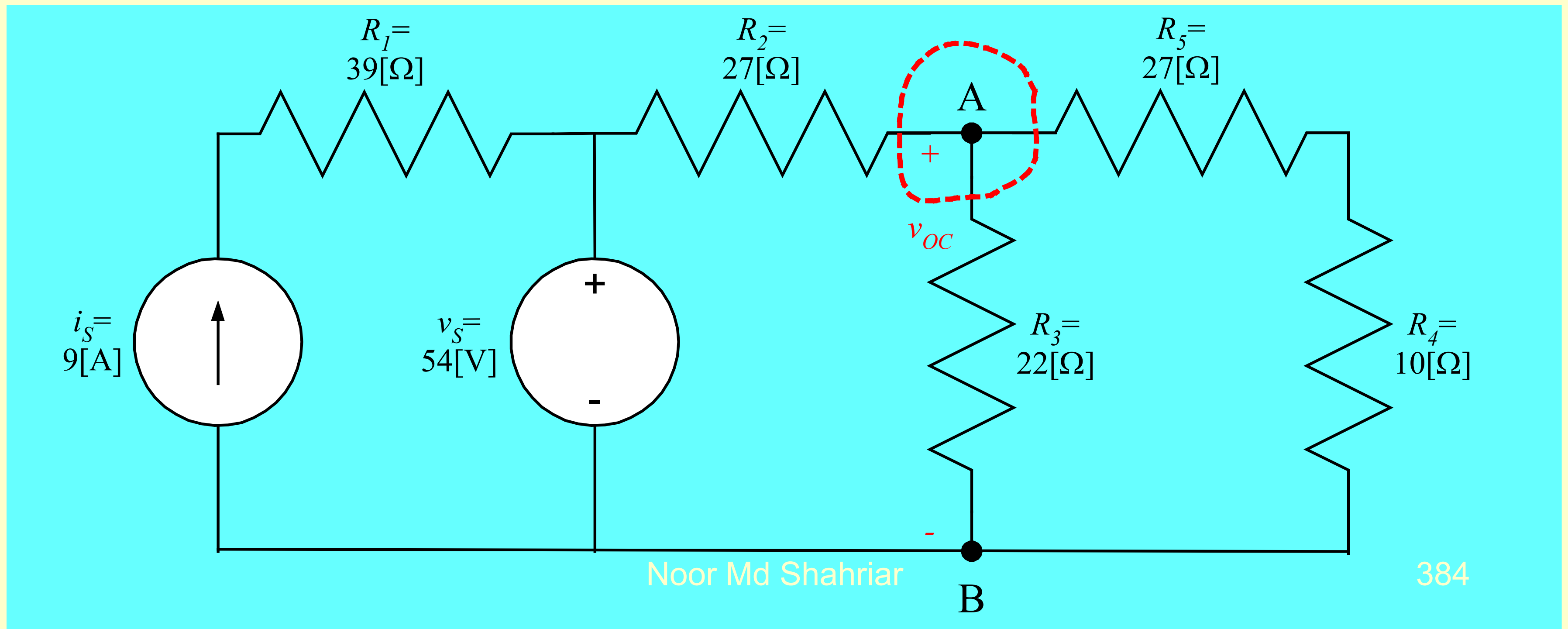
Some students may be tempted to remove resistor R_3 from this circuit. We should not do this. In future problems, **if** we are asked to find "the equivalent circuit seen by resistor R_3", then we assume that the resistor "does not see itself", and remove it. In this problem, we are not given this instruction. Leave the resistor in place, even though the open-circuit voltage is across it.



Example Problem – Step 2

We wish to find the voltage v_{OC}. Writing KCL at the node encircled with a dashed red line, we have

$$\frac{v_{OC}}{R_5 + R_4} + \frac{v_{OC}}{R_3} + \frac{v_{OC} - v_S}{R_2} = 0.$$



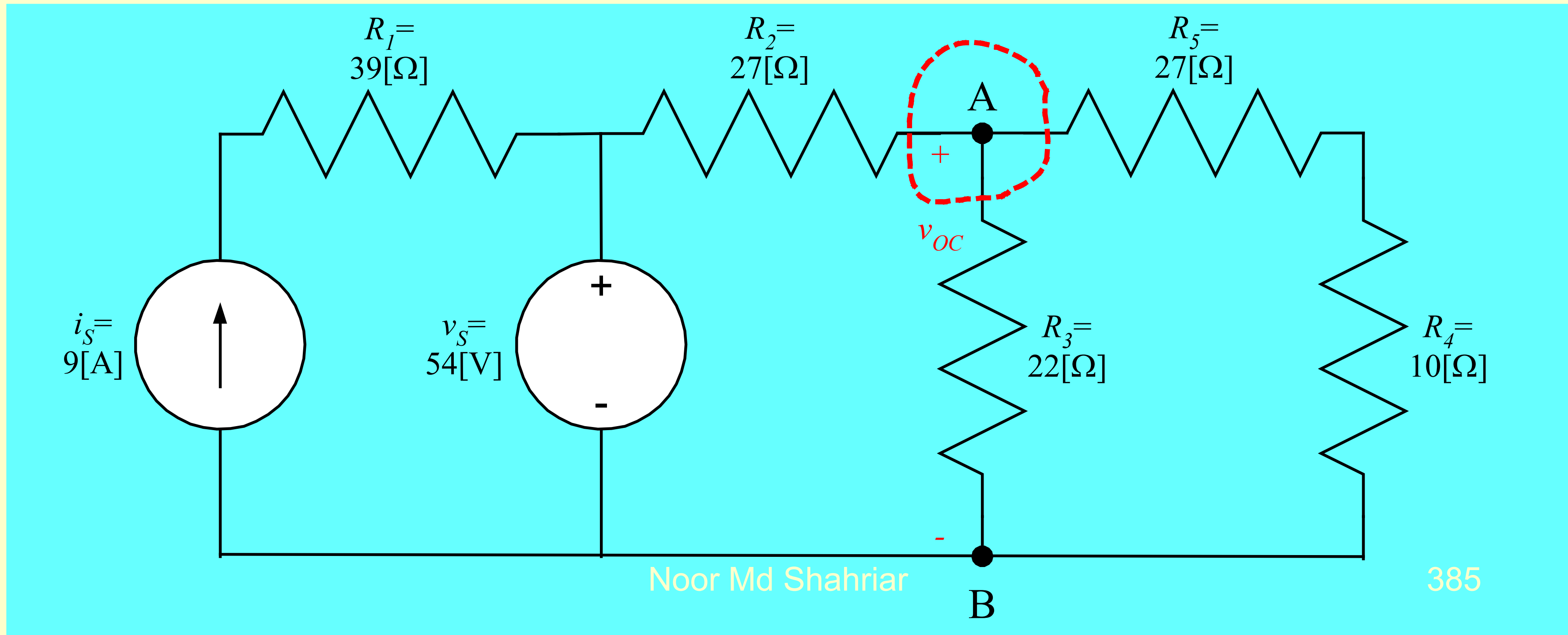
Example Problem – Step 3

Substituting in values, we have

$$\frac{v_{OC}}{37[\Omega]} + \frac{v_{OC}}{22[\Omega]} + \frac{v_{OC} - 54[\text{V}]}{27[\Omega]} = 0, \text{ or by solving}$$

$$v_{OC}\left(0.1095[\text{S}]\right) = 2[\text{A}], \text{ or}$$

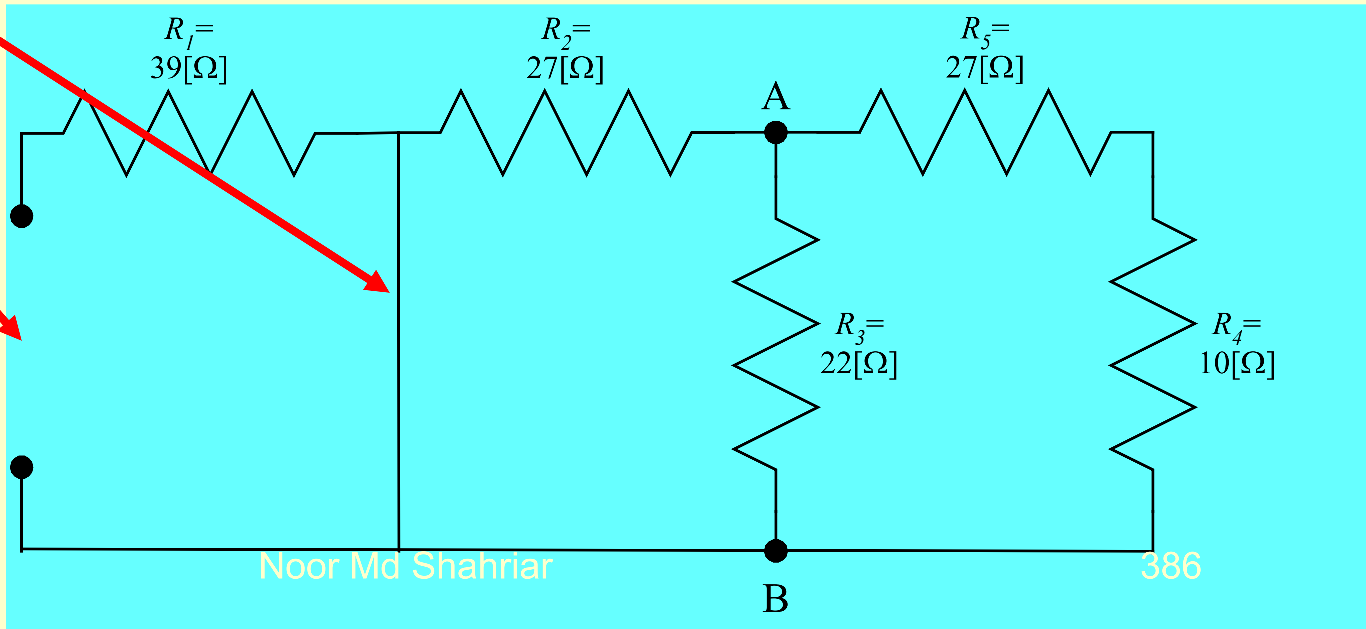
$$v_{OC} = 18.3[\text{V}].$$



Example Problem – Step 4

Next, we will find the equivalent resistance, R_{EQ}. The first step in this solution is to set the independent sources equal to zero. We then have the circuit below.

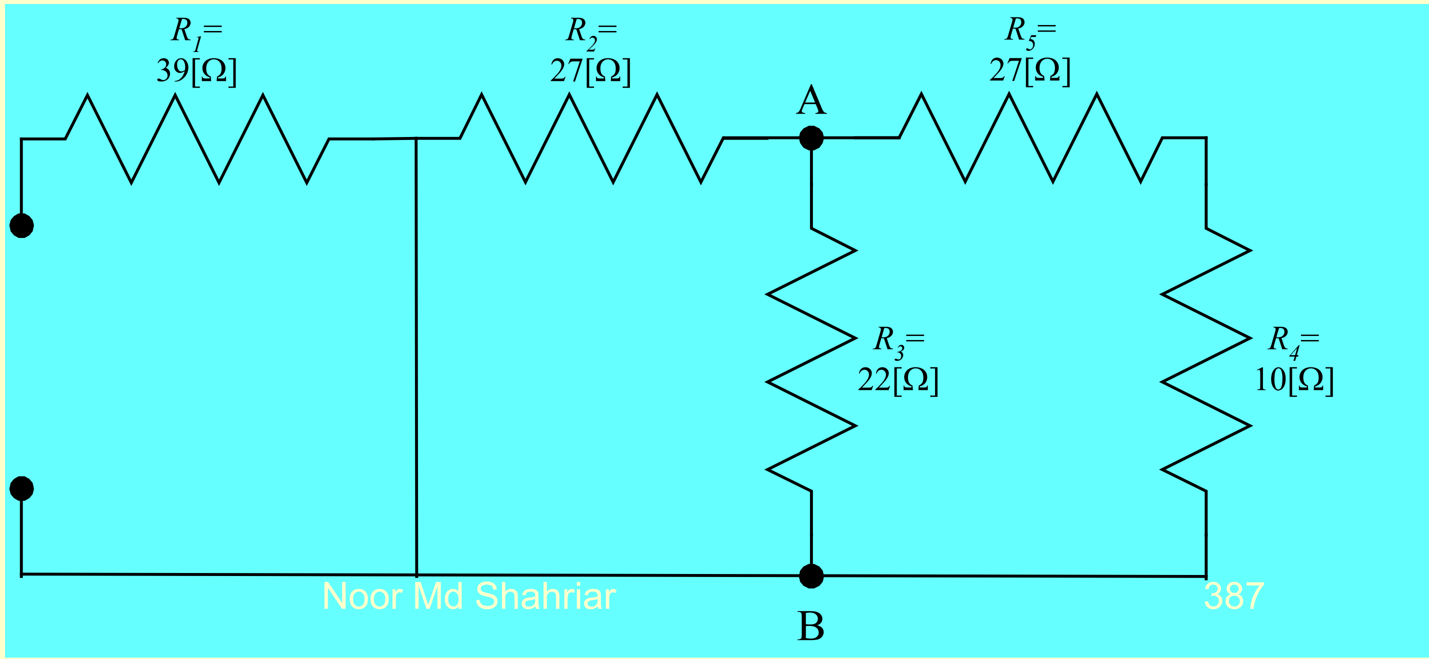
Note that the voltage source becomes a short circuit, and the current source becomes an open circuit. These represent zero-valued sources.



Example Problem – Step 5

To find the equivalent resistance, R_{EQ}, we simply combine resistances in parallel and in series. The resistance between terminals A and B, which we are calling R_{EQ}, is found by recognizing that R_5 and R_4 are in series. That series combination is in parallel with R_2. That parallel combination is in parallel with R_3. We have

$R_{EQ} = \left(R_5 + R_4\right) \| R_2 \| R_3 = 37[\Omega] \| 27[\Omega] \| 22[\Omega]$. Solving, we get $R_{EQ} = 9.13[\Omega]$.



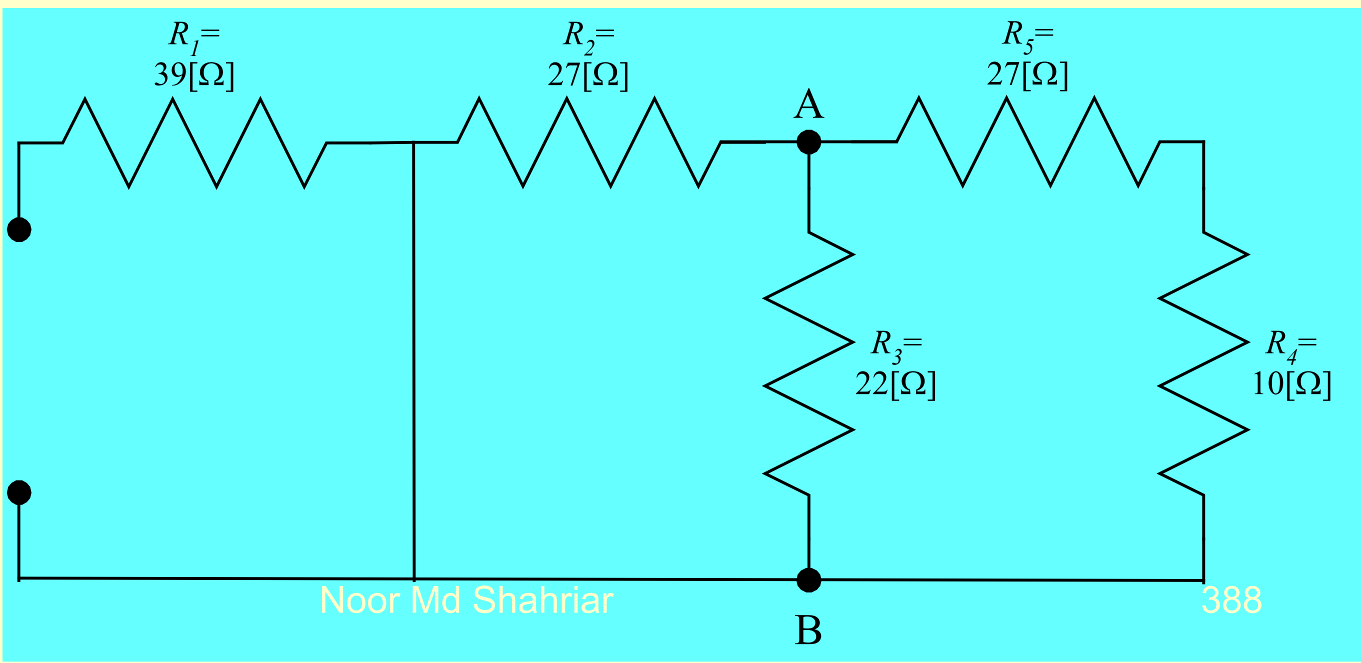
Example Problem – Step 5 (Note)

To find the equivalent resistance, R_{EQ}, we simply combine resistances in parallel and in series. The resistance between terminals A and B, which we are calling R_{EQ}, is found by recognizing that R_5 and R_4 are in series. That series combination is in parallel with R_2. That parallel combination is in parallel with R_3. We have

$$R_{EQ} = (R_5 + R_4) \,||\, R_2 \,||\, R_3 = 37[\Omega] \,||\, 27[\Omega] \,||\, 22[\Omega].$$ Solving, we get

$$= 9.13[\Omega].$$

Some students may have difficulty getting this expression. Remember that we are finding the resistance seen at these two terminals, A and B. The idea is that we would have this resistance if you connected a source to these two terminals.

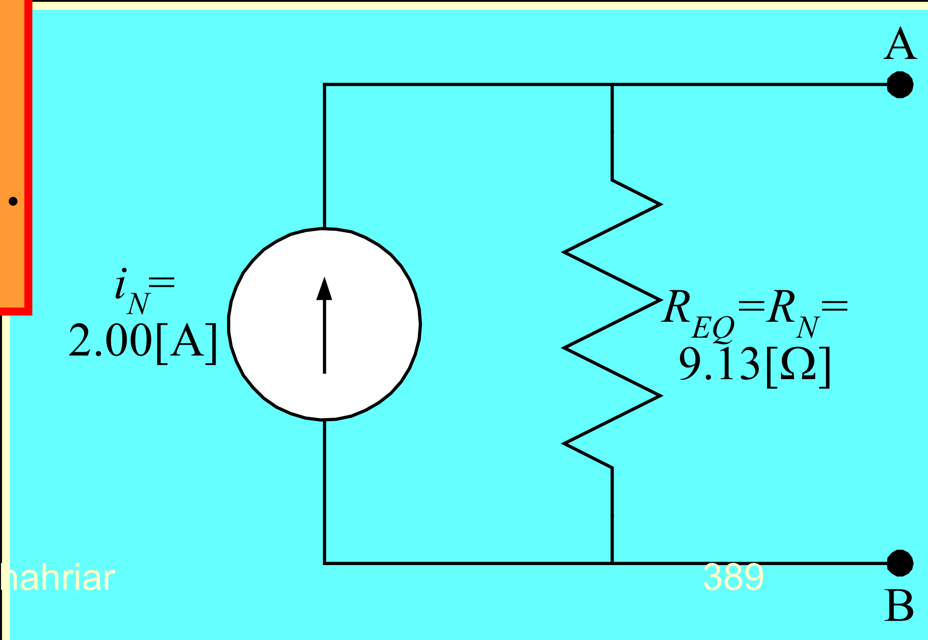


Example Problem – Step 6 (Solution)

To complete this problem, we would typically redraw the circuit, showing the complete Norton's equivalent, along with terminals A and B. This has been done here. To get this, we need to use our equation to get the Norton current,

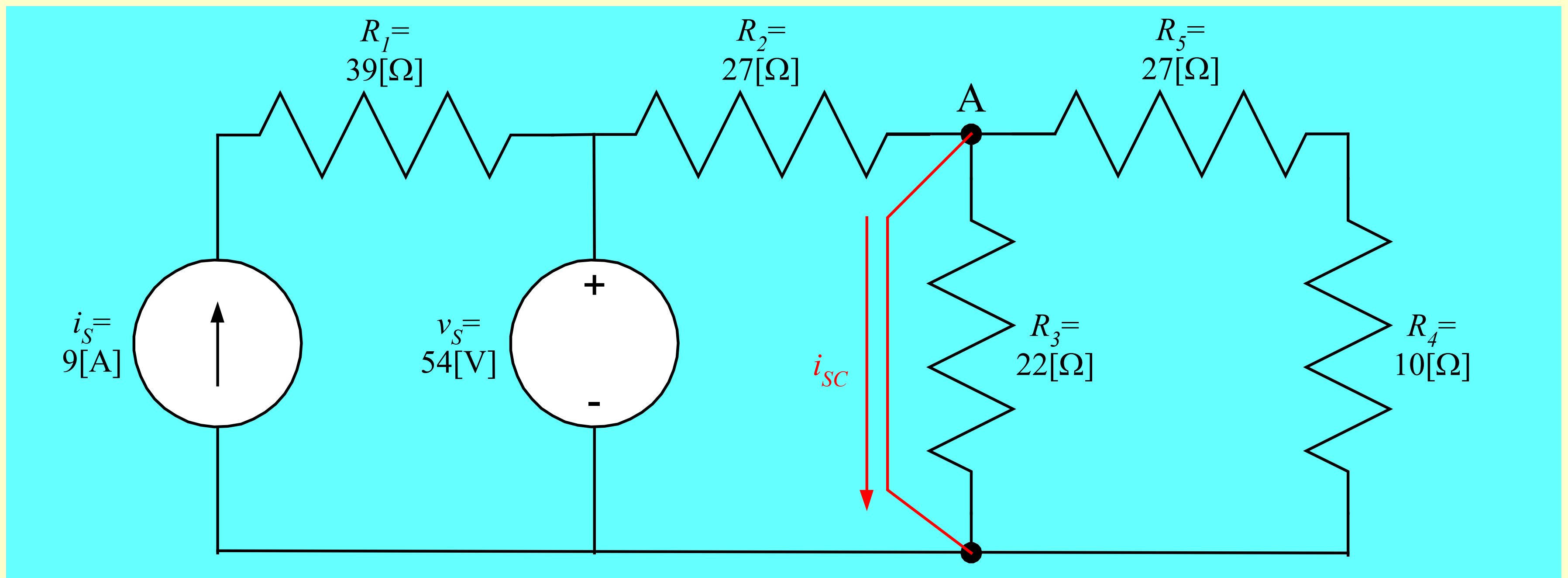
$v_{OC} = i_N R_{EQ}$. We will want to solve for i_N, so we write

$$i_N = \frac{v_{OC}}{R_{EQ}} = \frac{18.3[\text{V}]}{9.13[\Omega]} = 2.00[\text{A}].$$



Example Problem – Step 7 (Check)

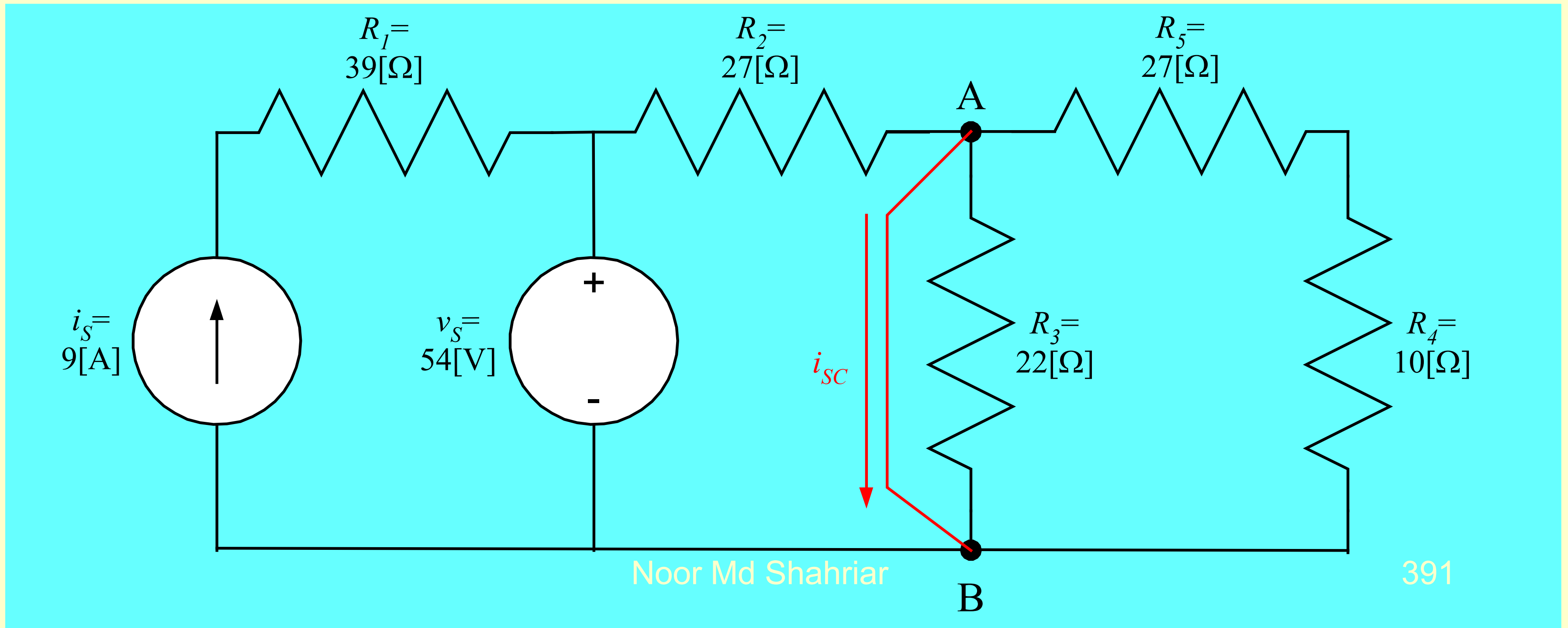
Let us check this solution, by finding the short-circuit current in the original circuit, and compare it to the short-circuit current in the Norton's equivalent. We redraw the original circuit, with the short circuit current shown. We wish to find this short circuit current, i_{SC}.



Example Problem – Step 8 (Check)

We start by noting that that there is no current through resistor R_3, since there is no voltage across it. Another way of saying this is that the resistor R_3 is in parallel with a short circuit. The parallel combination of the resistor and the short circuit, will be a short circuit.

The same exact argument can be made for the series combination of R_5 and R_4. This series combination is in parallel with a short circuit. Thus, we can simplify this circuit to the circuit on the next slide.

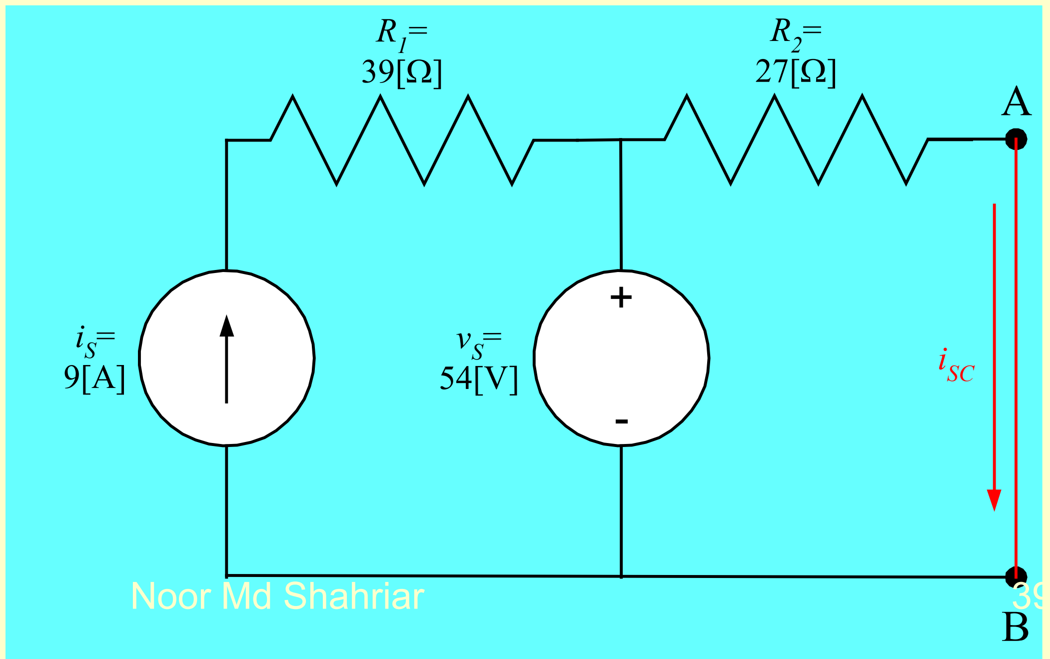


Example Problem – Step 9 (Check)

Here, we have removed resistors R_3, R_4 and R_5 since they do not affect the short circuit current, i_{SC}. When we look at this circuit, we note that the voltage source v_S is directly across the resistor R_2, and so we can write directly,

$$i_{SC} = \frac{v_S}{R_2} = \frac{54[\text{V}]}{27[\Omega]}.$$ Solving, we get

$$i_{SC} = 2.00[\text{A}].$$

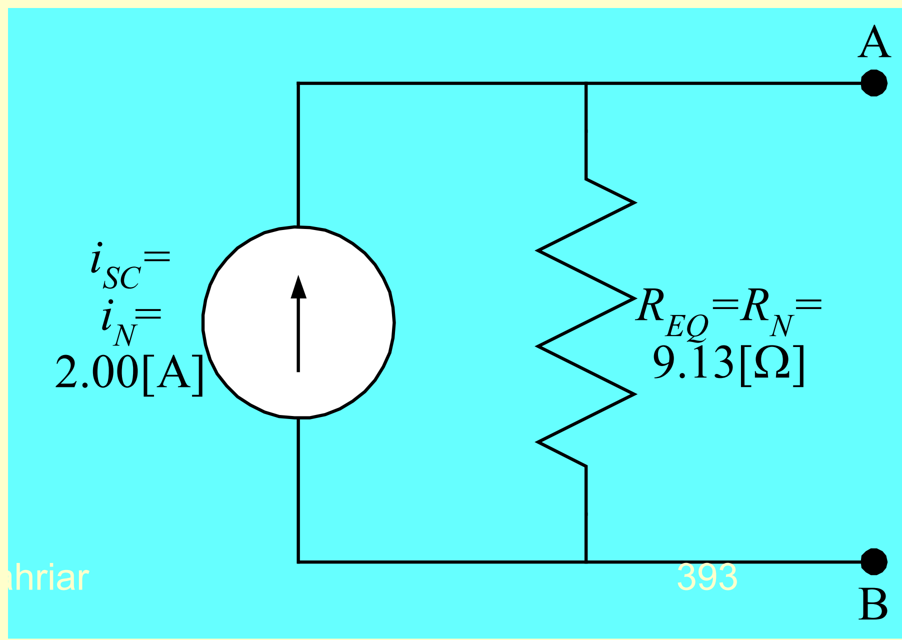


Example Problem – Step 10 (Check)

This short-circuit current is the same result that we found in the Norton's Equivalent earlier.

In retrospect, it is now clear that we did not take the best possible approach to this solution. If we had solved for the short-circuit current, and the equivalent resistance, we would have gotten the solution more quickly and more easily.

One of our goals is to be so good at circuit analysis that we can see ahead of time which approach will be the best for a given problem.

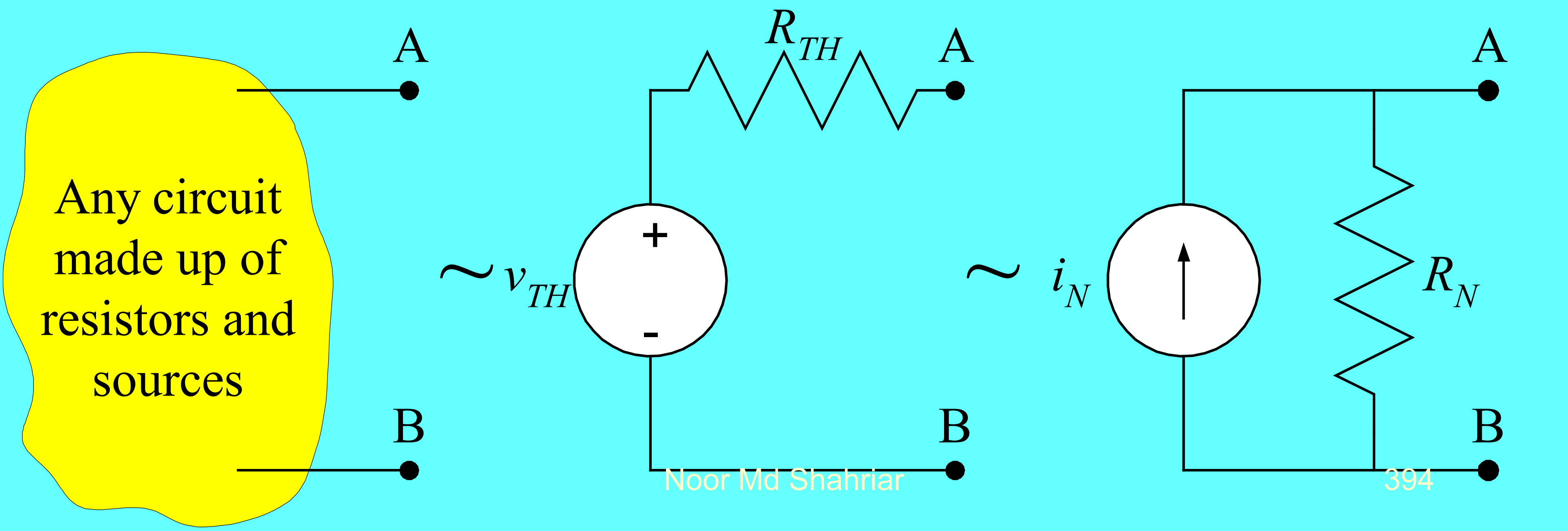


Thévenin's and Norton's Theorems Reviewed

Thévenin's Theorem and Norton's Theorem can be stated as follows:

Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance, or to a current source in parallel with a resistance.

The voltage source is equal to the open-circuit voltage for the two-terminal circuit, the current source is equal to the short-circuit current for that circuit, and the resistance is equal to the equivalent resistance of that circuit.

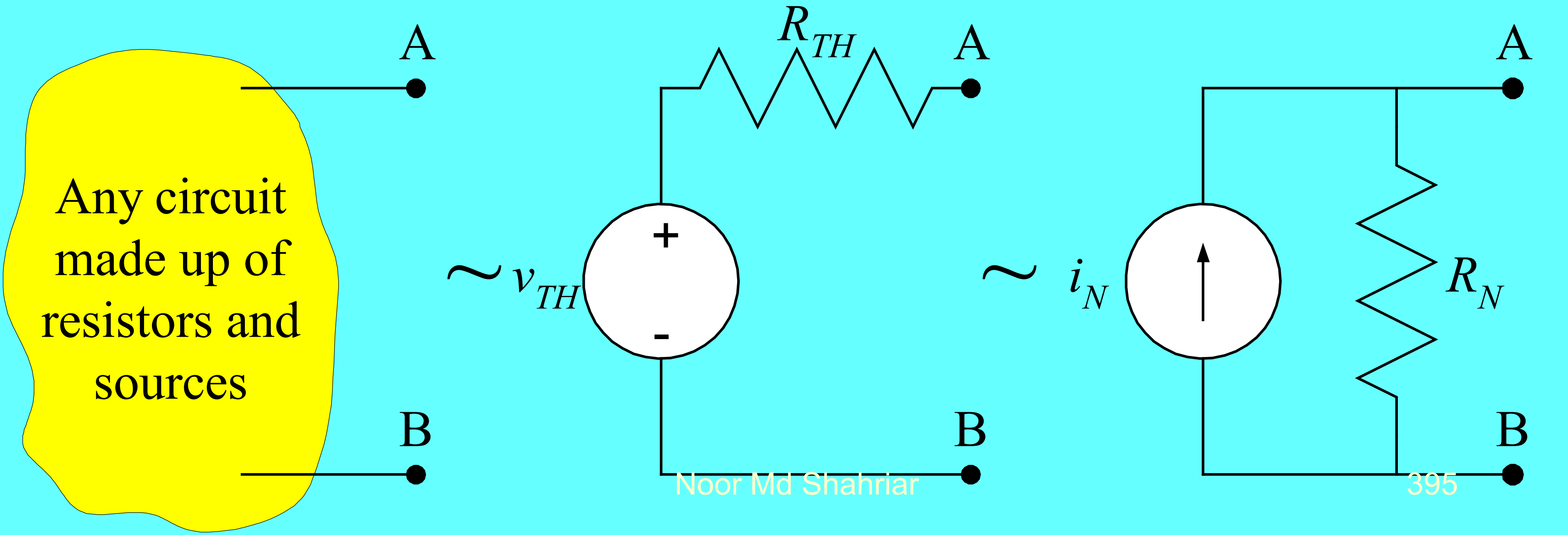


Equivalent Resistance Reviewed

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

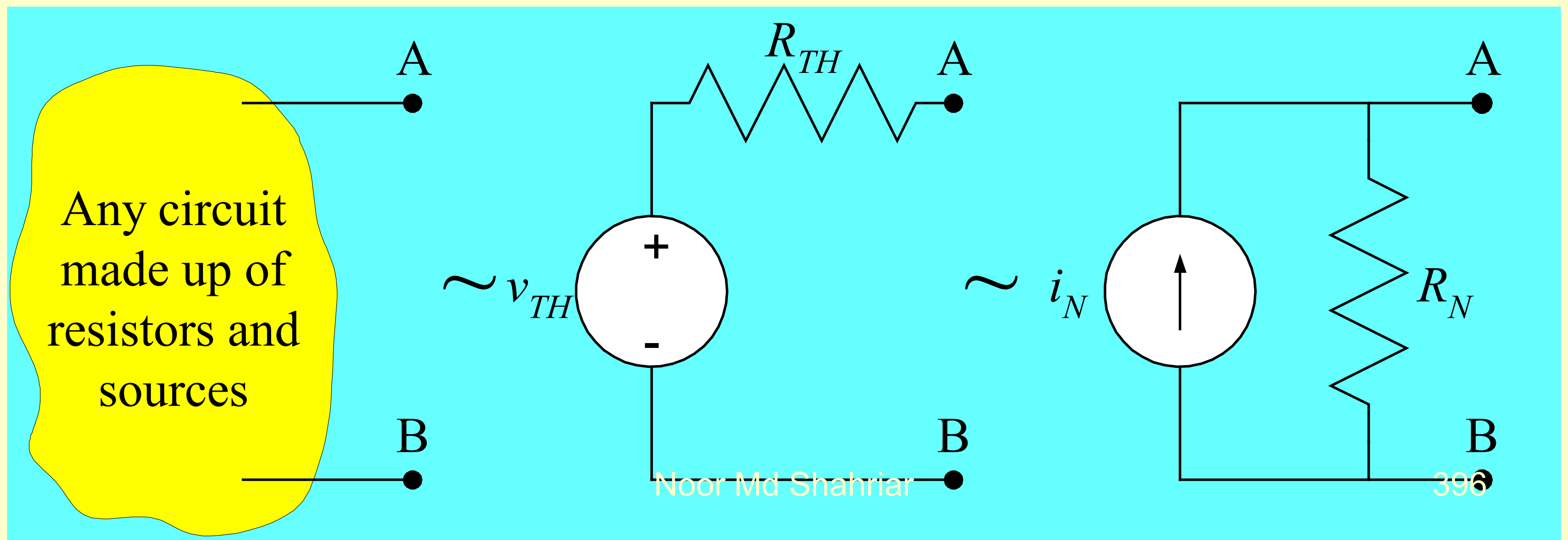
When a dependent source is present, trying to find the equivalent resistance results in a situation we have not dealt with yet. What do we mean by the equivalent resistance of a dependent source?

The answer must be stated carefully. If the ratio of voltage to current for something is a constant, then that something can be said to have an equivalent resistance, since it is behaving as a resistance.



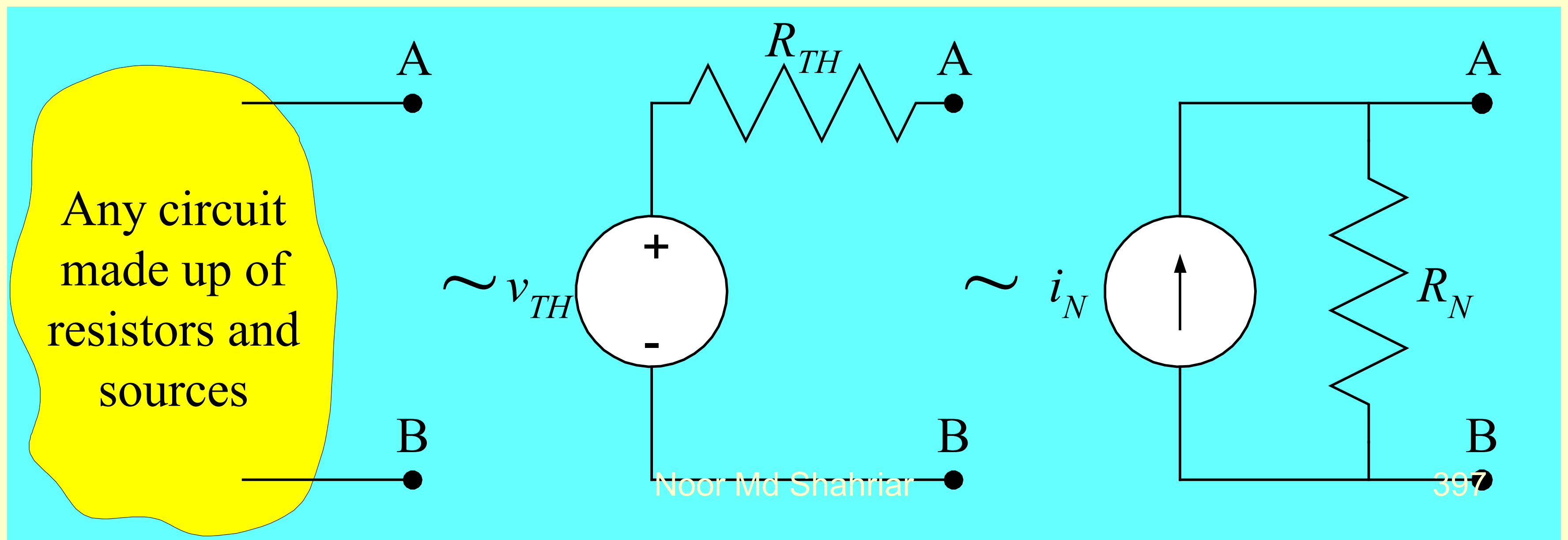
Equivalent Resistance of a Source

So, what we mean by the equivalent resistance of a dependent source is that in this case the ratio of voltage to current is a constant. Then the source can be said to have an equivalent resistance, since it is behaving as a resistance. The equivalent resistance of a dependent source depends on what voltage or current it depends on, and where that voltage or current is in the circuit. It is not easy to predict the answer.



No Equivalent Resistance for an Independent Source

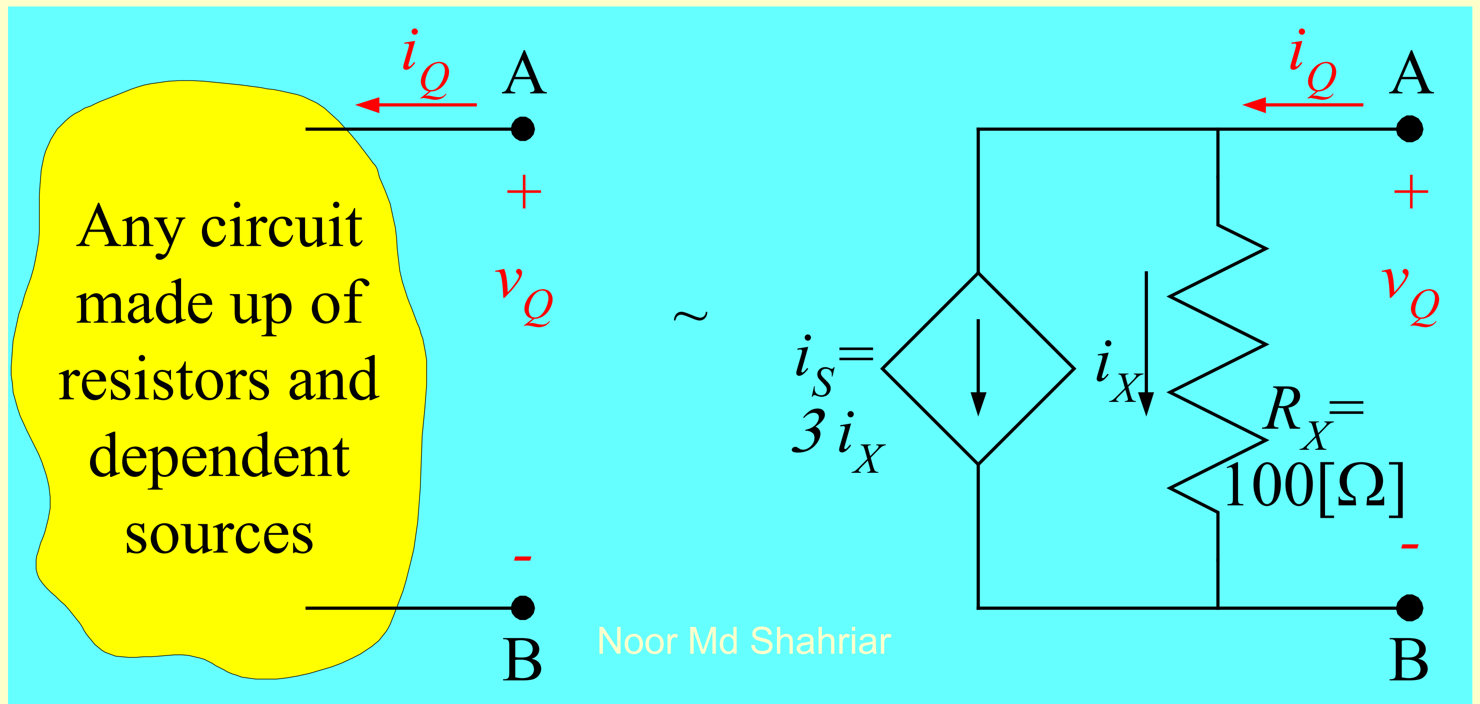
The equivalent resistance of a dependent source, in this case, is the ratio of voltage to current, which is a constant. Then the source can be said to have an equivalent resistance, since it is behaving as a resistance. This will only be meaningful for a dependent source. It is not meaningful to talk about the equivalent resistance of an independent source. The ratio of voltage to current will not be constant for an independent source.



Simple Example with a Dependent Source

We will try to explain this by starting with a simple example. We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B.

This will mean that the ratio of the voltage across the circuit, labeled v_Q, to the ratio of the current through the circuit, labeled i_Q, must be a constant. Let's find that constant by finding the ratio.



Simple Example with a Dependent Source – Step 1

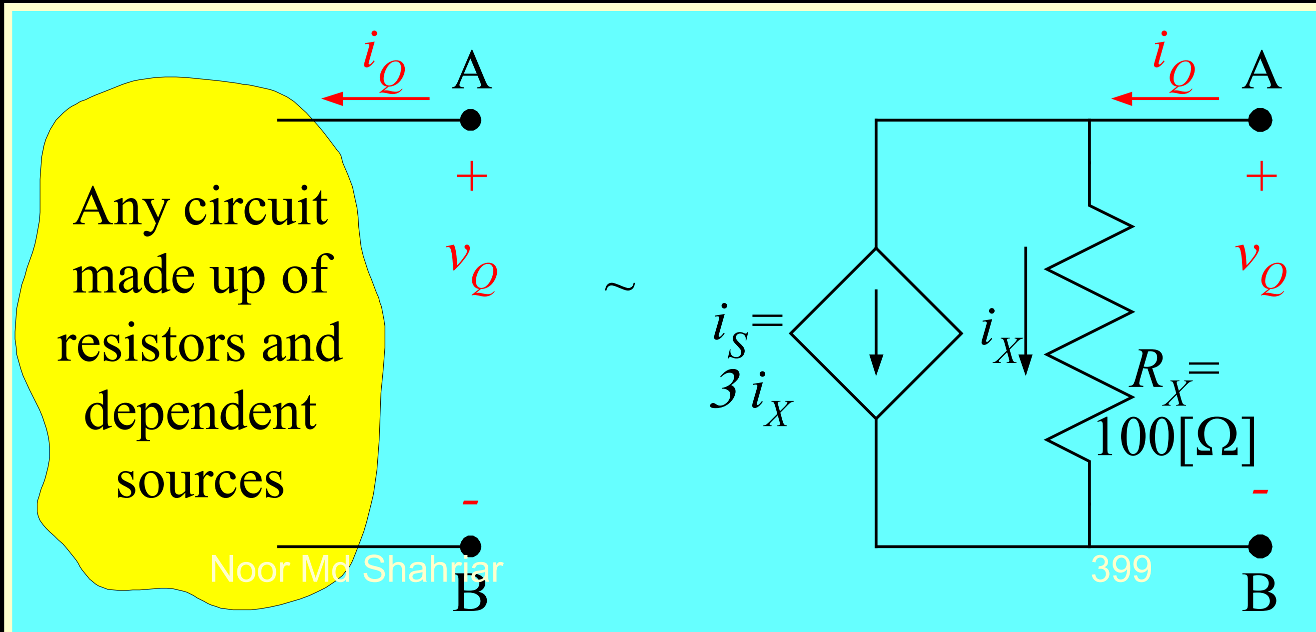
We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B.

Let's find the ratio of the voltage across the circuit, labeled v_Q, to the ratio of the current through the circuit, labeled i_Q. This must be a constant. Let's look first at the circuit equivalent on the right. We note that from Ohm's Law applied to R_X, we can say

$$v_Q = i_X R_X .$$

Next, we apply KCL at the A node to write that

$$i_Q = i_X + 3i_X .$$

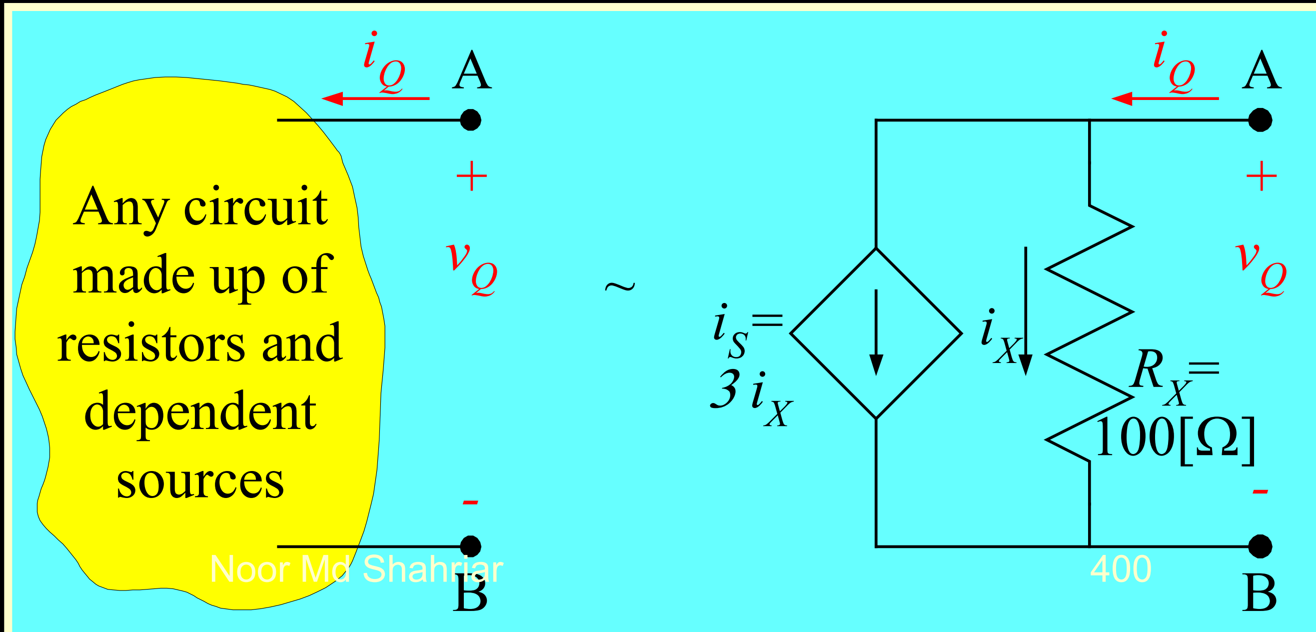


Simple Example with a Dependent Source – Step 2

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. On the last slide we found v_Q, and we found i_Q. We take the ratio of them, and plug in the expressions that we found for each. When we do this, we get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X + 3i_X} = \frac{i_X R_X}{4i_X} = \frac{R_X}{4} = \frac{100[\Omega]}{4} = 25[\Omega].$$

Note that ratio is a constant. The ratio has units of resistance, which is what we expect when we take a ratio of a voltage to a current.

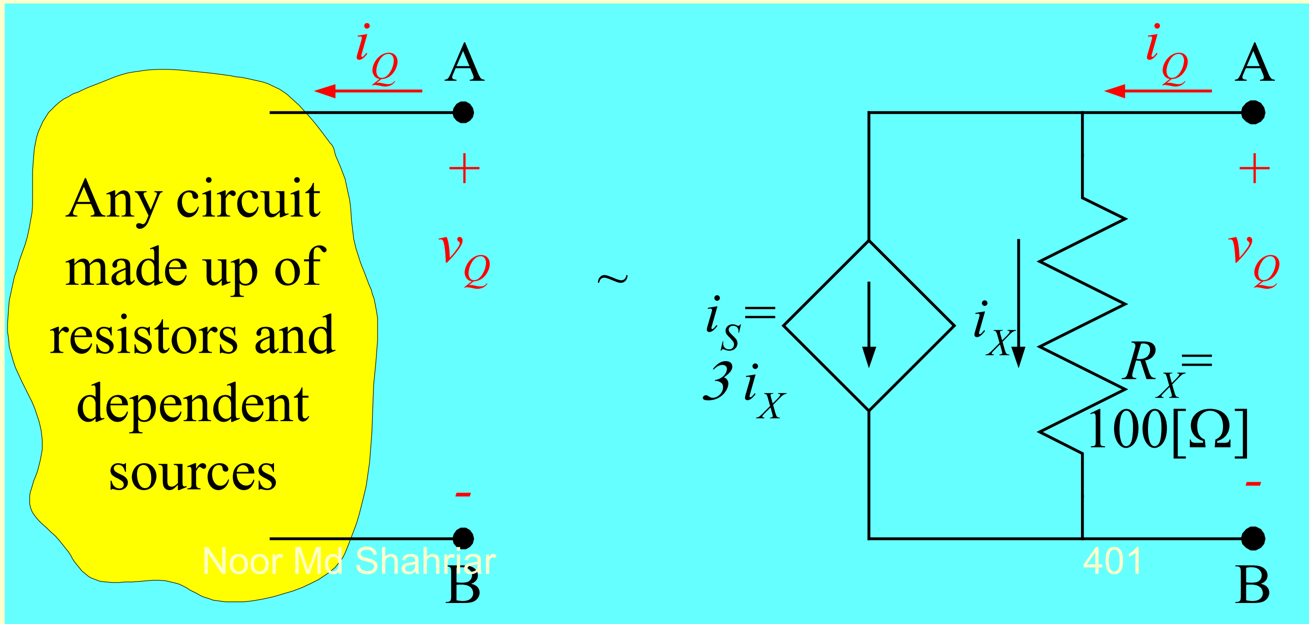


Simple Example with a Dependent Source – Step 2 (Note)

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. Let's find the ratio of the voltage across the circuit, labeled v_Q, to the ratio of the current through the circuit, labeled i_Q. We take the ratio of them, and get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X + 3i_X} = \frac{i_X R_X}{4i_X} = \frac{R_X}{4} = \frac{100[\Omega]}{4} = 25[\Omega].$$

The dependent source is in parallel with the resistor R_X. Since the parallel combination is 25[Ω], the dependent source must be behaving as if it were a 33.33[Ω] resistor. However, this value depends on R_X; in fact, it is $R_X/3$.



2^{nd} Simple Example with a Dependent Source – Step 1

We wish to find the equivalent resistance of a second circuit, given below, as seen at terminals A and B.

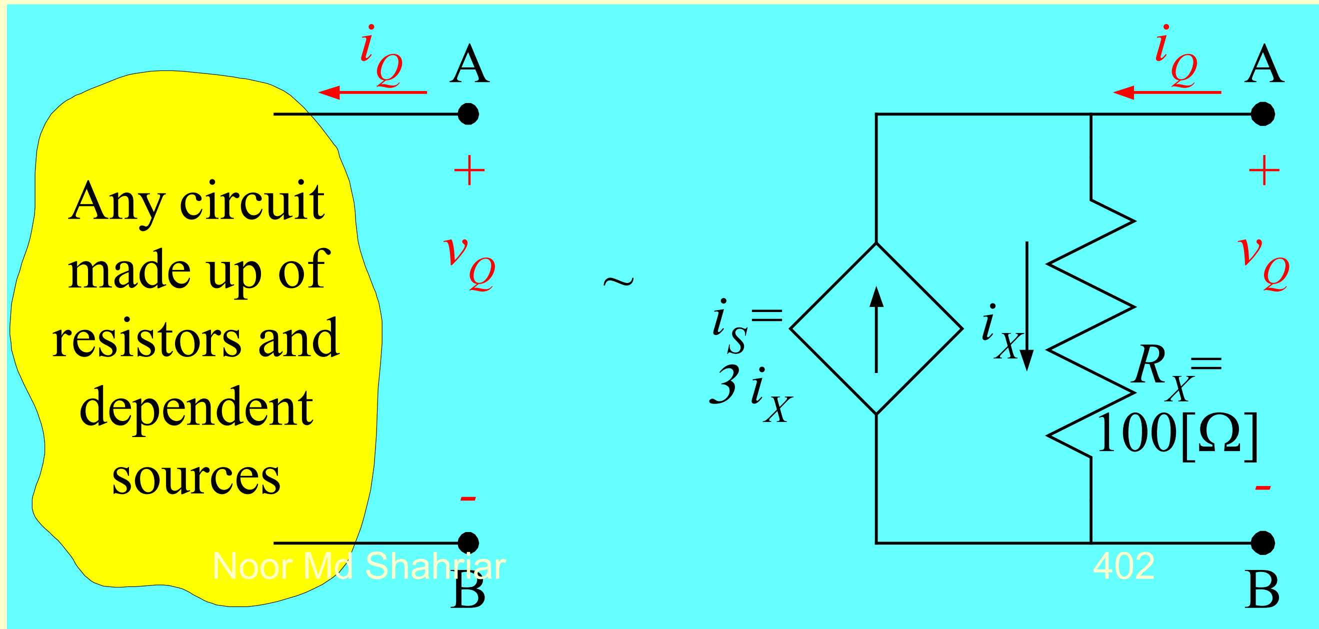
Let's find the ratio of the voltage across the circuit, labeled v_Q, to the ratio of the current through the circuit, labeled i_Q. This must be a constant. We note that from Ohm's Law applied to R_X, we can say that

$$v_Q = i_X R_X.$$

Next, we apply KCL at the A node to write that

$$i_Q = i_X - 3i_X.$$

Note the change in polarity for the source, from the previous example.

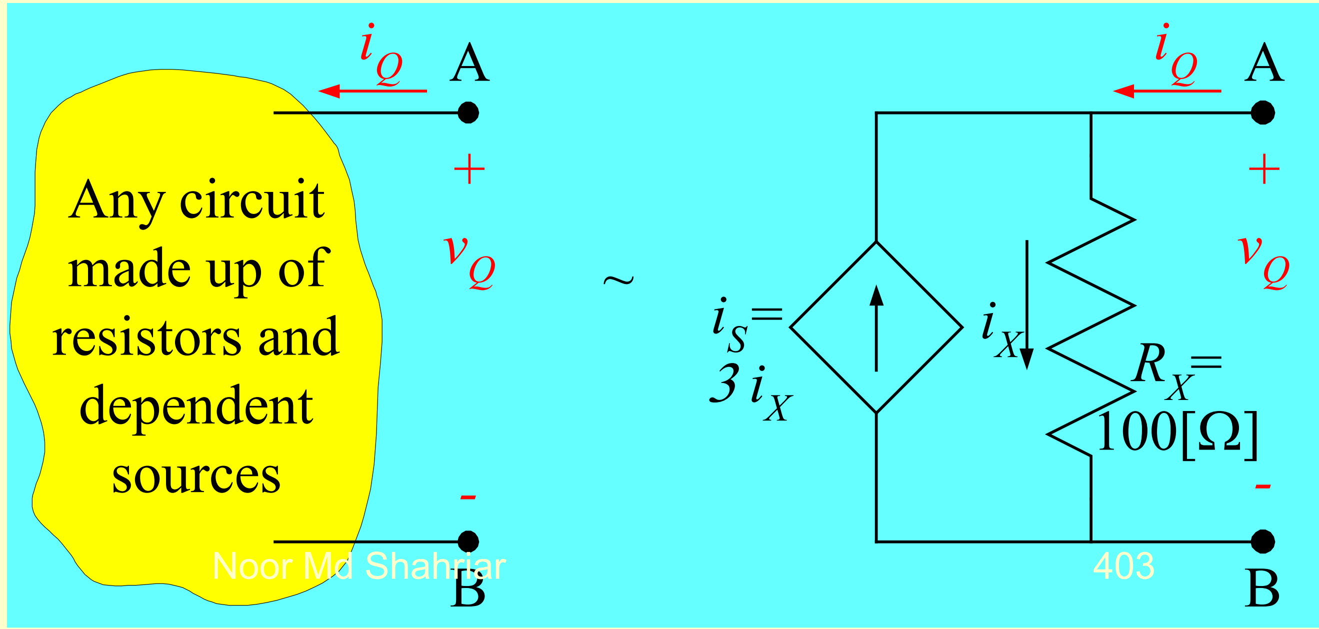


2^{nd} Simple Example with a Dependent Source – Step 2

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. On the last slide we found v_Q, and we found i_Q. We take the ratio of them, and plug in the expressions that we found for each. When we do this, we get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X - 3i_X} = \frac{i_X R_X}{-2i_X} = \frac{R_X}{-2} = \frac{100[\Omega]}{-2} = -50[\Omega].$$

Note that ratio has changed when we simply changed the polarity of the dependent source. The magnitude is not the only thing that changed; the equivalent resistance is now **negative**!

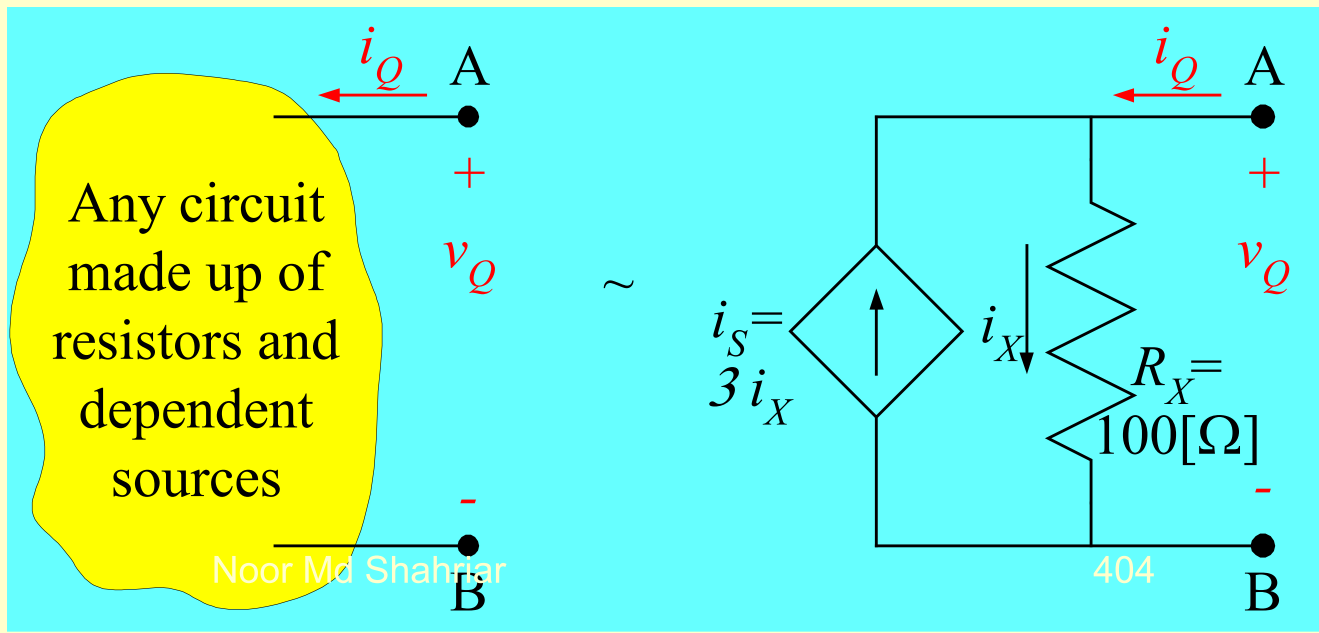


Simple Example with a Dependent Source – Step 2 (Note)

We wish to find the equivalent resistance of the circuit below, as seen at terminals A and B. Let's find the ratio of the voltage across the circuit, labeled v_Q, to the ratio of the current through the circuit, labeled i_Q. We take the ratio of them, and get

$$\frac{v_Q}{i_Q} = \frac{i_X R_X}{i_X - 3i_X} = \frac{i_X R_X}{-2i_X} = \frac{R_X}{-2} = \frac{100[\Omega]}{-2} = -50[\Omega].$$

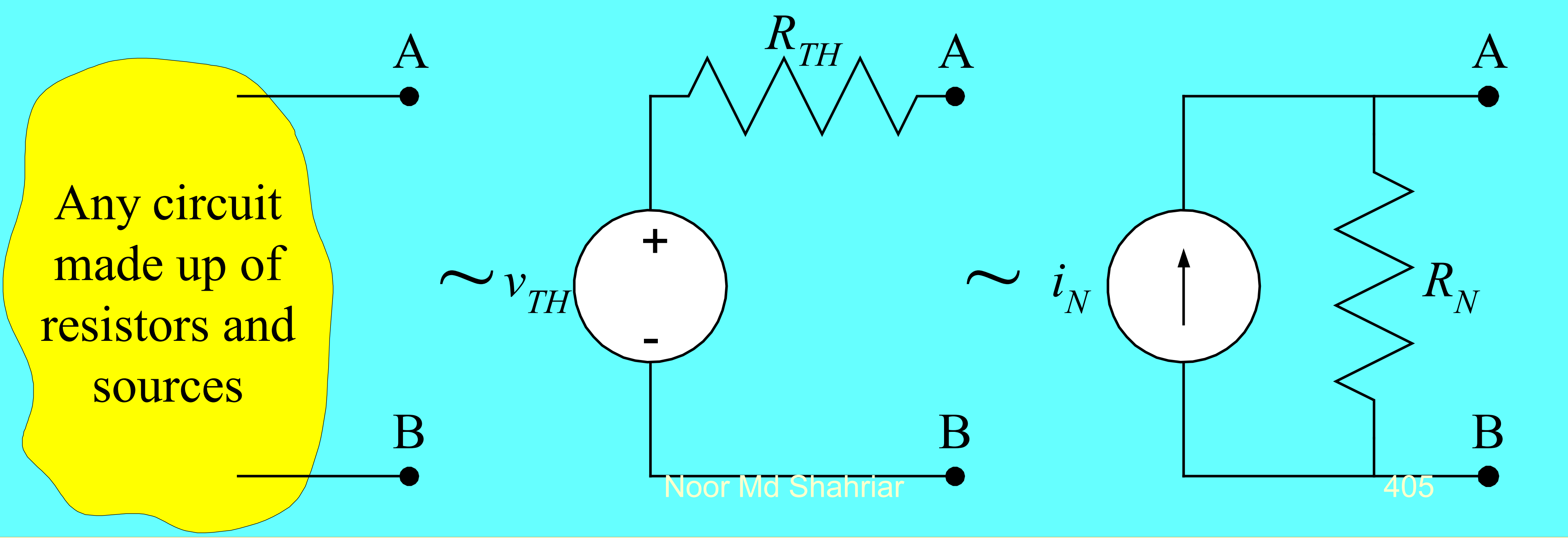
The dependent source is in parallel with the resistor R_X. Since the parallel combination is -50[Ω], the dependent source must be behaving as if it were a -33.33[Ω] resistor. This value depends on R_X; in fact, it is $-R_X/3$.



Note 1

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

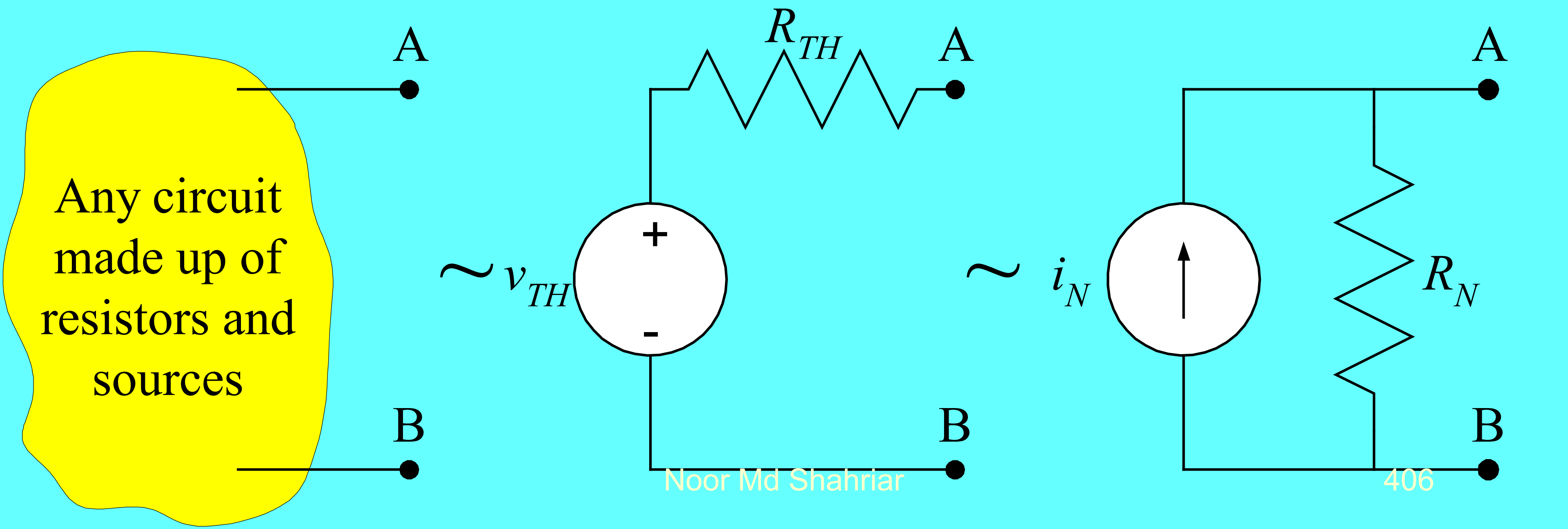
We can see that the equivalent resistance can be negative. This is one reason why we have been so careful about polarities all along. We need to get the polarities right to be able to get our signs right.



Note 2

When we find the equivalent resistance for a Thévenin's equivalent or a Norton's equivalent, we set the independent sources equal to zero, and find the equivalent resistance of what remains.

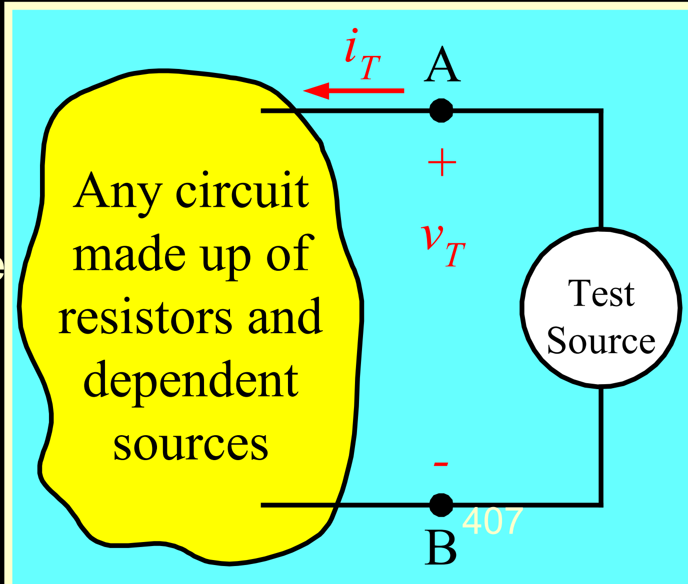
In the simple examples that we just did, we were effectively applying a source to the terminals of the circuit. This results in a circuit like others that we have solved before, and we can find the ratio of voltage to current. This is usually easier to think about for most students. It is as if we were applying a source just to test the circuit; we call this method the Test-Source Method.



Test-Source Method – Defined

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to zero.
2) Find the equivalent resistance.
   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

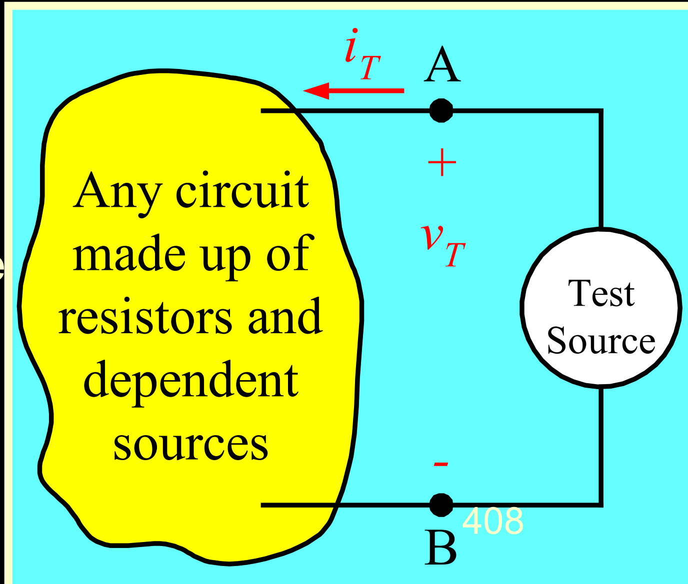


Test-Source Method – Note 1

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to zero.
2) Find the equivalent resistance.
   a) If there are no dependent sources, use the equivalent resistance rules. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

Don't forget this step. It is **always** applied when finding equivalent resistance.



Test-Source Method – Note 2

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z

2) Find the equivalent resistance.

   a) If there are no dependent sources, find this equivalent resistance using the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.

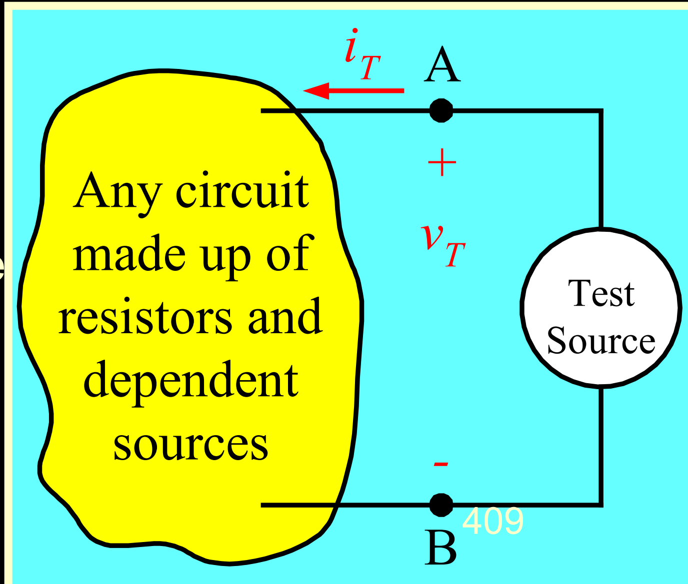
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.

      1) If you apply a voltage source, find the current through that voltage source.

      2) If you apply a current source, find the voltage across that current source.

      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

Note that step 2 has two options (a or b). Pick one. You don't need to do both.

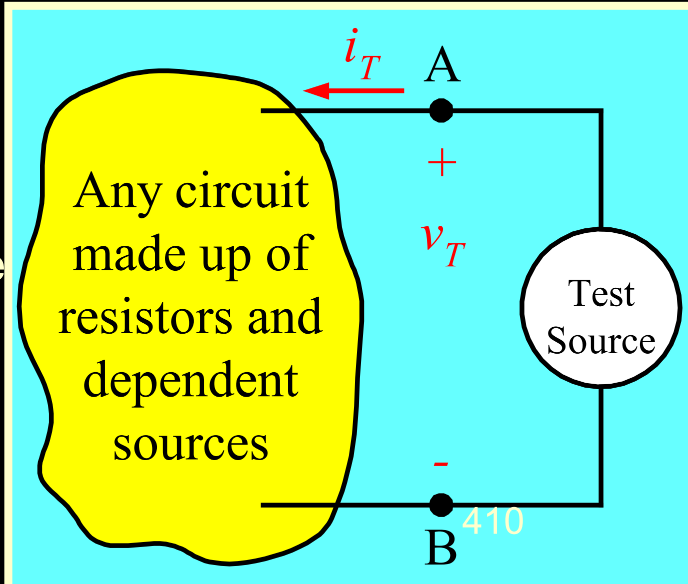


Test-Source Method – Note 3

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z[ero.]
2) Find the equivalent resistance.
   a) If there are no dependent sour[ces, find this equivalent resistance using] the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

You could actually pick option b) every time, but option a) is easier. Use it if you can.



Test-Source Method – Note 4

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z

2) Find the equivalent resistance.

   a) If there are no dependent sour the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.

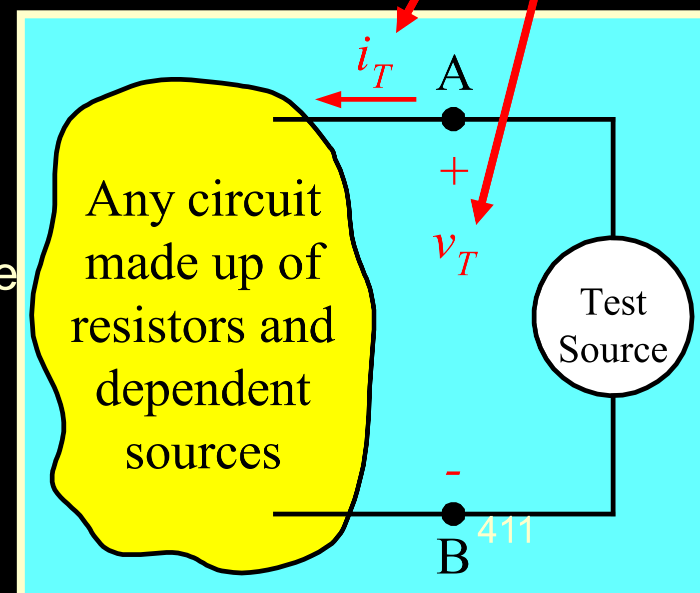
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.

      1) If you apply a voltage source, find the current through that voltage source.

      2) If you apply a current source, find the voltage across that current source.

      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

When you apply these voltages and currents, we suggest that you apply them in the active sign relationship for the source. This gives the sign relationship we prefer.

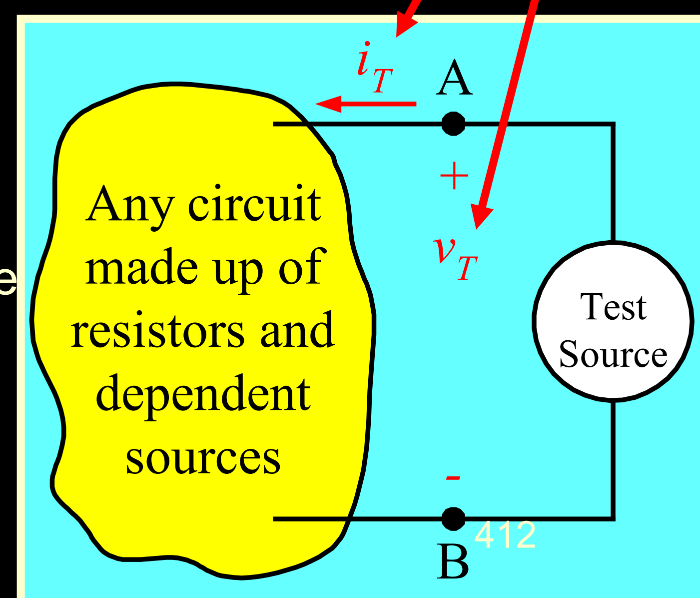


Test-Source Method – Note 5

To get the equivalent resistance of a circuit, as seen by two terminals of that circuit, we follow these steps.

1) Set all independent sources equal to z
2) Find the equivalent resistance.
   a) If there are no dependent sour the equivalent resistance rules that have been used before. These include series combinations, parallel combinations, and delta-to-wye equivalents.
   b) If there are dependent sources present, apply a test source to the two terminals. It can be either a voltage source or a current source.
      1) If you apply a voltage source, find the current through that voltage source.
      2) If you apply a current source, find the voltage across that current source.
      3) Then, find the ratio of the voltage to the current, which will be the equivalent resistance.

The active sign relationship for the test source gives the passive sign relationship for the circuit, which gives the resistance, by Ohm's Law, with a positive sign in the equation.



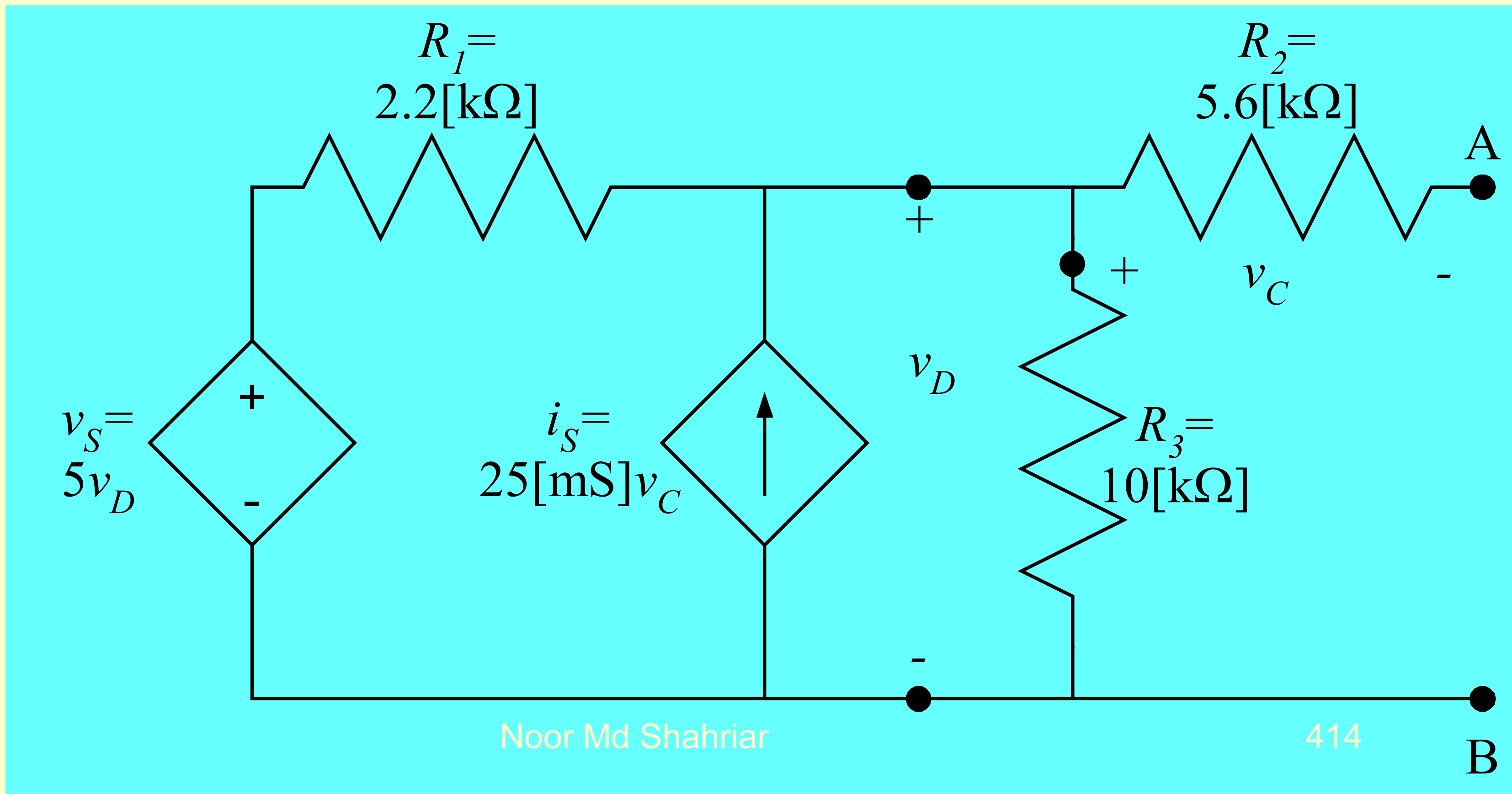
Go back to Overview slide.

Notes

1. The Test-Source Method usually requires some practice before it becomes natural for students. It is important to work several problems to get this practice in.
2. There is a tendency to assume that one could just ignore the Test-Source Method, and just find the open-circuit voltage and short-circuit current whenever a dependent source is present. However, sometimes this does not work. In particular, when the open-circuit voltage and short-circuit current are zero, we must use the Test-Source Method. Learn how to use it.

Example Problem

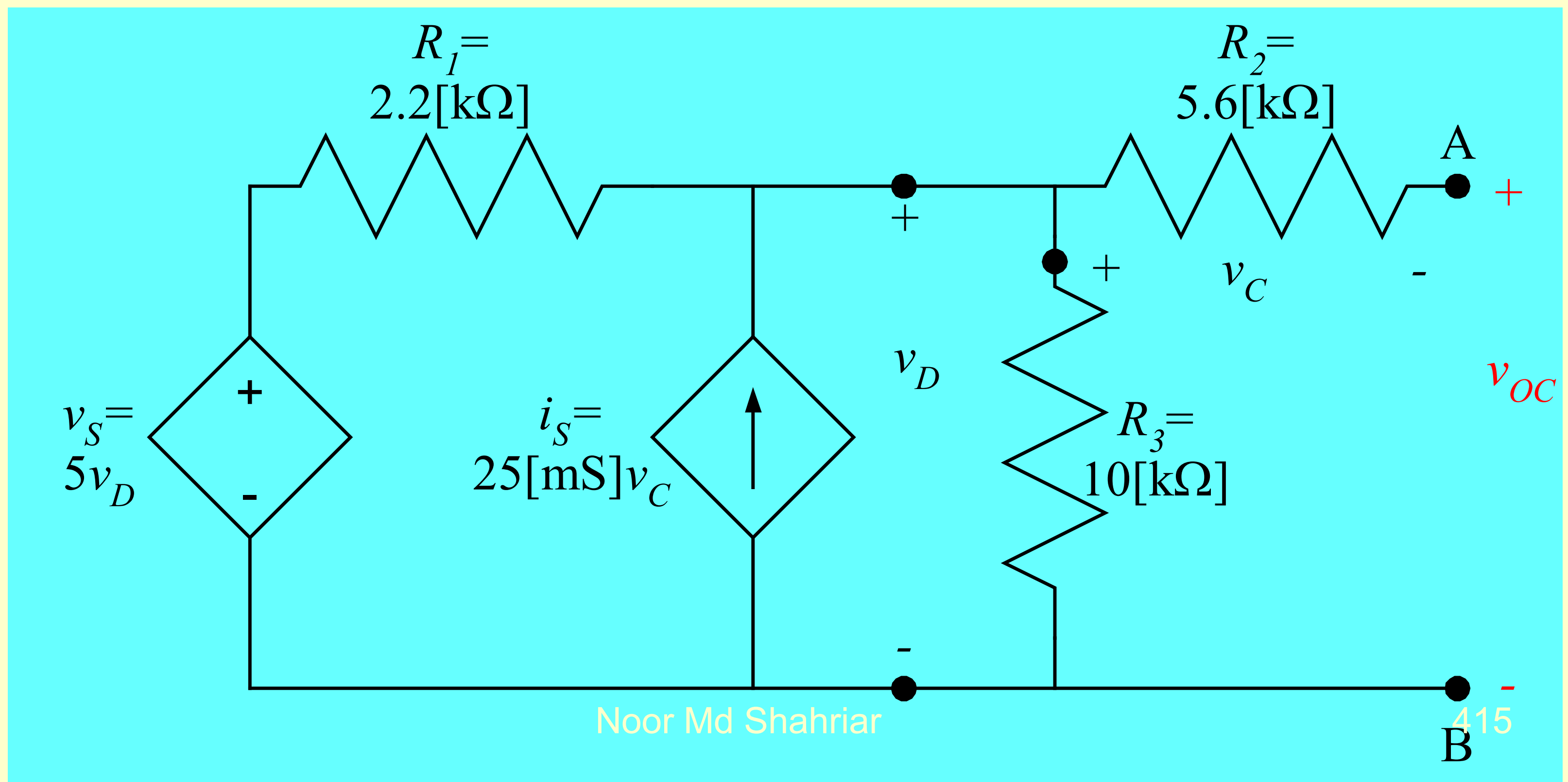
We wish to find the Thévenin equivalent of the circuit below, as seen from terminals A and B.



Example Problem – Step 1

We wish to find the Thévenin equivalent of the circuit below, as seen from terminals A and B.

We will start by find the open-circuit voltage at the terminals, as defined below.

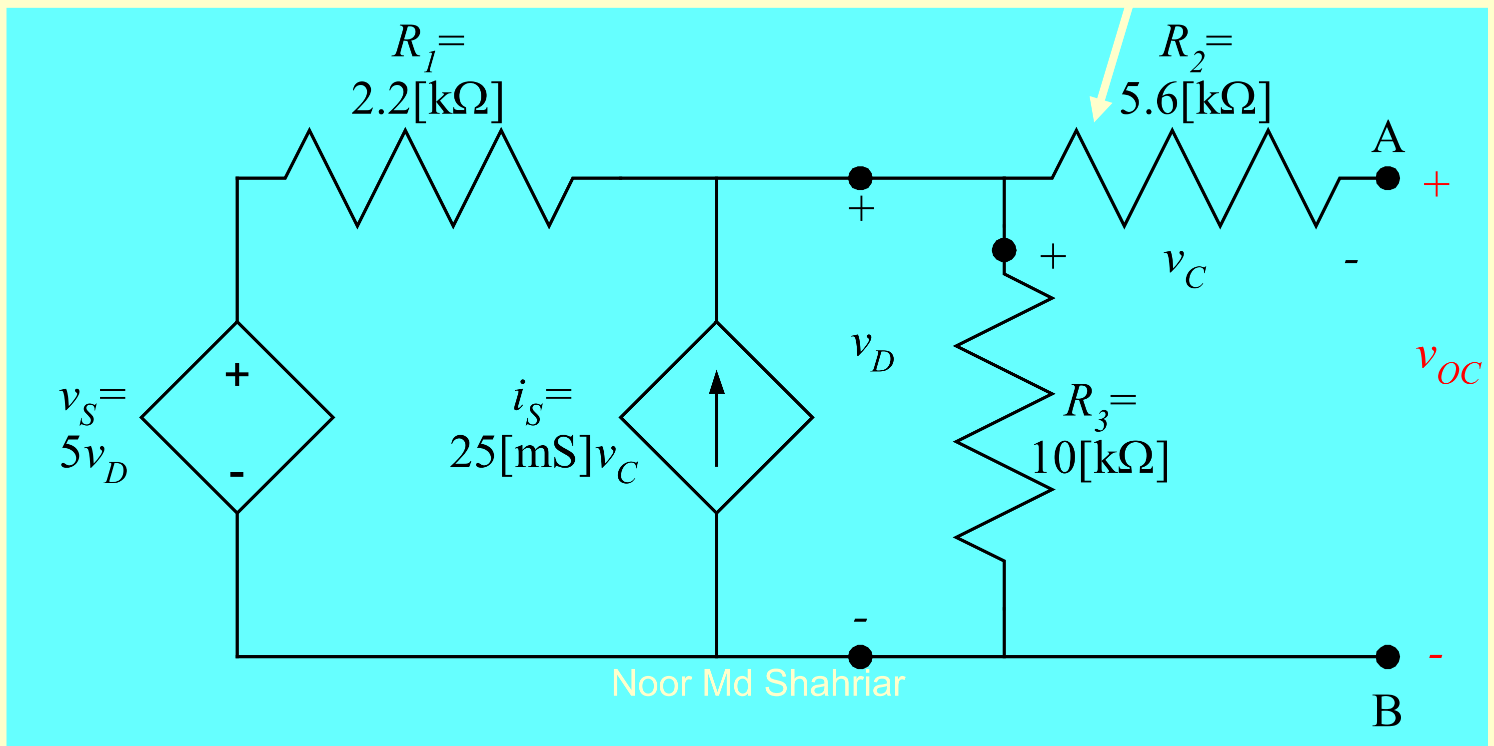


Example Problem – Step 2

To find v_{OC}, we will first find v_D, by writing KCL at the top center node. We have

$$\frac{v_D - 5v_D}{R_1} + 0 + \frac{v_D}{R_3} - i_S = 0.$$

Note that we recognize that the current through R_2 must be zero since R_2 is in series with an open circuit.

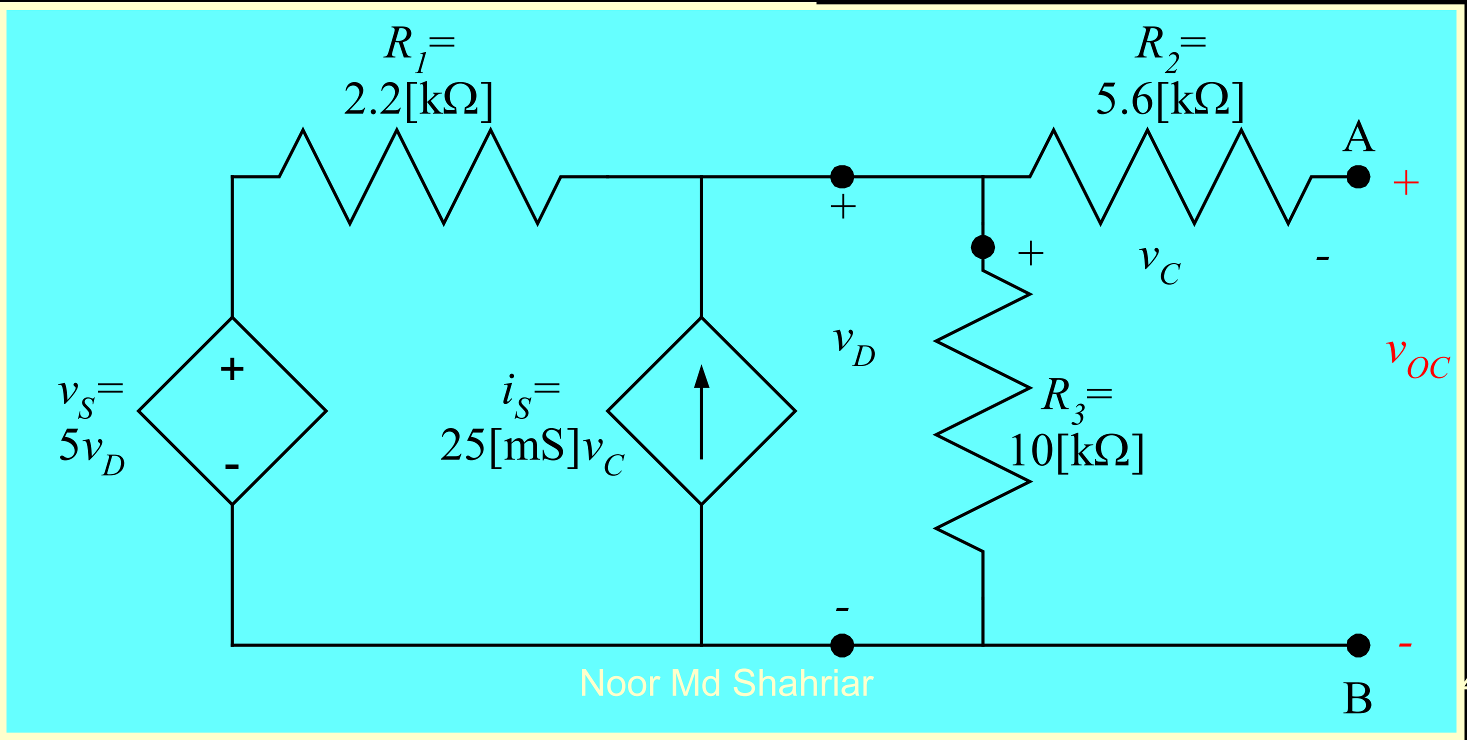


Example Problem – Step 3

We can substitute in the value for i_S, 25[mS]v_C. We note that since the current through R_2 is zero, the voltage across it is zero, so v_C is zero. So, we write

$$\frac{v_D - 5v_D}{R_1} + 0 + \frac{v_D}{R_3} - 25[\text{mS}]v_C = 0, \text{ or}$$

$$\frac{v_D - 5v_D}{R_1} + \frac{v_D}{R_3} = 0.$$

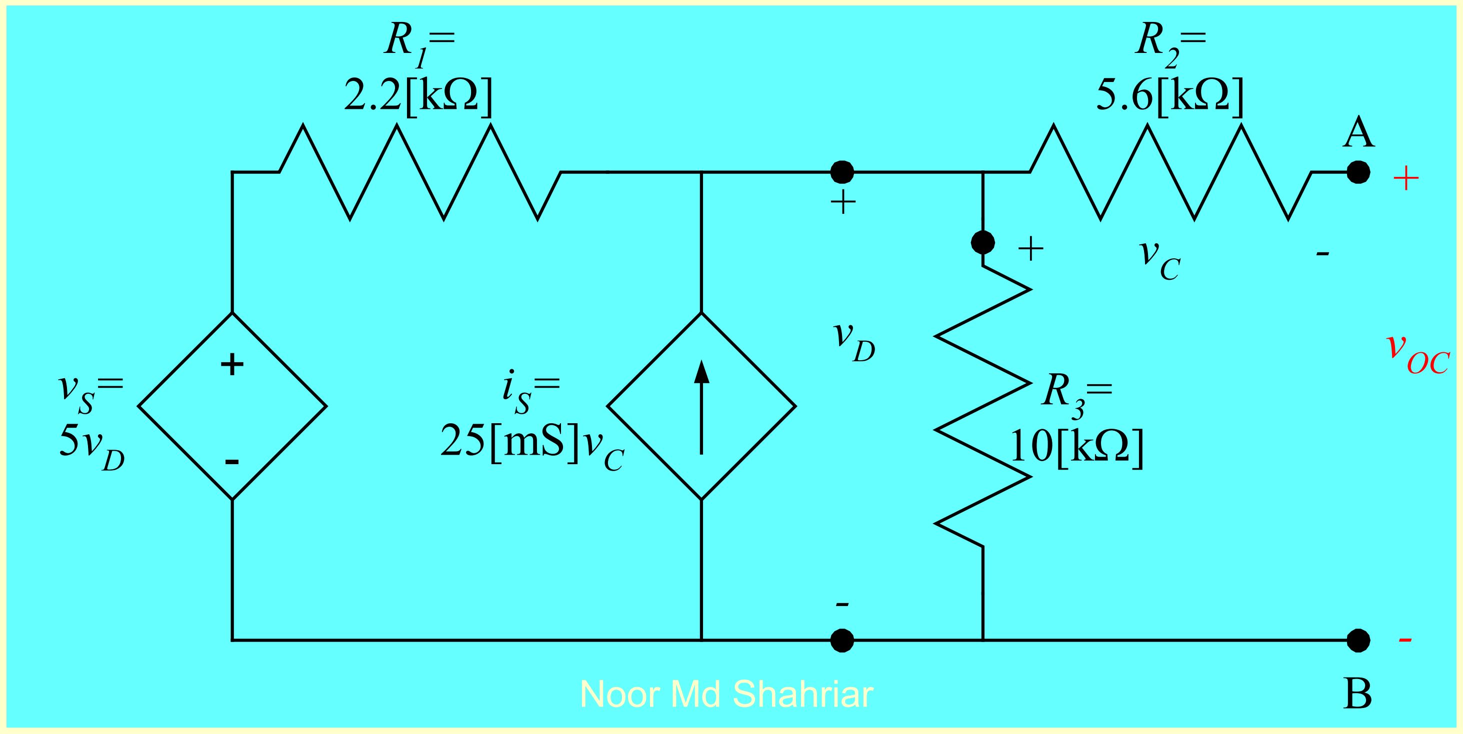


Example Problem – Step 4

Next, we substitute in values and solve for v_D. We write

$$\frac{-4v_D}{2.2[\text{k}\Omega]} + \frac{v_D}{10[\text{k}\Omega]} = 0.$$ With some math, we find

$v_D = 0.$

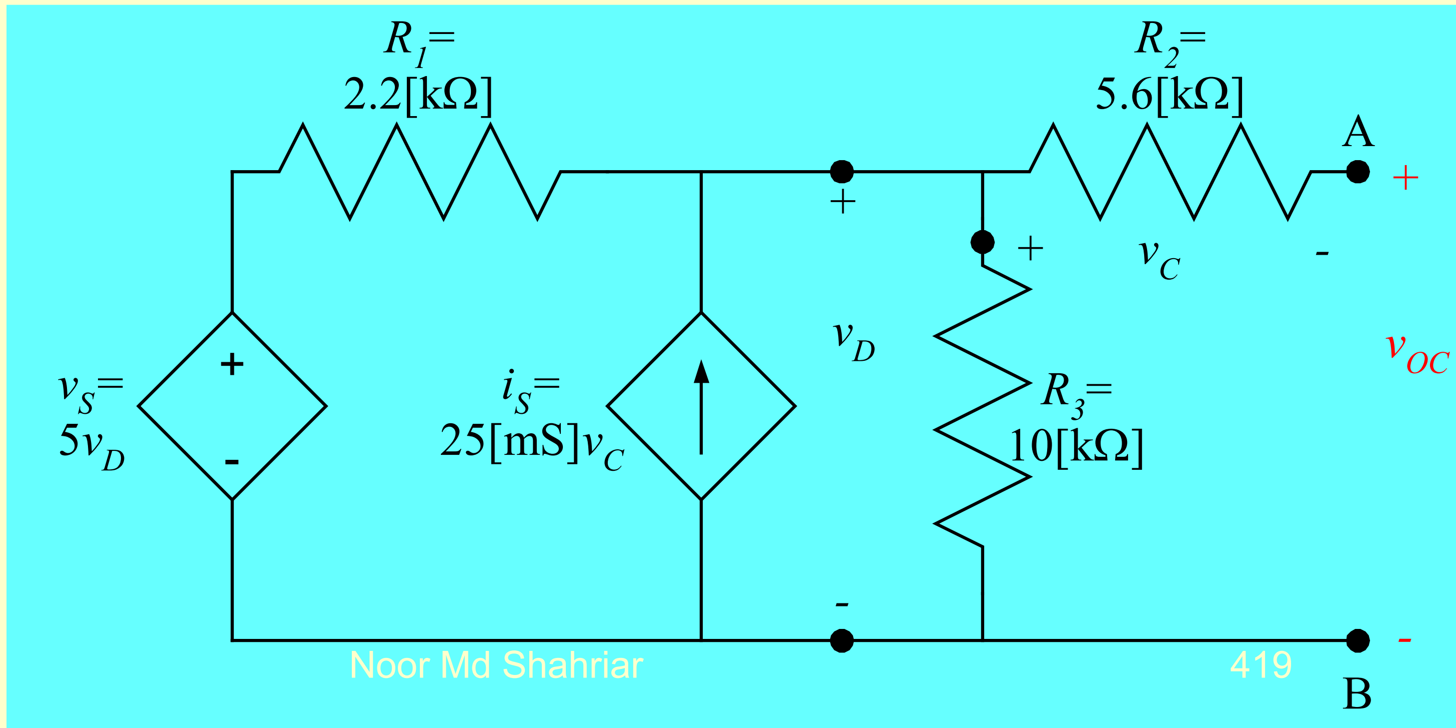


Example Problem – Step 5

Now, we can take KVL around the loop, and we write

$$-v_D + v_C + v_{OC} = 0, \text{ and so}$$
$$v_{OC} = 0.$$

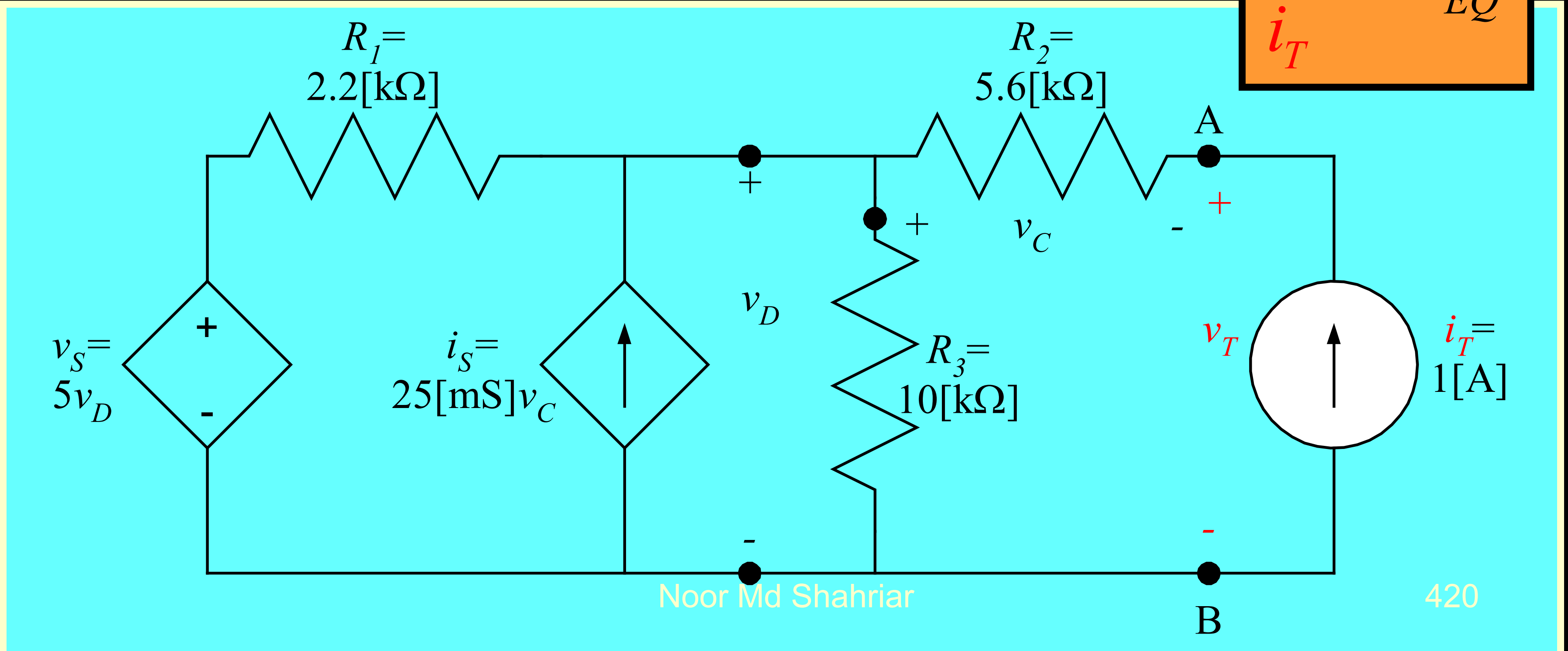
The Thévenin voltage is equal to this open-circuit voltage, so the Thévenin voltage must be zero. The short-circuit current will also be zero. To get the resistance, we need to use the Test-Source Method.



Example Problem – Step 6

We have applied a test current source to the two terminals. We have also labeled a voltage across this current source, v_T. This voltage has been defined in the active sign relationship for the current source. As noted earlier, this will give us the passive sign relationship for v_T and i_T for the circuit that we are finding the equivalent resistance of. Thus, we will have

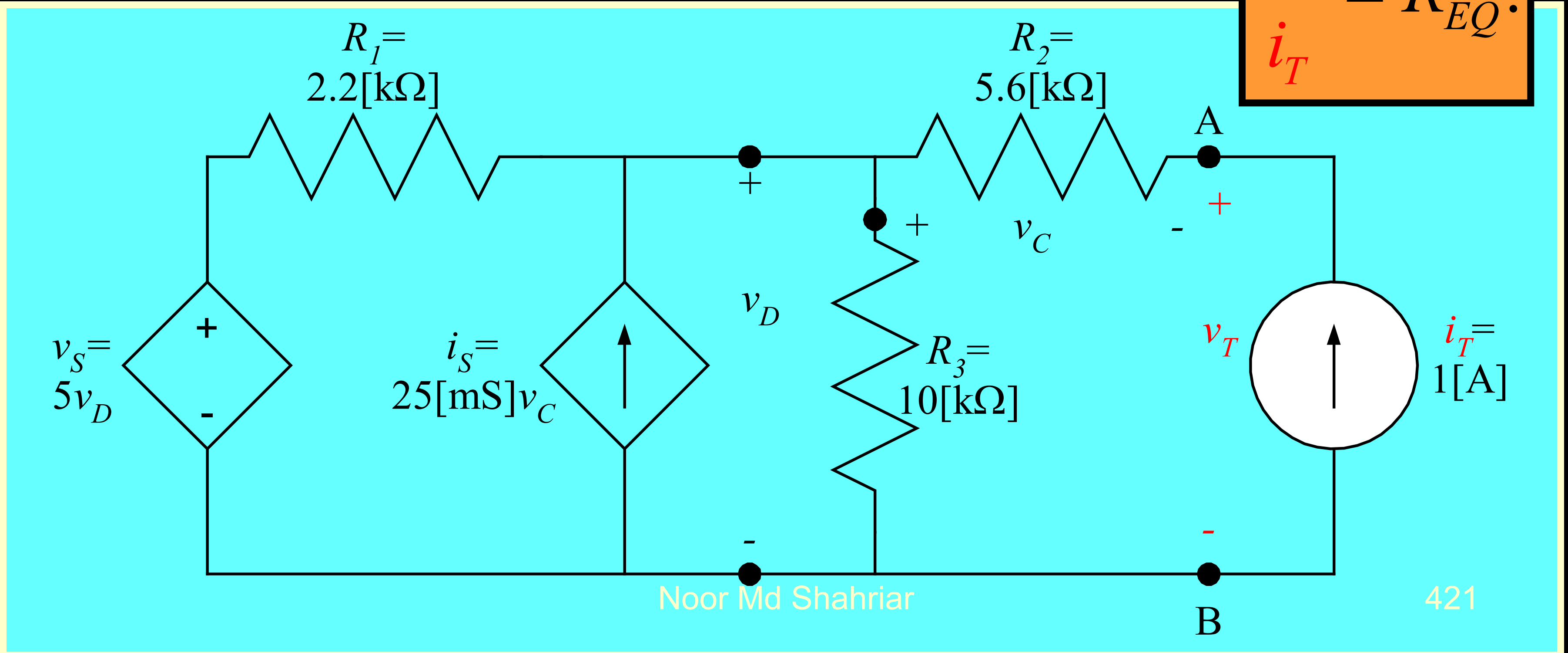
$$\frac{v_T}{i_T} = R_{EQ}.$$



Example Problem – Step 7

We have applied a ***test*** current ***source*** to the two terminals. We don't need to do this, but doing so makes it clear that we are now just solving another circuit, like the many that we have solved before. We have even given the source a value, in this case, 1[A]. This is just a convenience. Many people choose to leave this as an arbitrary source. We choose to use a value, an easy value like 1[A], to allow us to find an actual value for v_T.

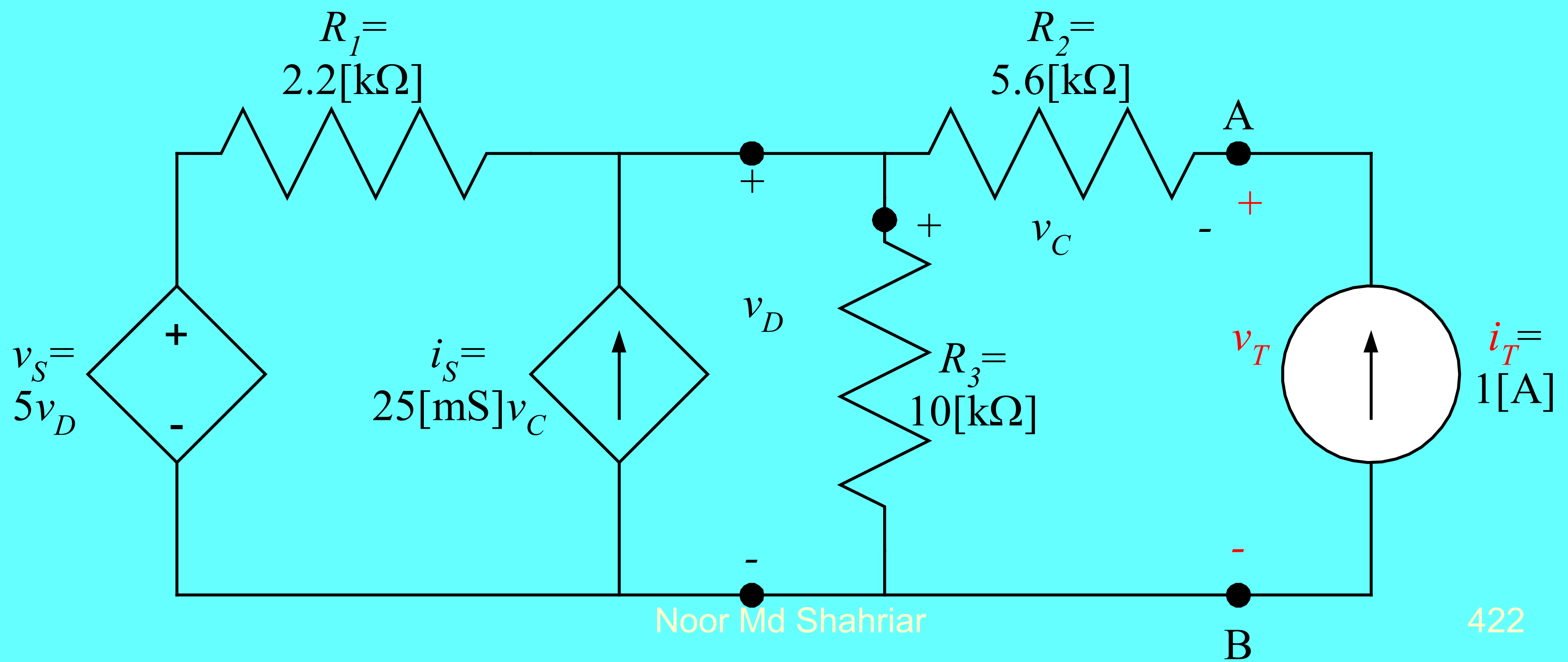
$$\frac{v_T}{i_T} = R_{EQ}.$$



Example Problem – Step 8

We have applied a test ***current*** source to the two terminals. A test ***voltage*** source would have been just as good. We chose a current source because we thought it might make the solution a little easier, since we can find v_C so easily now. But it really does not matter. Don't worry about which one to choose. Let us solve.

$$\frac{v_T}{i_T} = R_{EQ}.$$

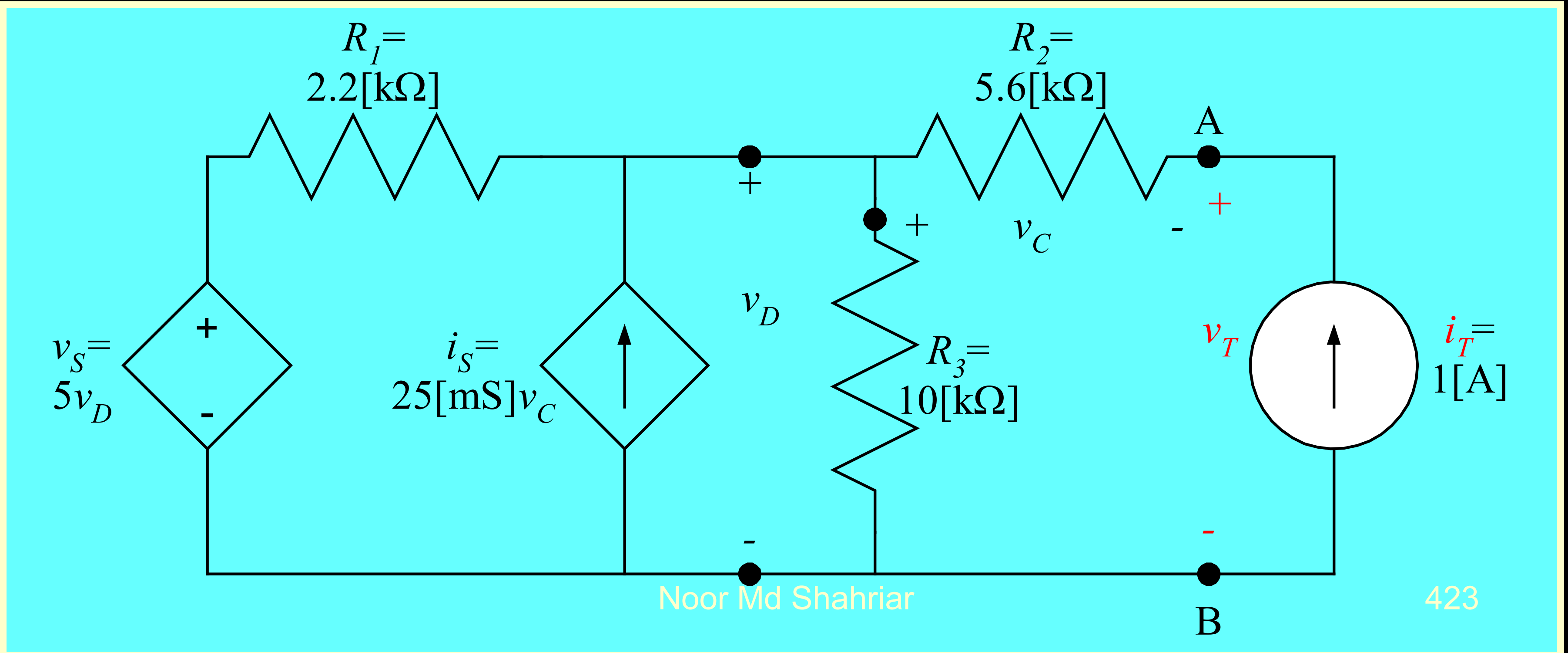


Example Problem – Step 9

Let us solve for v_T. We note that we can write an expression for v_C using Ohm's Law, and get

$$v_C = -1[\text{A}]R_2 = -5600[\text{V}].$$

This voltage may seem very large. Don't let this bother you. We do not actually have this voltage; it is just for calculating the resistance.

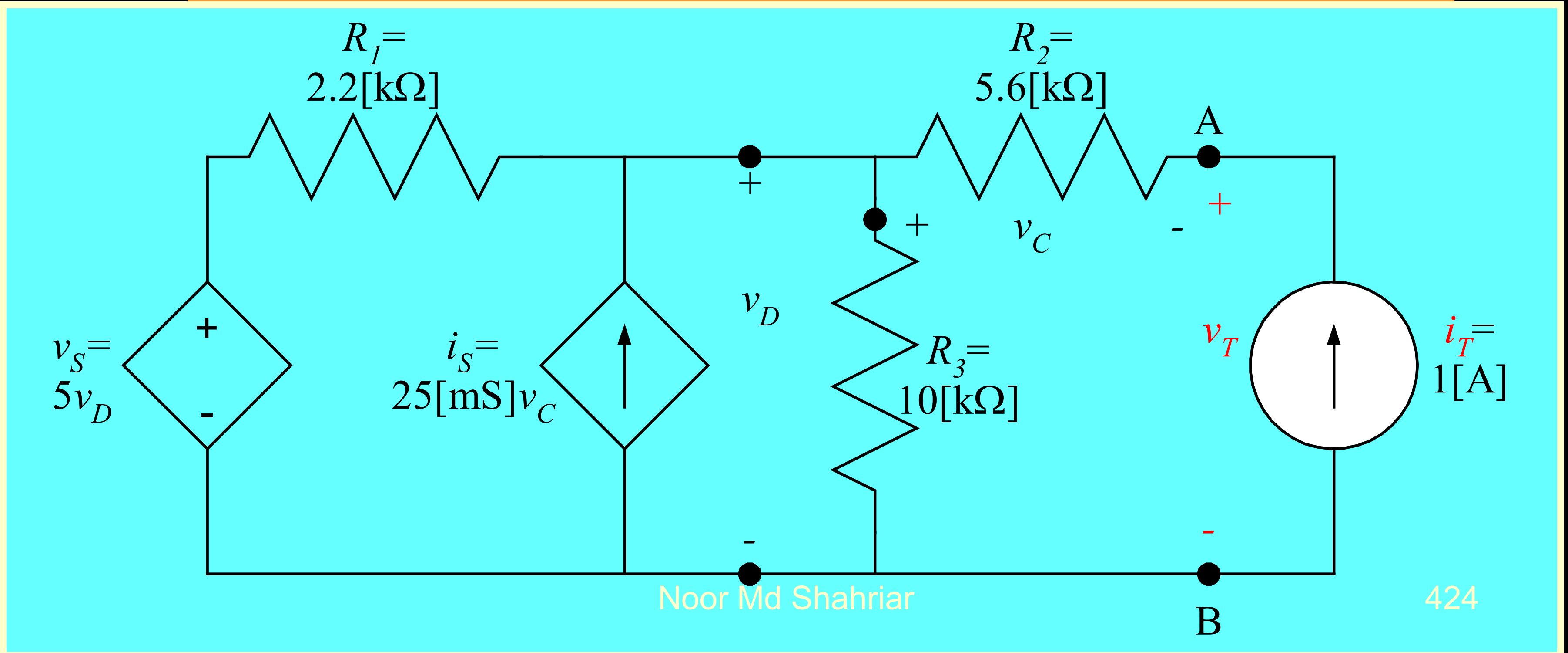


Example Problem – Step 10

Next, let's write KCL for the top center node. We get

$$\frac{v_D - 5v_D}{R_1} - i_T + \frac{v_D}{R_3} - i_S = 0, \text{ or by substituting,}$$

$$\frac{-4v_D}{2.2[\text{k}\Omega]} - 1[\text{A}] + \frac{v_D}{10[\text{k}\Omega]} - 25[\text{mS}]\left(-5600[\text{V}]\right) = 0.$$



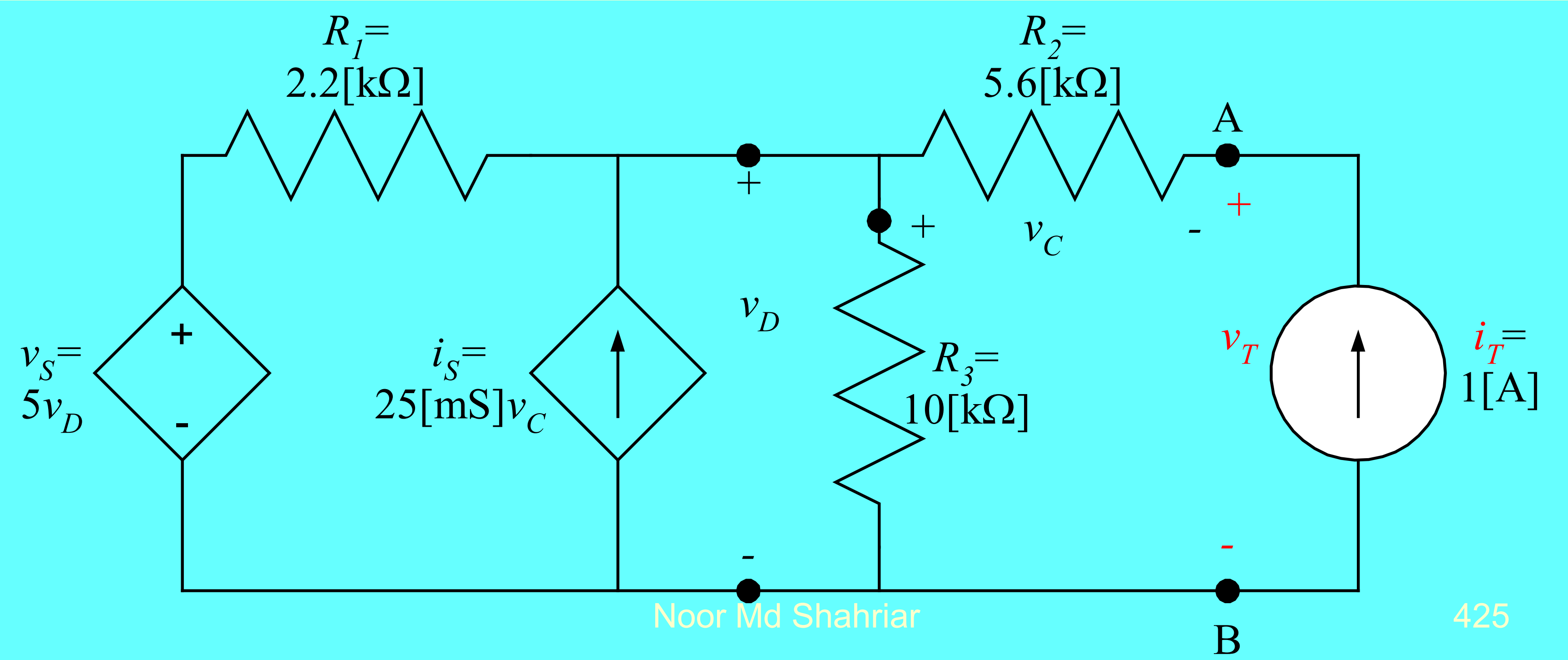
Example Problem – Step 11

Solving for v_D yields

$$\frac{-4v_D}{2.2[\text{k}\Omega]} + \frac{v_D}{10[\text{k}\Omega]} = -139[\text{A}], \text{ or}$$

$$\left(-1.72[\text{mS}]\right)v_D = -139[\text{A}], \text{ or}$$

$$v_D = 80,900[\text{V}].$$

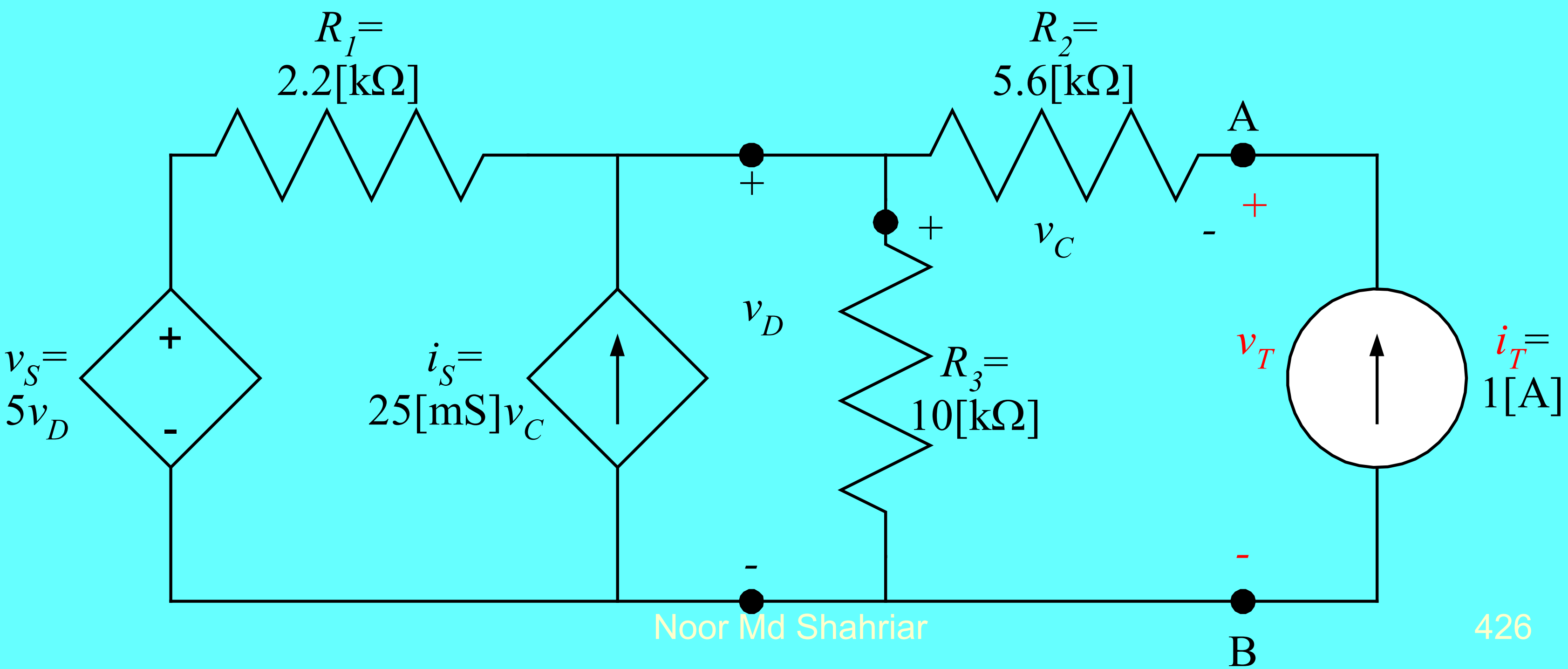


Example Problem – Step 12

Taking KVL, we get

$$-v_D + v_C + v_T = 0, \text{ or}$$

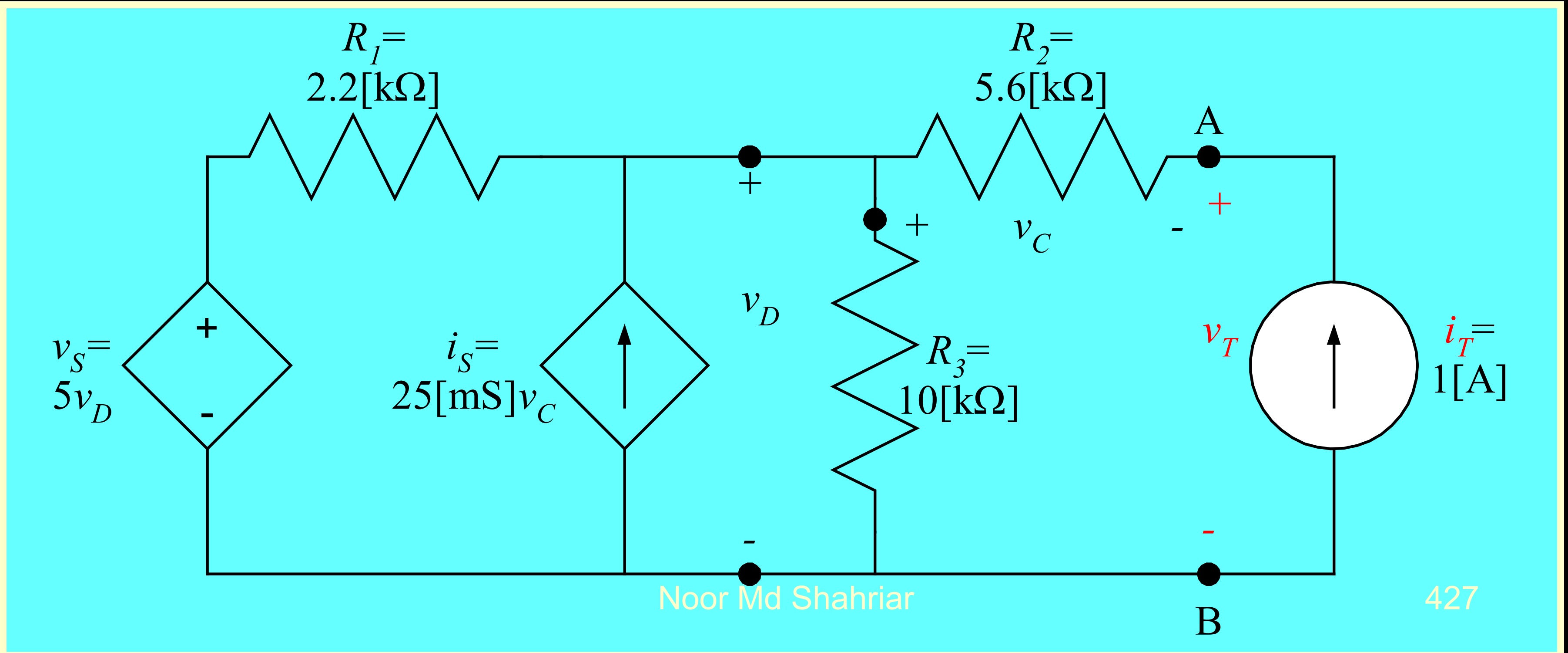
$$v_T = v_D - v_C = 80,900[\text{V}] - (-5600[\text{V}]) = 86,500[\text{V}].$$



Example Problem – Step 13

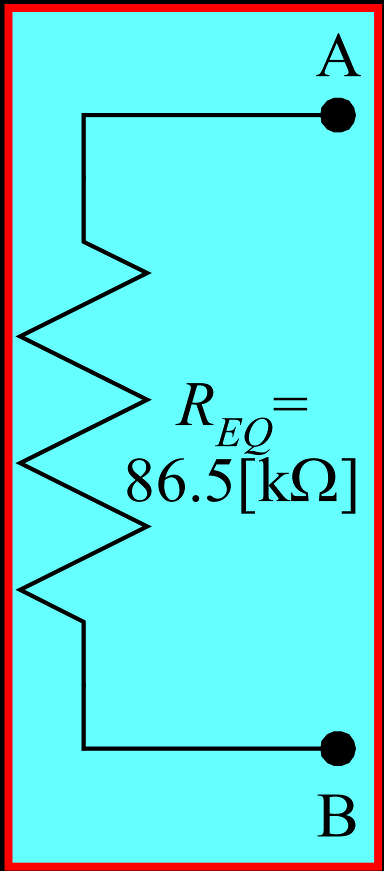
So, we can find the equivalent resistance by finding

$$R_{EQ} = \frac{v_T}{i_T} = \frac{86,500[\text{V}]}{1[\text{A}]} = 86.5[\text{k}\Omega].$$



Example Problem – Step 14

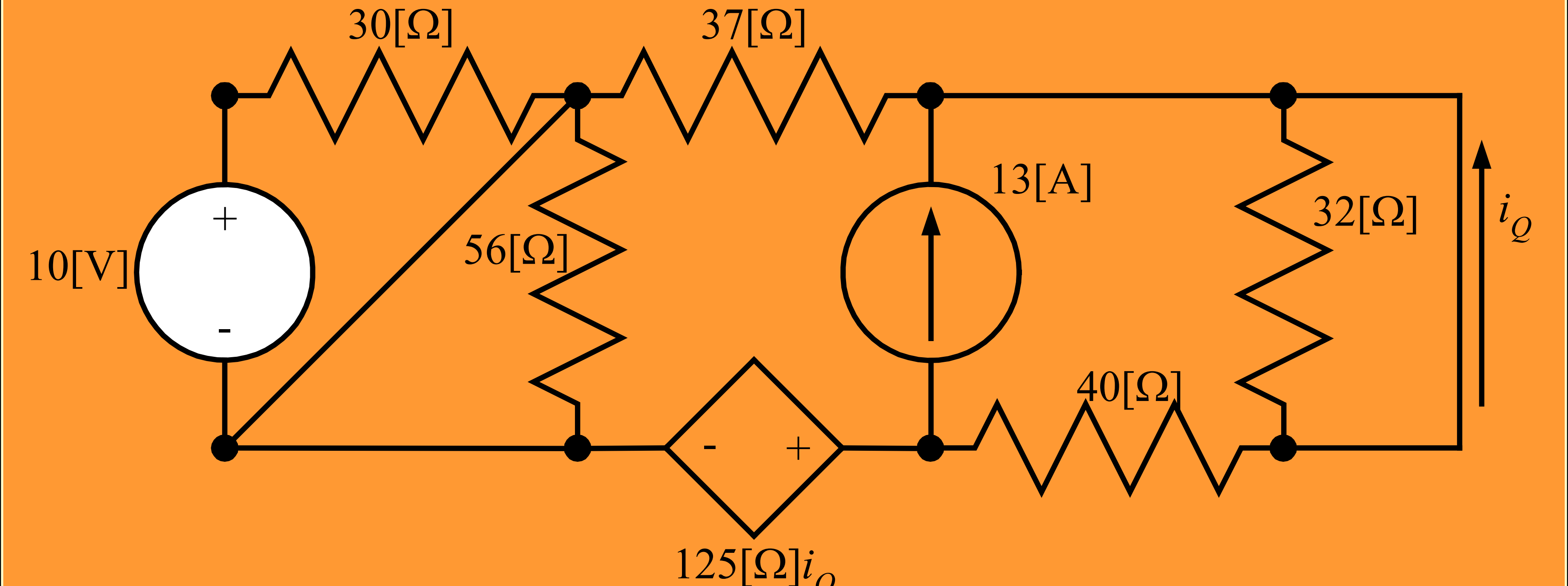
So, the Thévenin equivalent is given in the circuit below. Note that the Thévenin voltage is zero, and so we don't even show the voltage source at all. The Thévenin resistance is shown, and in this case, it is the Thévenin equivalent.



Example Problem #1

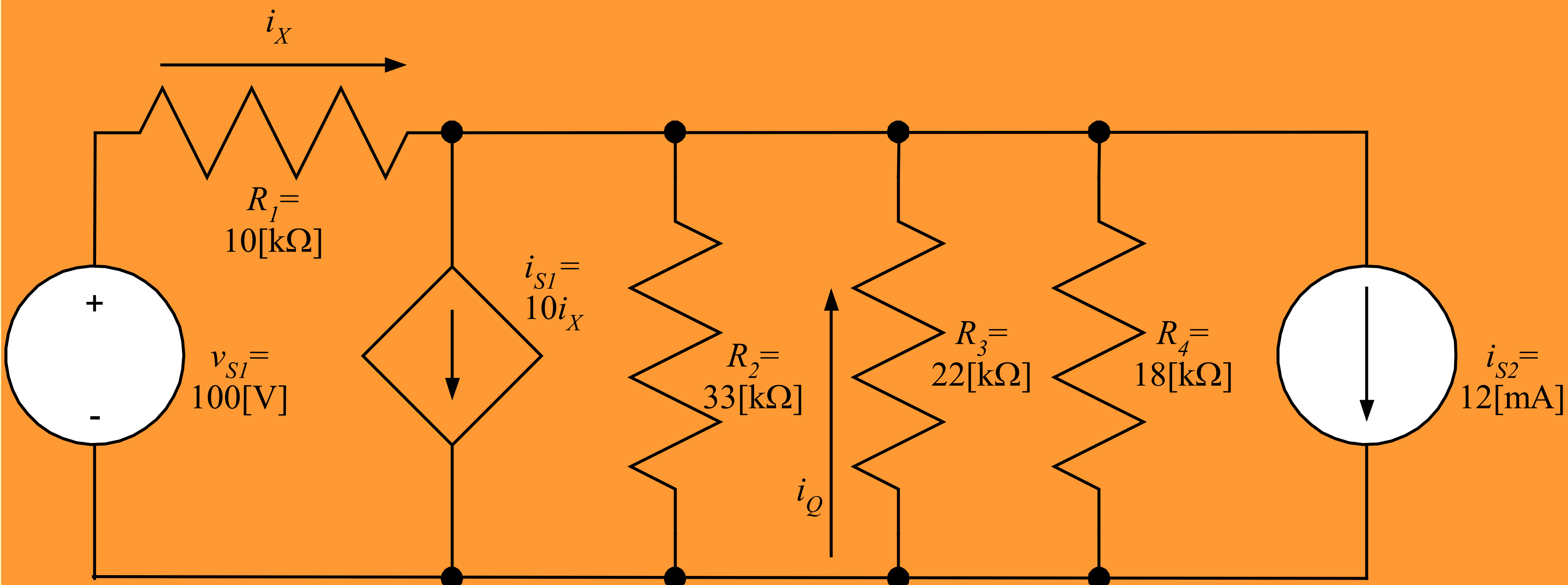
For the circuit given below, find the Norton equivalent as seen by the current source.

Find the power delivered by the current source in this circuit.



Sample Problem #2

a) Find the Norton equivalent as seen by the 22[kΩ] resistor.
b) Use this circuit to solve for i_Q.

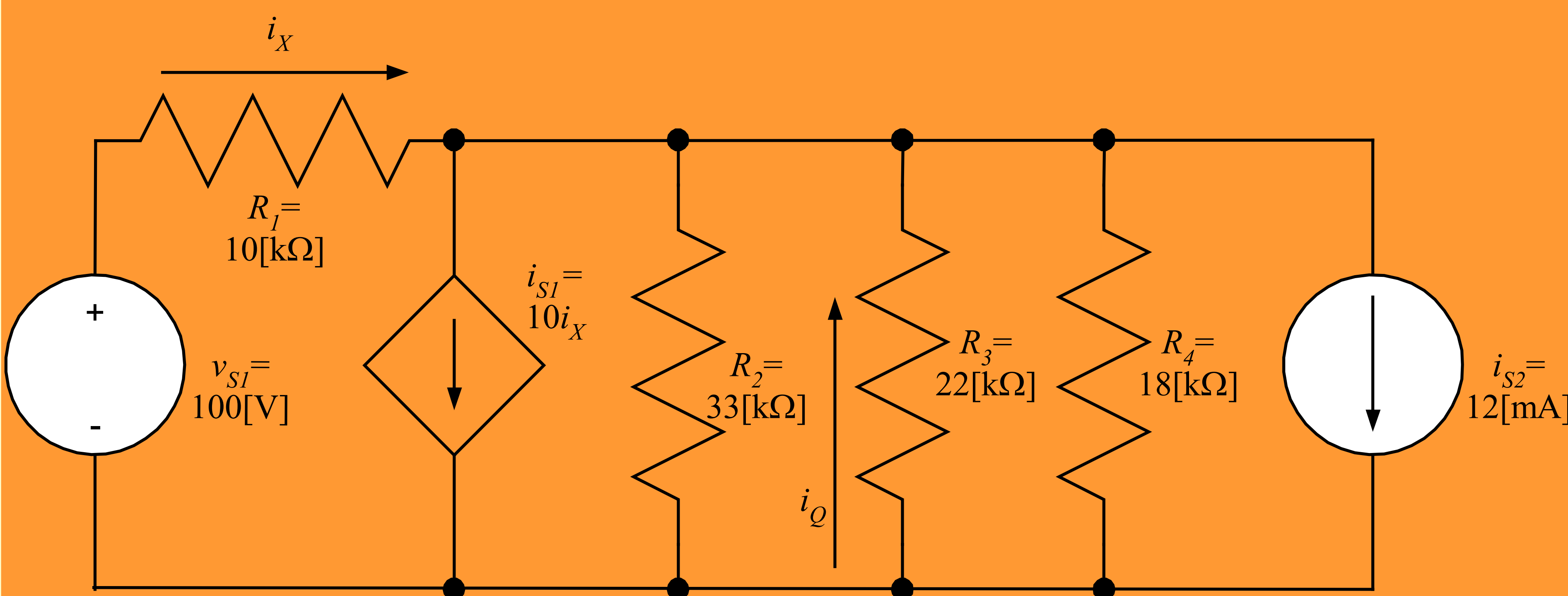


Sample Problem #2

3. a) Find the Norton equivalent as seen by the 22[kΩ] resistor.
   b) Use this circuit to solve for i_Q.

Soln: a) i_N = -102[mA], R_N = -1.228[kΩ]

b) -6.03[mA]



Week -13

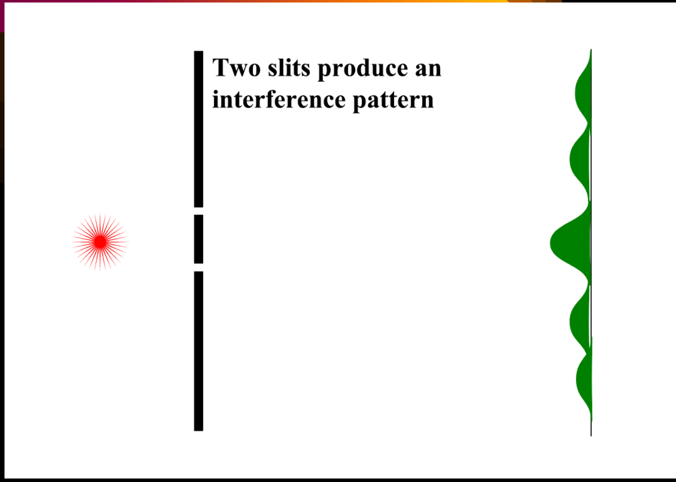
Page- (433-452)

Superposition

Superposition

The circuits we cover in this course fit into the category that are called Linear Circuits. This will be true as long as the circuits are made up of only the five basic circuit elements that we introduce in this course.

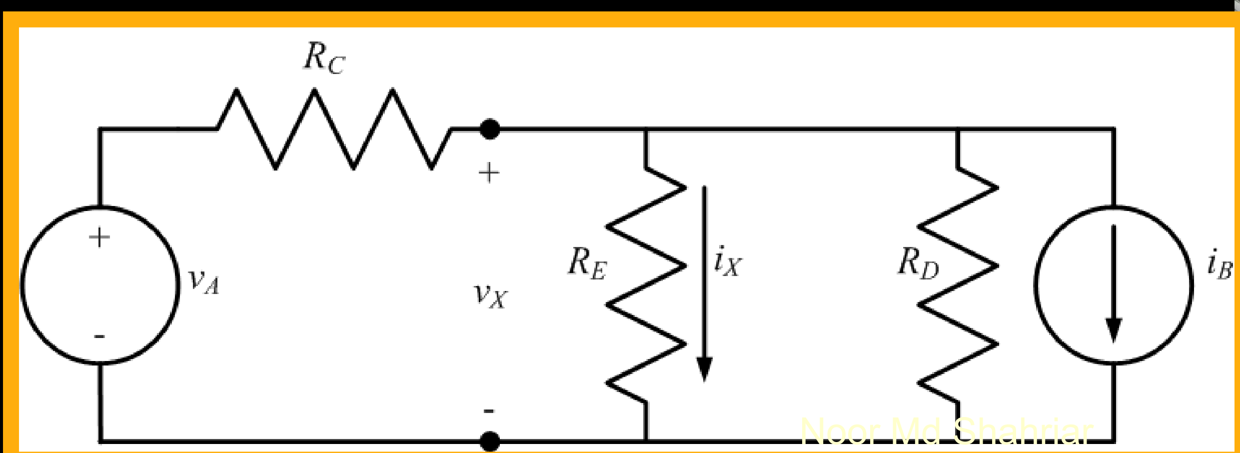
One of the definitions of Linear Circuits is that Linear Circuits are the circuits where superposition holds. If for no other reason, we should know what superposition is, so that we can understand this definition.



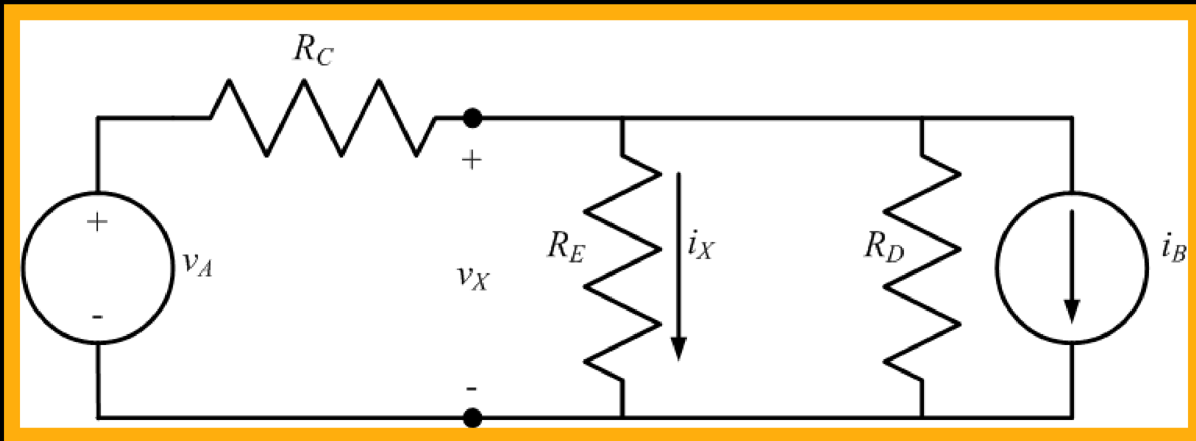
Superposition – Statement

Superposition can be stated in the following way, in the context of Circuit Analysis.

If there are more than one independent sources in a circuit, then any voltage or current in that circuit can be found by taking one independent source at a time, setting all other independent sources to zero, and solving for that voltage or current. This process is then repeated for all of the independent sources. Then, all of the obtained voltages or currents, for each independent source, can be added to find the desired voltage or current.



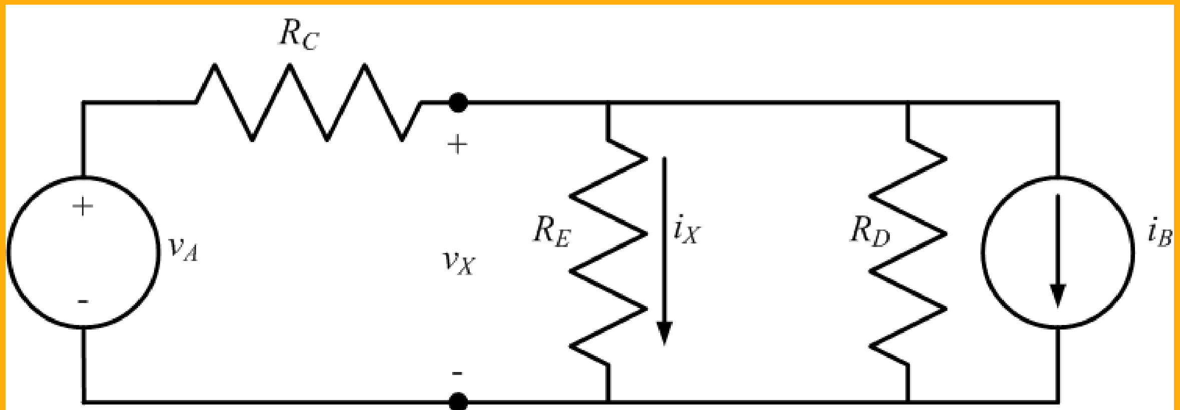
*Superposition – Emphasis on **Independent** Sources*



Superposition, in the context of Circuit Analysis, says that if there are more than one **independent** sources in a circuit, then any voltage or current in that circuit can be found by taking one **independent** source at a time, setting all other **independent** sources to zero, and solving for that voltage or current. This process is then repeated for all of the **independent** sources. Then, all of the obtained voltages or currents, for each **independent** source, can be added to find the desired voltage or current.

We have **bolded** the word **independent** in this statement to emphasize that it does not apply to dependent sources.

Superposition – General Example

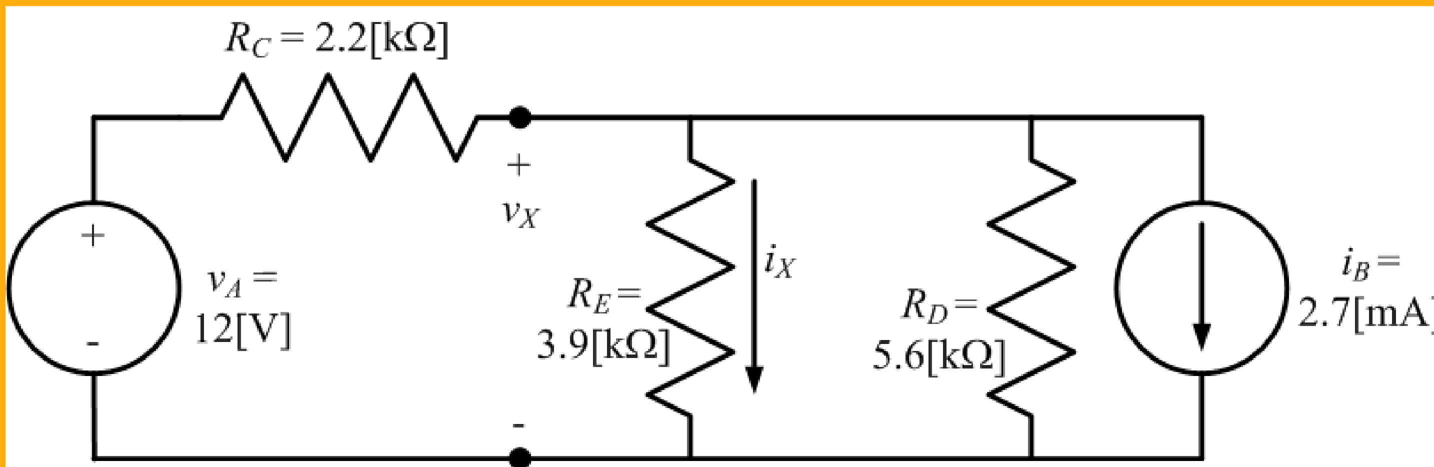


Superposition, then, means that in the circuit above, the current i_X can be found by taking v_A, setting i_B equal to zero, and solving for the current i_{XA} that results. Then, we would take i_B, setting v_A equal to zero, and solving for the current i_{XB} that results. Then, we would find i_X by using the equation

$$i_X = i_{XA} + i_{XB}.$$

We could do the same kind of thing for the voltage v_X.

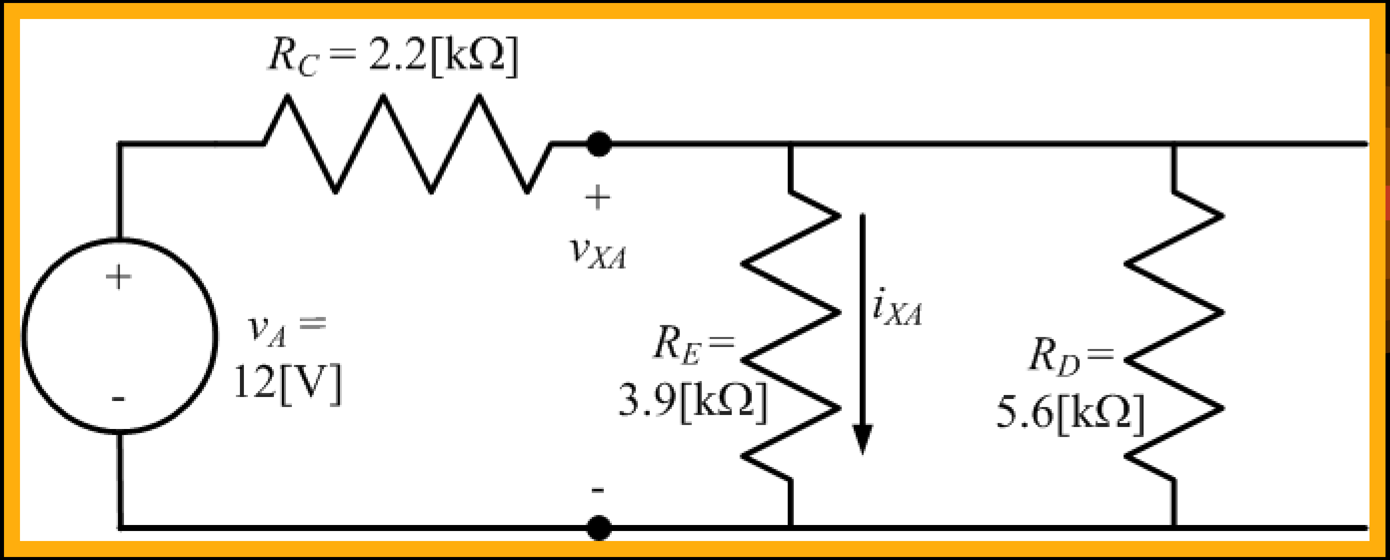
Superposition – Numerical Example



We will try to make this more clear by doing a specific, numerical example. Consider the circuit shown here, with numerical values for the components. We will solve for i_X and v_X using superposition. We will use the equations

$$i_X = i_{XA} + i_{XB}, \text{ and}$$
$$v_X = v_{XA} + v_{XB}.$$

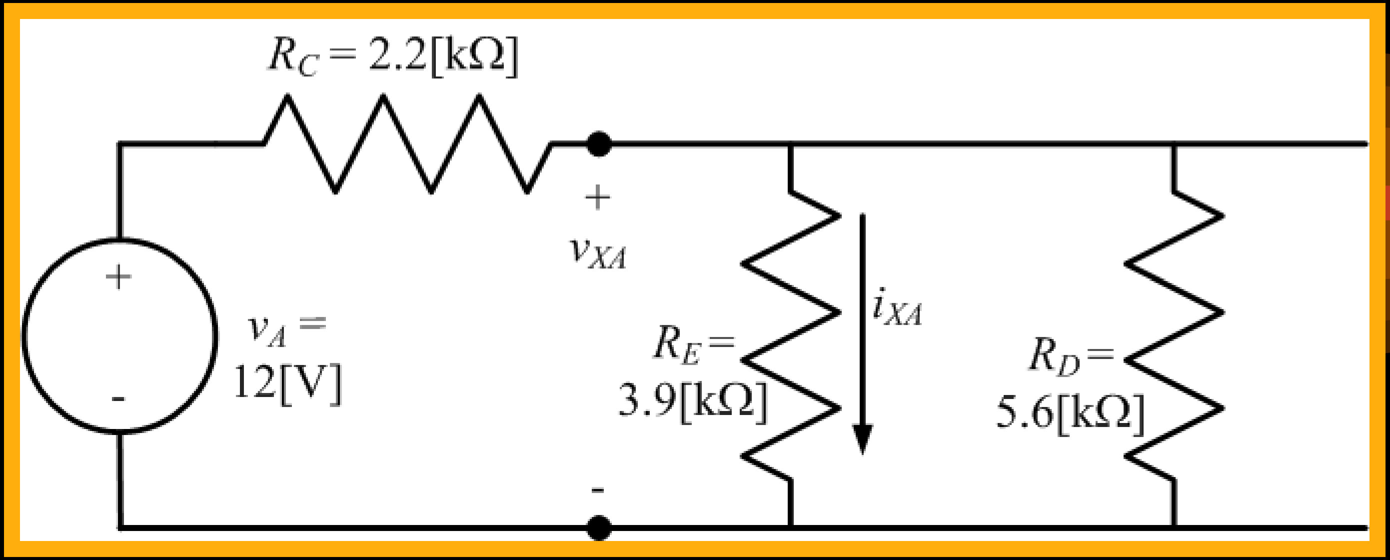
Superposition – Numerical Example
Step 1



We begin by taking the v_A source, and setting the i_B source equal to zero. We obtain the circuit above, and solve by writing VDR,

$$v_{XA} = v_A \frac{(R_E \parallel R_D)}{(R_E \parallel R_D) + R_C} = 6.132[\text{V}].$$

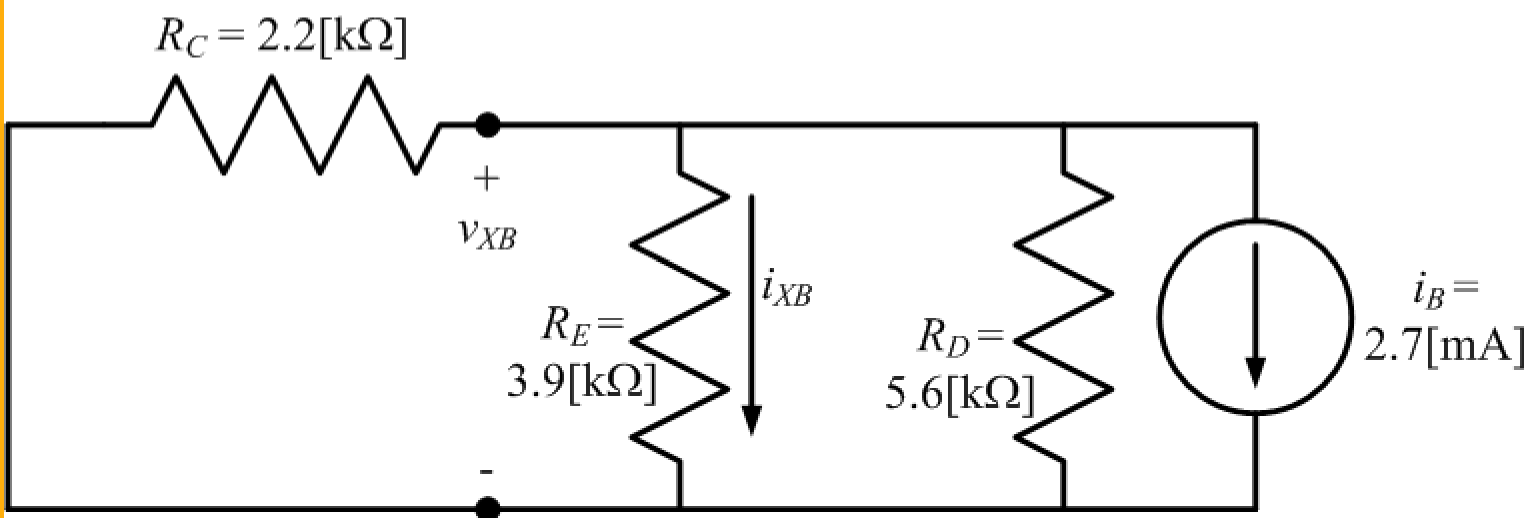
Superposition – Numerical Example Step 2



We can next find i_{XA} through Ohm's Law as

$$i_{XA} = \frac{v_{XA}}{R_E} = \frac{6.132[\text{V}]}{3.9[\text{k}\Omega]} = 1.572[\text{mA}].$$

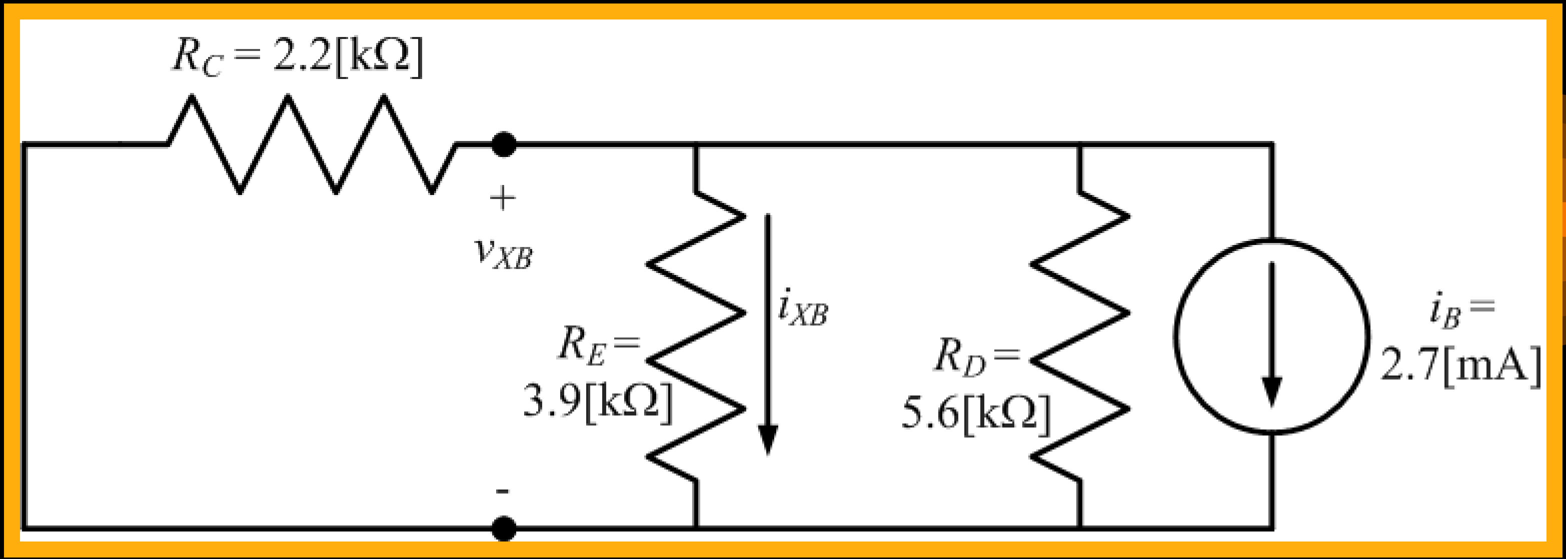
Superposition – Numerical Example Step 3



We continue by taking the i_B source, and setting the v_A source equal to zero. We obtain the circuit above, and solve by writing CDR,

$$i_{XB} = -i_B \frac{\left(R_C \parallel R_D\right)}{\left(R_C \parallel R_D\right) + R_E} = -778.3[\mu\text{A}].$$

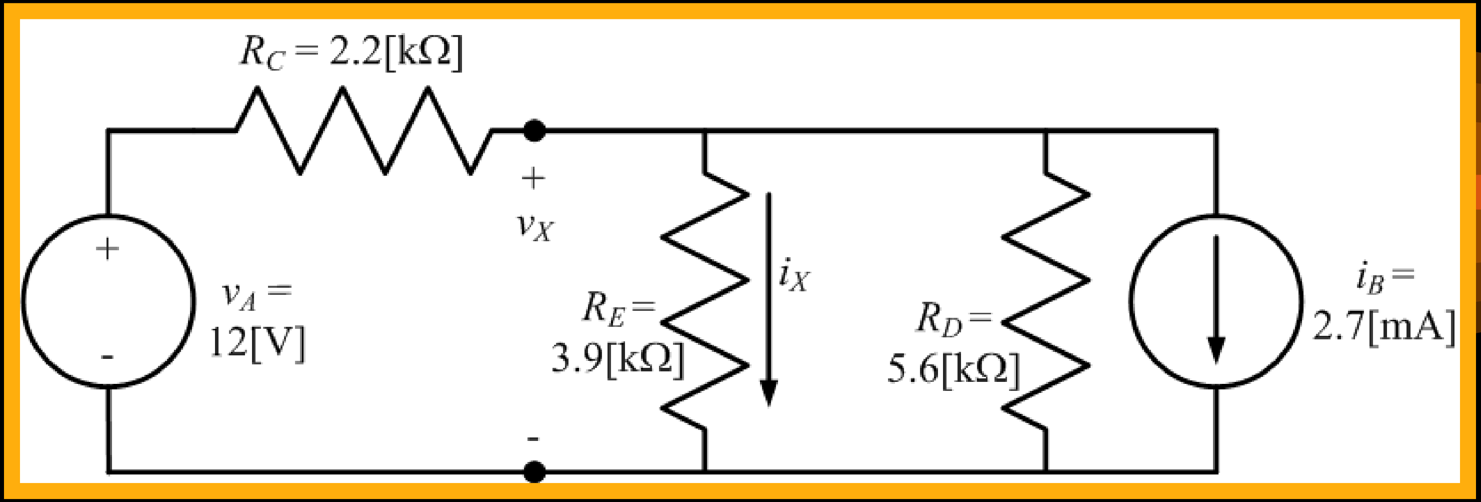
Superposition – Numerical Example Step 4



We can next find v_{XB} through Ohm's Law as

$$v_{XB} = i_{XB} R_E = \left(-778.3[\mu\text{A}]\right) \times \left(3.9[\text{k}\Omega]\right), \text{ or}$$
$$v_{XB} = -3.035[\text{V}].$$

Superposition – Numerical Example i_X Solution

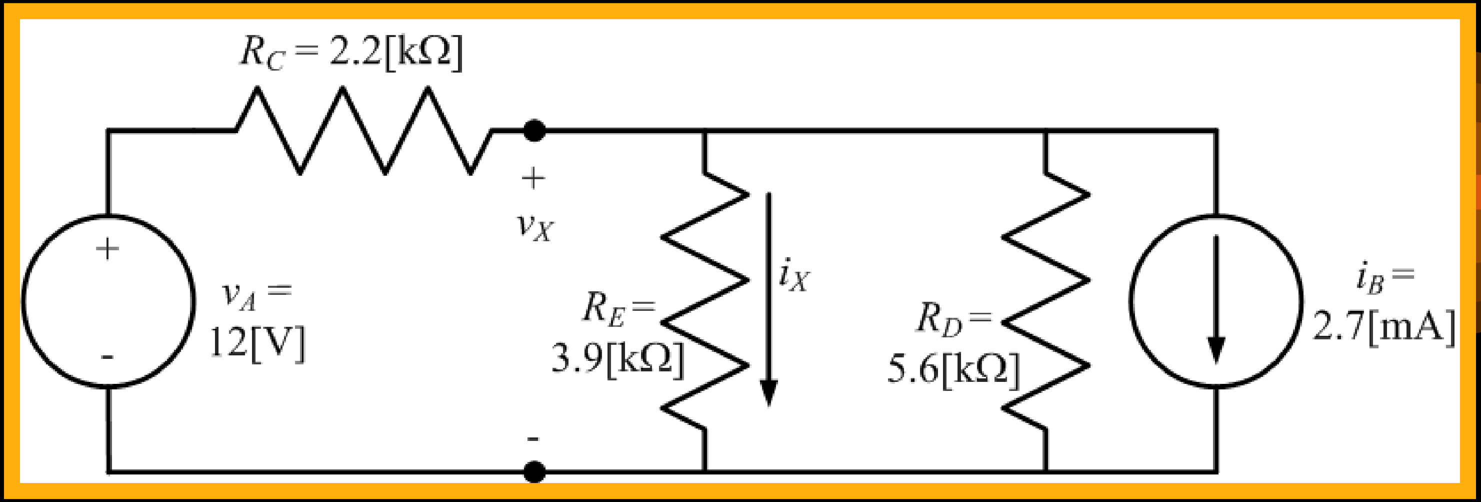


We can now say that

$$i_X = i_{XA} + i_{XB} = 1.572[\text{mA}] - 0.7783[\text{mA}], \text{ or}$$

$$i_X = 794[\mu\text{A}].$$

Superposition – Numerical Example v_X Solution

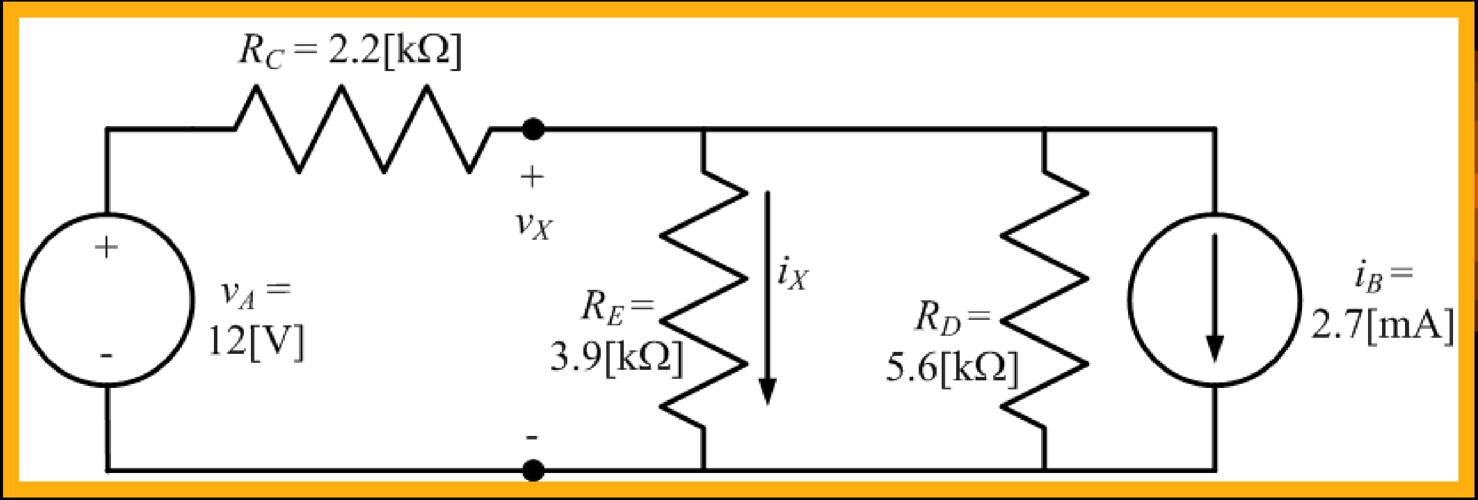


We can now say that

$$v_X = v_{XA} + v_{XB} = 6.132[\text{V}] - 3.035[\text{V}], \text{ or}$$

$$v_X = 3.097[\text{V}].$$

Solving without Superposition – Numerical Example



We now note we could have written KCL in this circuit to get that

$$\frac{v_X}{3.9[\text{k}\Omega]} + \frac{v_X - 12[\text{V}]}{2.2[\text{k}\Omega]} + 2.7[\text{mA}] + \frac{v_X}{5.6[\text{k}\Omega]} = 0, \text{ or}$$

$$v_X = 3.097[\text{V}].$$

This would give us the same answer, more easily than by using superposition.

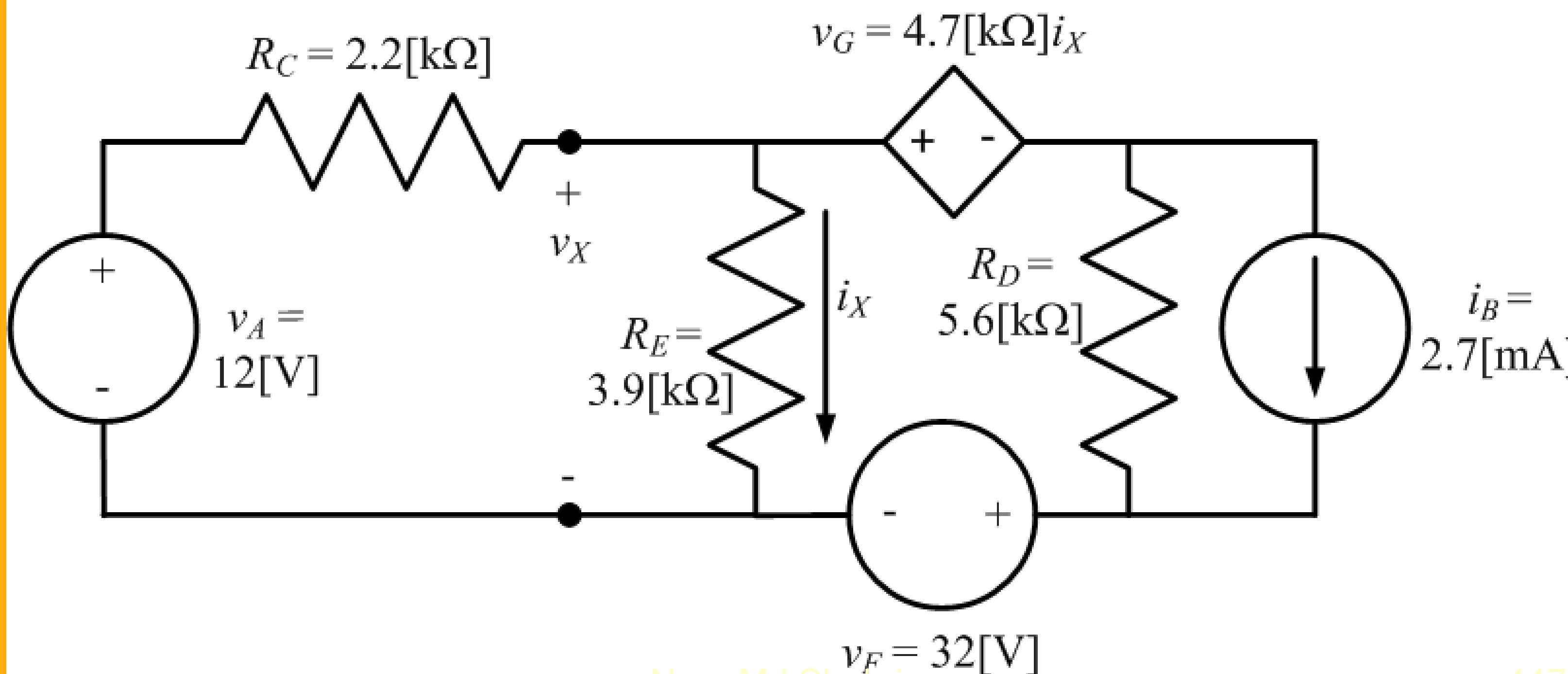
Notes

1. We found that superposition means that we can find voltages and currents by adding the inputs of each of the independent sources, taking each independent source one at a time.
2. This superposition approach, however, is not really a very efficient way to solve the problems we have at this point.
3. Later in this course, we will introduce a situation where superposition allows us to use a technique we will call phasor analysis in places where we can take a much more efficient approach using that superposition concept. So, soon it will be very valuable.

Example Problem

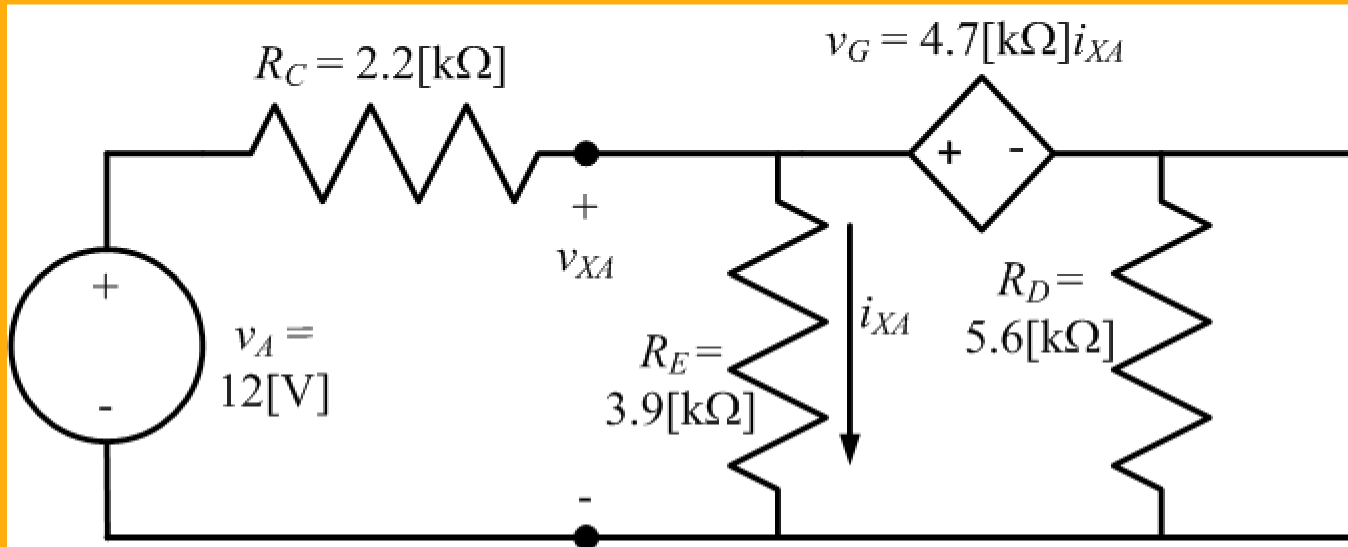
We wish to use superposition to find, v_X, in the circuit below.

This will give us a chance to show what having three independent sources means, and how to handle dependent sources.



Example Problem – Step 1

We begin by taking v_A, and setting i_B and v_F equal to zero. Note that we do **not** set the dependent source v_G to zero.

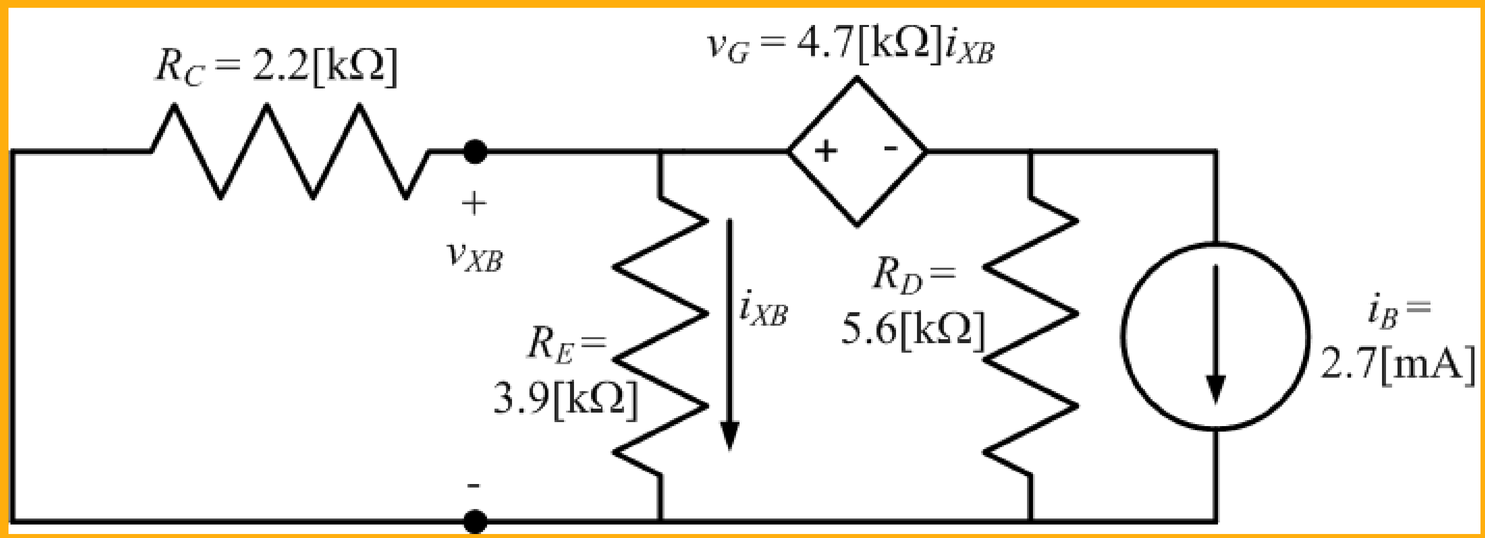


$$\frac{v_{XA}}{3.9[\text{k}\Omega]} + \frac{v_{XA} - 12[\text{V}]}{2.2[\text{k}\Omega]} + \frac{v_{XA} - 4.7[\text{k}\Omega] i_{XA}}{5.6[\text{k}\Omega]} = 0, \text{ and}$$

$$i_{XA} = \frac{v_{XA}}{3.9[\text{k}\Omega]}. \text{ Solving, } v_{XA} = 8.089[\text{V}].$$

Example Problem – Step 2

Our next step involves taking i_B, and setting v_A and v_F equal to zero. Note that we do **not** set the dependent source v_G to zero.

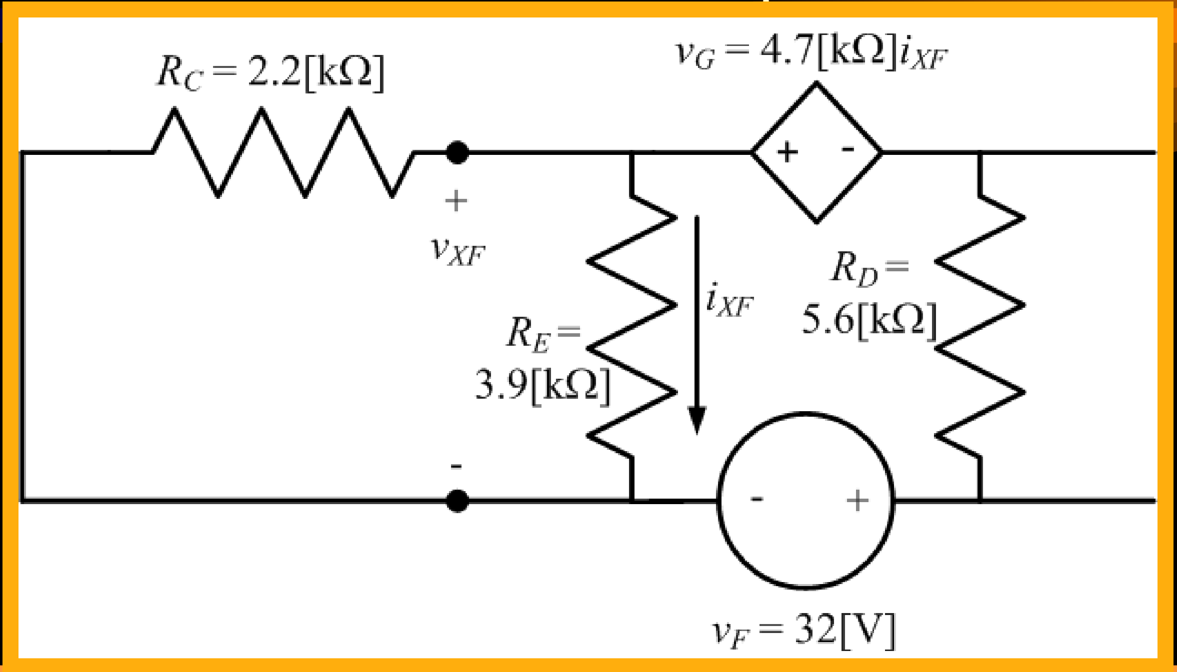


$$\frac{v_{XB}}{3.9[\text{k}\Omega]} + \frac{v_{XB}}{2.2[\text{k}\Omega]} + 2.7[\text{mA}] + \frac{v_{XB} - 4.7[\text{k}\Omega]i_{XB}}{5.6[\text{k}\Omega]} = 0, \text{ and}$$

$$i_{XB} = \frac{v_{XB}}{3.9[\text{k}\Omega]}. \text{ Solving, } v_{XB} = -4.004[\text{V}].$$

Example Problem – Step 3

Finally, by taking v_F, and setting v_A and i_B equal to zero. Note that we do **not** set the dependent source v_G to zero.



$$\frac{v_{XF}}{3.9[\text{k}\Omega]}+\frac{v_{XF}}{2.2[\text{k}\Omega]}+\frac{v_{XF}-4.7[\text{k}\Omega]i_{XF}-32[\text{V}]}{5.6[\text{k}\Omega]}=0, \text{ and}$$

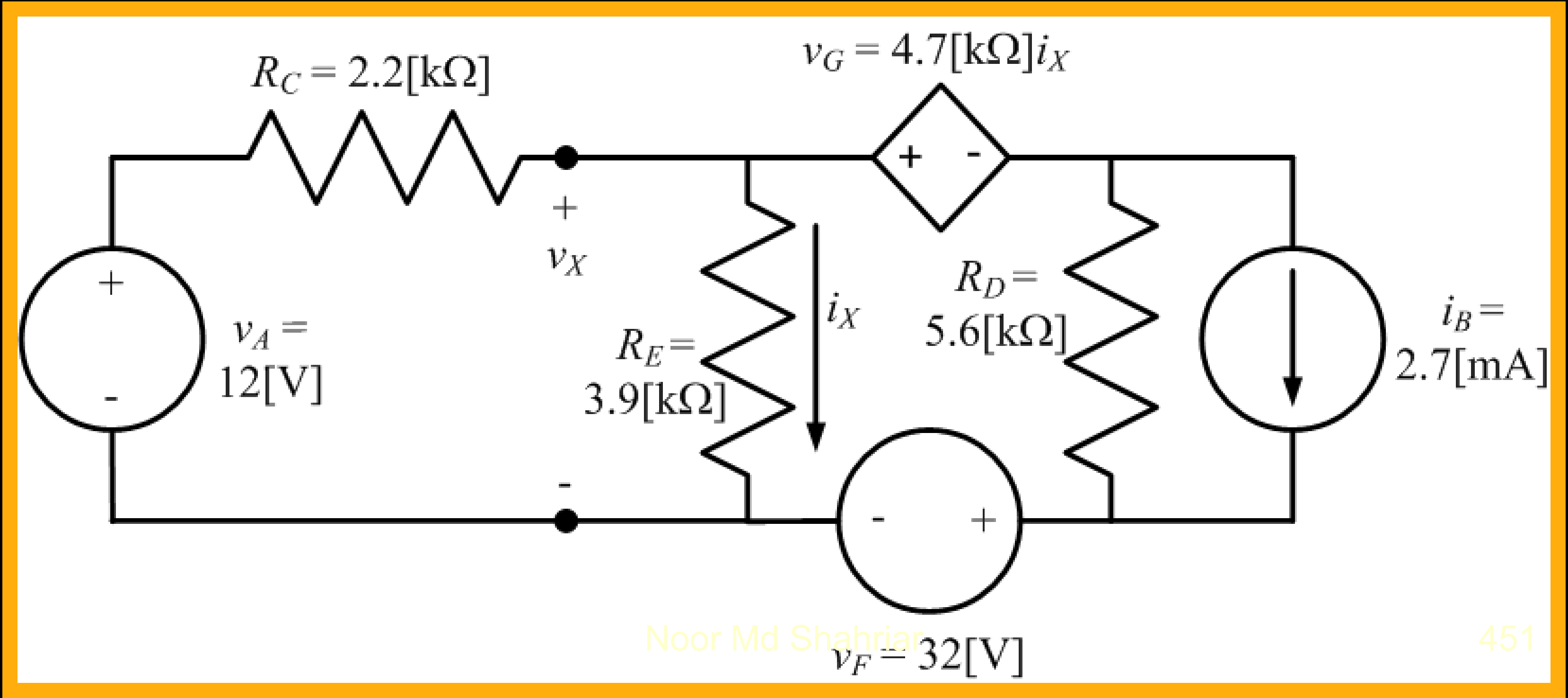
$$i_{XF}=\frac{v_{XF}}{3.9[\text{k}\Omega]}. \text{ Solving, } v_{XF}=8.474[\text{V}].$$

Example Problem – Step 4

We now solve for v_X, writing

$$v_X = v_{XA} + v_{XB} + v_{XF}, \text{ or}$$

$$v_X = 8.089[\text{V}] - 4.004[\text{V}] + 8.474[\text{V}] = 12.559[\text{V}].$$



Week -14

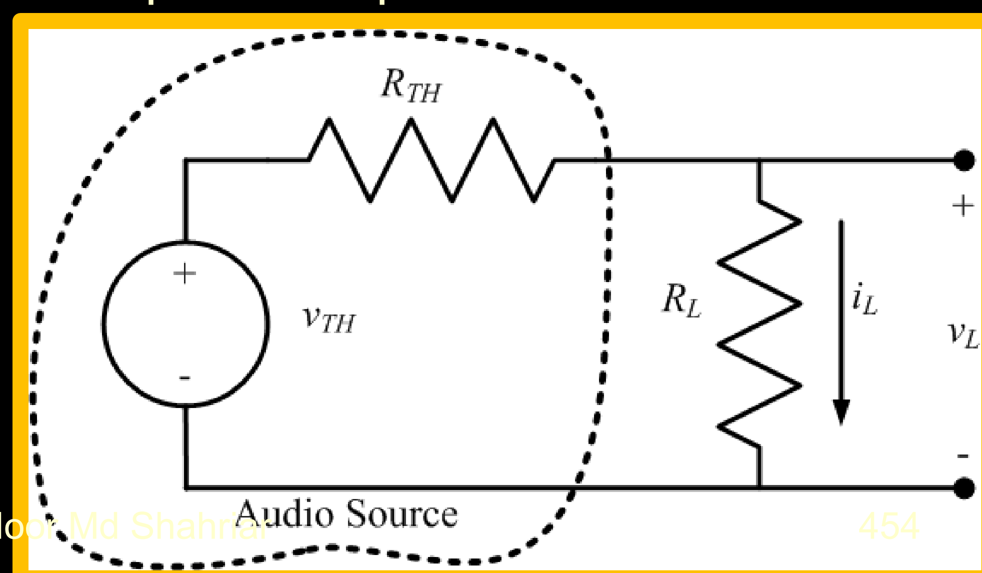
Page- (453-470)

Maximum Power Transfer

Maximum Power Transfer

Imagine a situation where the goal is to determine what load to attach to a source, so that as much power as possible can be extracted from that source. As just one practical example, imagine that you had an audio source in your vehicle. You wanted to get as much sound as possible out of that audio source, so that you could play your music as loud as possible.

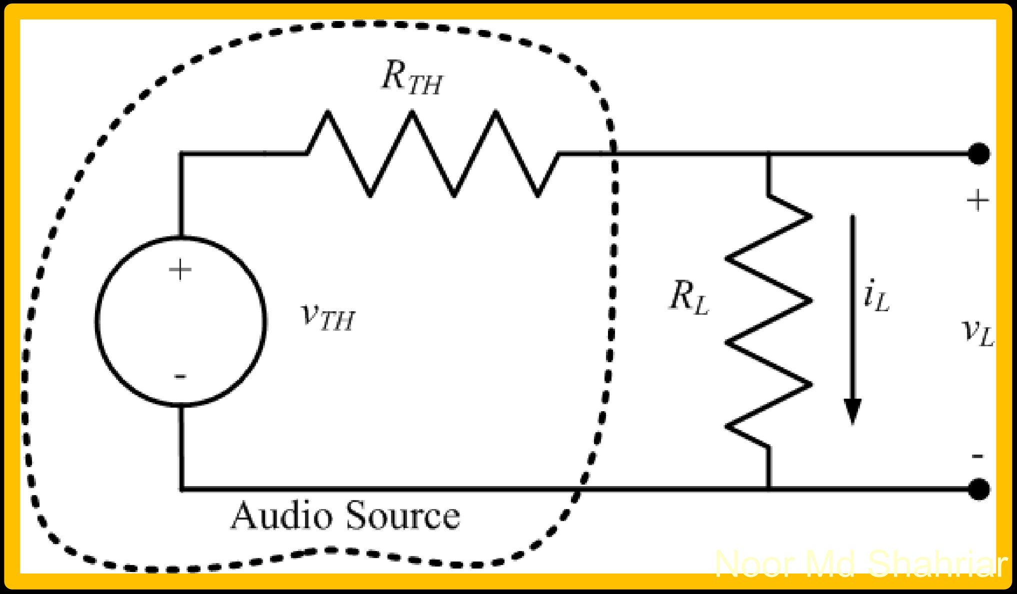
We could think of this with the following circuit assumptions. Assume that your audio source can be modeled with a Thevenin equivalent. Assume that this Thevenin equivalent has a positive value for the Thevenin equivalent resistance. Thus, R_{TH} is positive. Assume that your load, in this case, your speaker, could be modeled by a resistor, which means that R_L is positive. The question would then translate to this: How can you pick the load resistor value (R_L) to get as much power as possible out of the audio source?

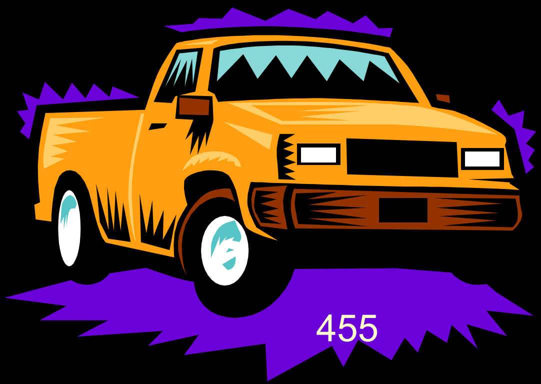


Maximum Power Transfer – Guess 1

How can you pick the load resistor value (R_L) to get as much power as possible out of the audio source?

Guess #1. Let us imagine that we decided to get maximum power absorbed by the load, (R_L), by **maximizing the current through the load**. We could maximize the current, i_L, by picking R_L = 0. Let us consider what would happen.





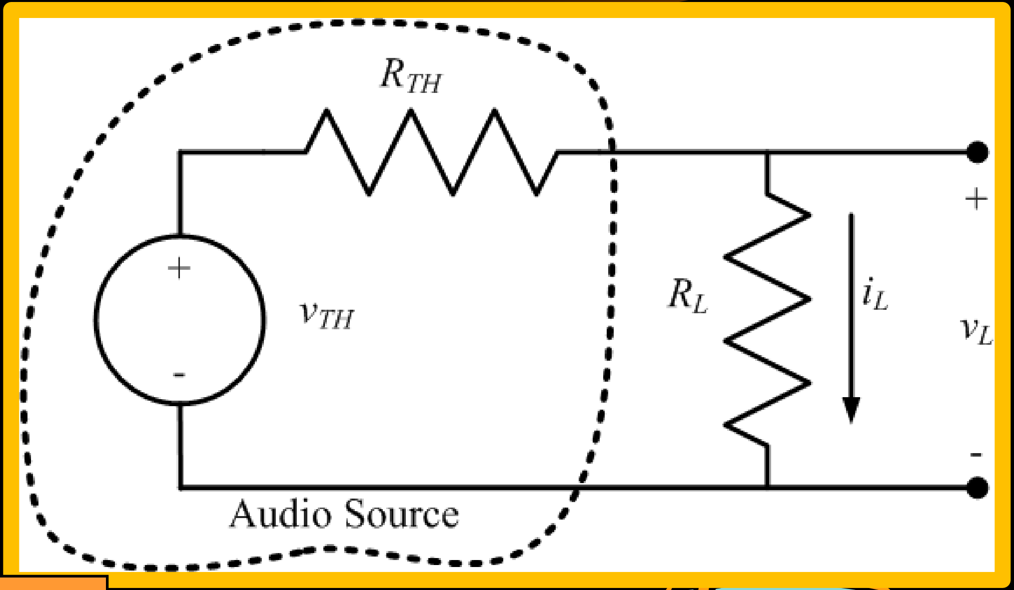
Maximum Power Transfer – Guess 1

How can you pick the load resistor value (R_L) to get as much power as possible out of the audio source?

With R_L = 0, we would have the following. The equation for v_L would be

$$v_L = v_{TH}\left(\frac{R_L}{R_L + R_{TH}}\right), \text{ so}$$

$$v_L = v_{TH}\left(\frac{0}{0 + R_{TH}}\right) = 0.$$



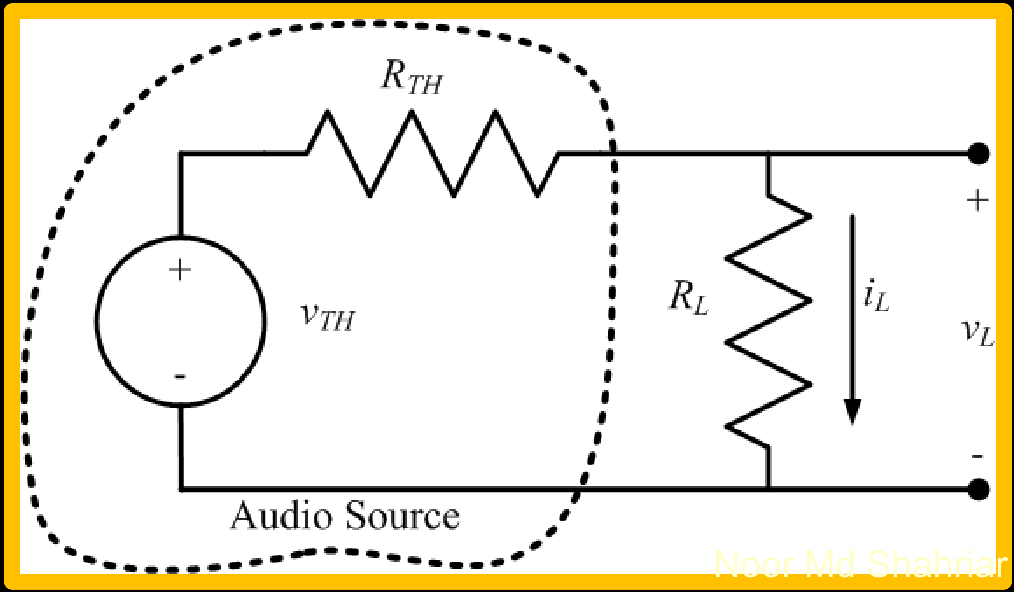
$$\text{Thus, } p_{ABS.BY.R_L} = v_L i_L = 0 i_L = 0.$$

Clearly, that was not the correct guess. Let us try again.

Maximum Power Transfer – Guess 2

How can you pick the load resistor value (R_L) to get as much power as possible out of the audio source?

Guess #2. Let us imagine that we decided to get maximum power absorbed by the load, (R_L), by **maximizing the voltage across the load**. We could maximize the voltage, v_L, by picking $R_L = \infty$. Let us consider what would happen.



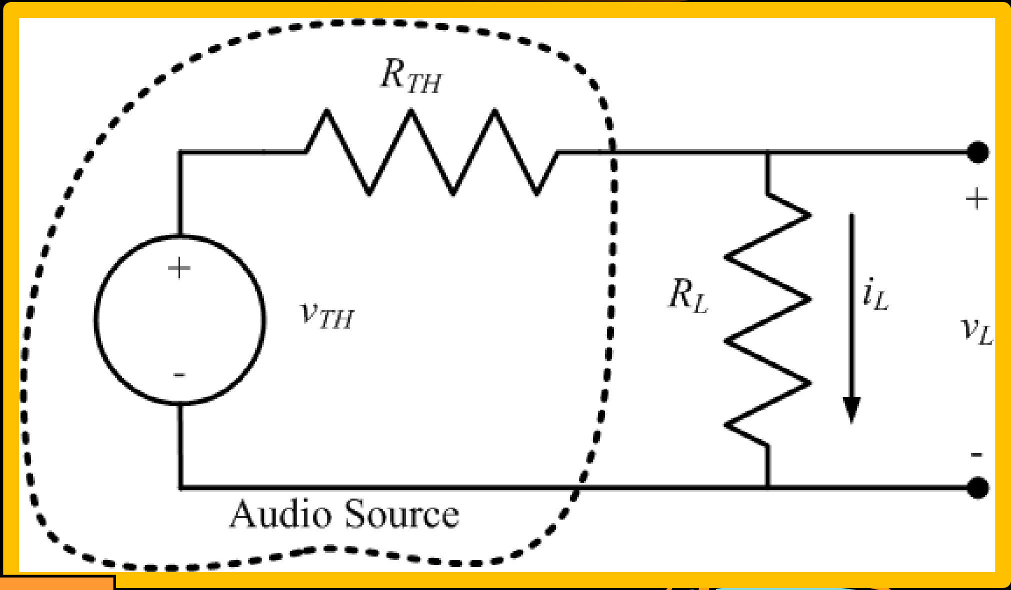
Maximum Power Transfer – Guess 2

How can you pick the load resistor value (R_L) to get as much power as possible out of the audio source?

With $R_L = \infty$, we would have the following. The equation for i_L would be

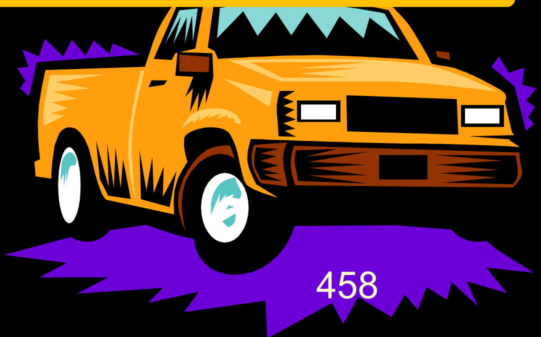
$$i_L = \left(\frac{v_{TH}}{R_L + R_{TH}}\right), \text{ so}$$

$$i_L = \left(\frac{v_{TH}}{\infty + R_{TH}}\right) = 0.$$



Thus, $p_{ABS.BY.R_L} = v_L i_L = v_L 0 = 0.$

Clearly, that was not the correct guess, either. Let us try again.



Maximum Power Transfer – Maxima and Minima Problem

It is probably obvious to you that this is a problem we should approach with the techniques we learned in calculus to determine the maxima and minima of a function. We begin by setting up the formula for the power absorbed by the load. We have

$$p_{ABS.BY.R_L} = v_L i_L = v_{TH}\left(\frac{R_L}{R_L + R_{TH}}\right)\left(\frac{v_{TH}}{R_L + R_{TH}}\right), \text{ or}$$

$$p_{ABS.BY.R_L} = v_L i_L = (v_{TH})^2\left(\frac{R_L}{(R_L + R_{TH})^2}\right).$$

Next, we differentiate the power expression, with respect to R_L. We get

$$\frac{d\left(p_{ABS.BY.R_L}\right)}{dR_L} = \left(v_{TH}\right)^2 \left(\frac{\left(R_L + R_{TH}\right)^2 - R_L 2\left(R_L + R_{TH}\right)}{\left(R_L + R_{TH}\right)^4}\right).$$

After that, we set this derivative equal to zero and solve, to get

$$R_L = R_{TH}.$$

Then, we examine the second derivative, and find out it is negative, so this is a local maximum.

Maximum Power Transfer – Maxima and Minima Problem

So, we have

$$R_L = R_{TH}.$$

as a local maximum. To complete the process, we examine the end points of the possible range of values, which we actually already did with our Guess 1 and Guess 2. Those end points, where R_L = 0 and R_L = ∞, were both zero values for power, so they were not the maximum value.

Finally, we look for discontinuities in the function, but there are none for positive values of R_L and R_{TH}.

This value is our maximum value.

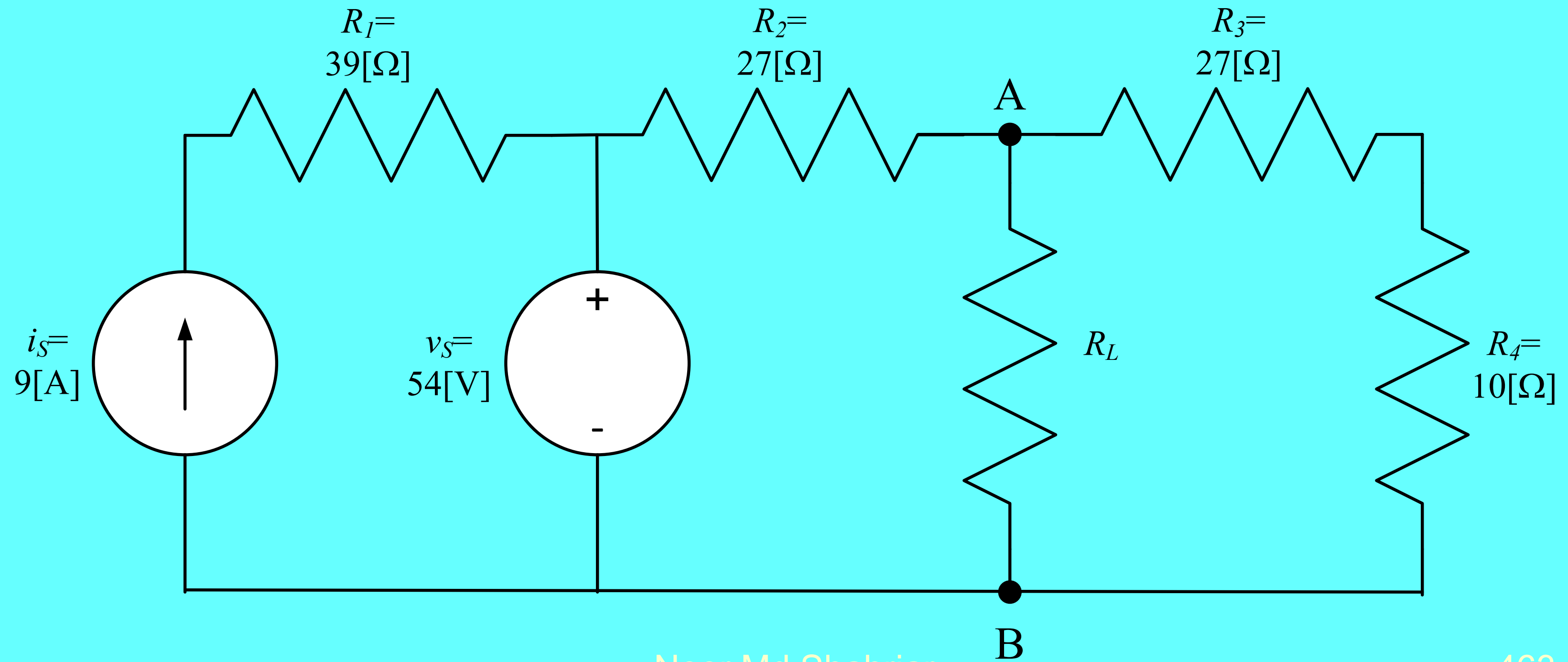
Notes

1. **We found that the maximum power is extracted from the source, when the load resistance is equal to the Thevenin resistance of the source.**
2. So the answer is that we should pick the resistance of the speaker in our vehicle to be equal to the Thevenin resistance of our audio source, to get the maximum power out of that audio source.
3. However, this conclusion is generally valid, and therefore significantly valuable. We call the rule stated in note 1 as the Maximum Power Transfer rule.
4. This will be useful in a wide range of applications.

Example Problem

We wish to find the maximum power that can be delivered to the load resistor, R_L, in the circuit below.

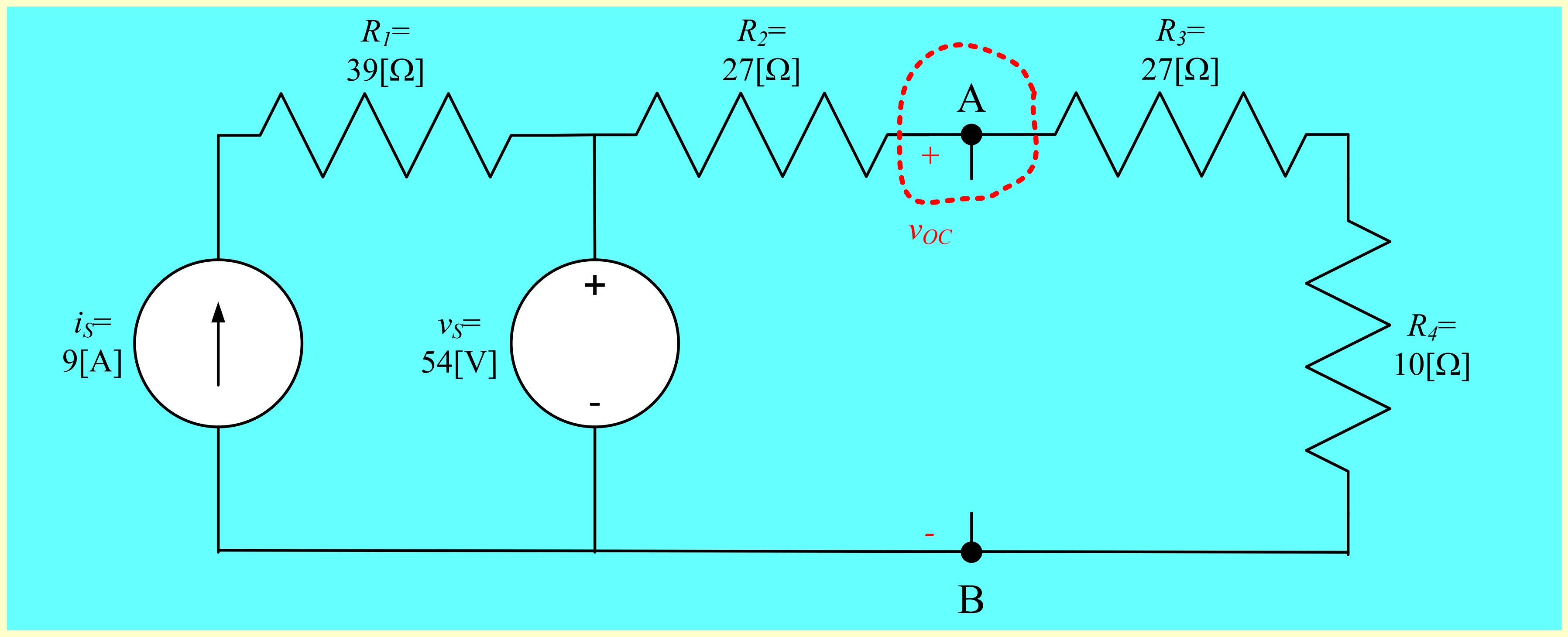
We will find the Thevenin equivalent as seen by the load resistor, R_L, and use it to get the solution. We begin by naming the terminals of the resistor R_L in the diagram, as A and B.



Example Problem – Step 1

We begin by finding the open-circuit voltage v_{OC} with the polarity defined in the circuit given below.

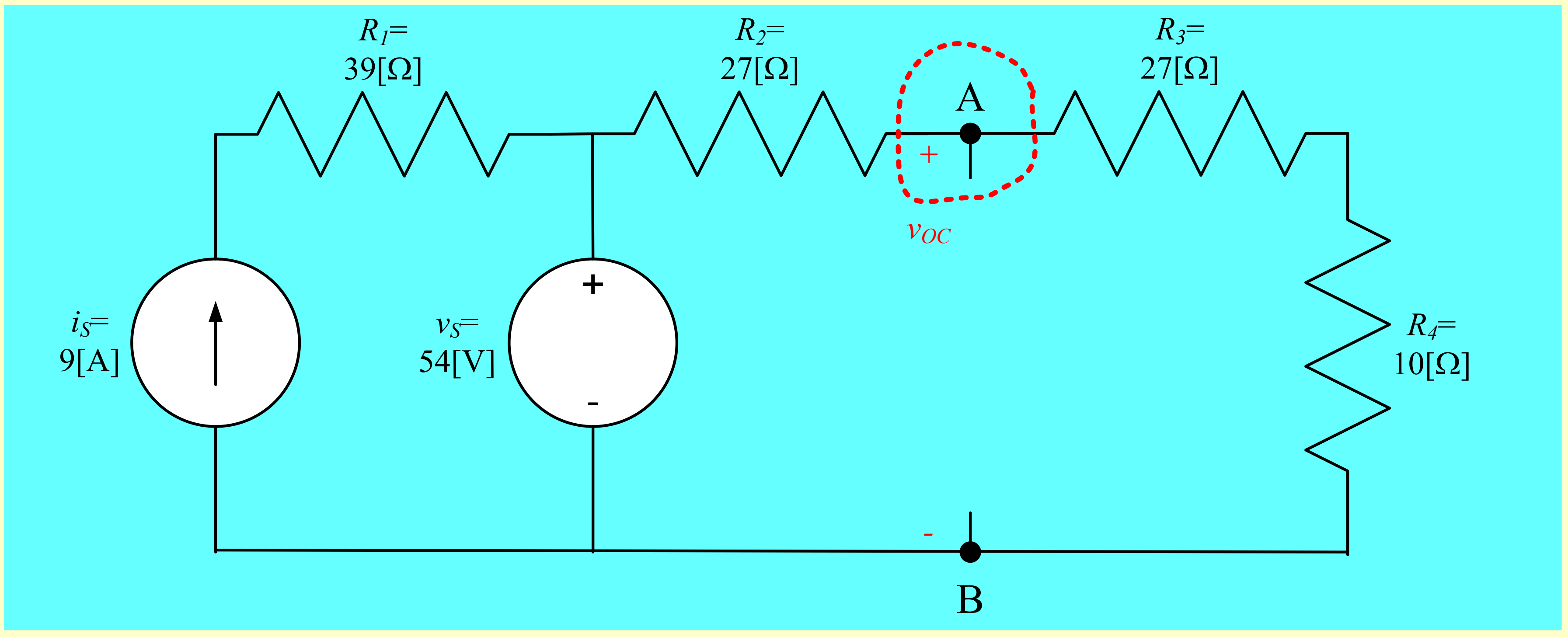
We remove R_L, since we are finding the Thevenin equivalent with respect to it.



Example Problem – Step 2

We find the voltage v_{OC}. Writing VDR as

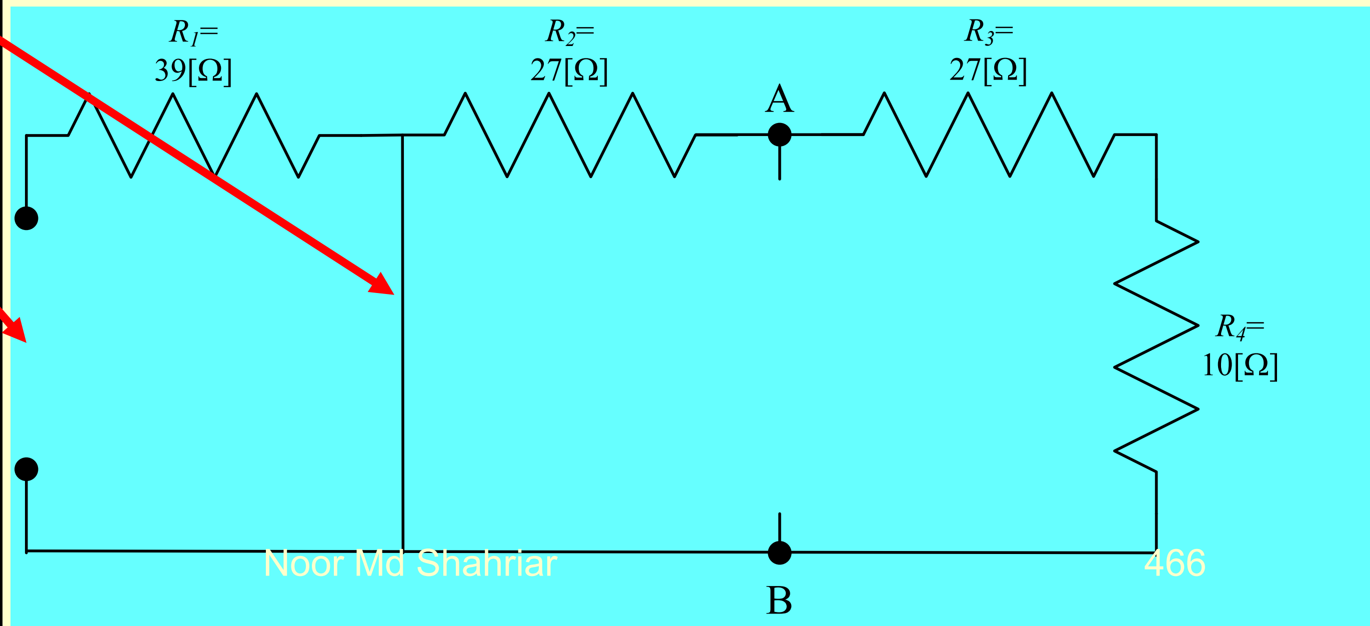
$$v_{OC} = 54[\text{V}]\frac{(10+27)[\Omega]}{(10+27+27)[\Omega]} = 31.22[\text{V}].$$



Example Problem – Step 3

Next, we will find the equivalent resistance seen by the load resistor. We will call this equivalent resistance R_{EQ}. The first step in this solution is to set the independent sources equal to zero. We get this circuit, shown below.

Note that the voltage source becomes a short circuit, and the current source becomes an open circuit. These represent zero-valued sources.

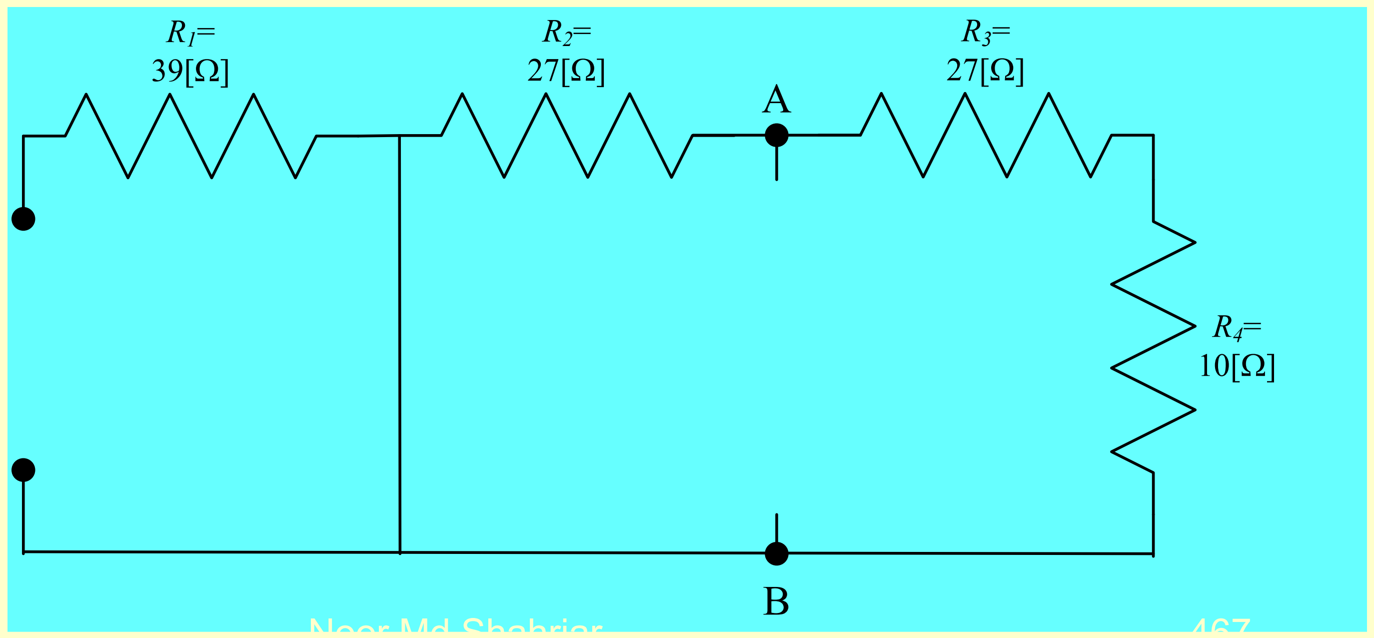


Example Problem – Step 4

To find the equivalent resistance, R_{EQ}, we simply combine resistances in parallel and in series. We have

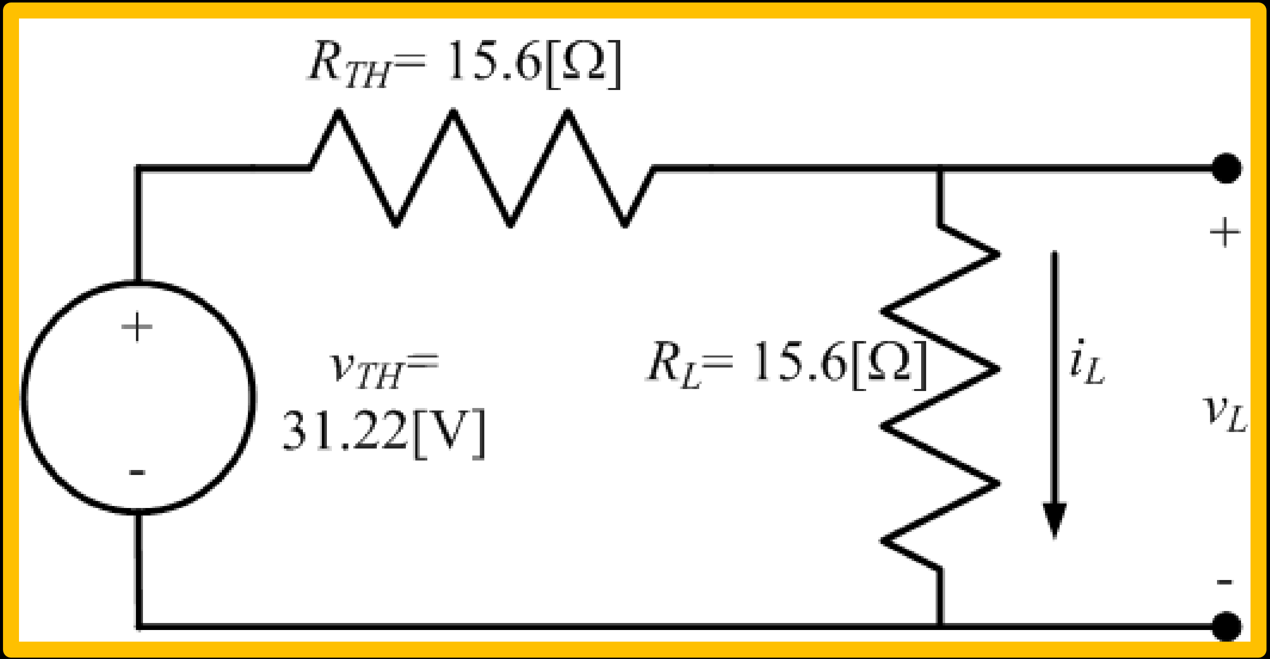
$$R_{EQ} = (R_3 + R_4) \| R_2 = 37[\Omega] \| 27[\Omega].$$ Solving, we get

$$R_{EQ} = 15.6[\Omega].$$



Example Problem – Step 5

To complete this problem, we would redraw the circuit, showing the complete Thevenin's equivalent, connected to the load. Also, to get maximum power transfer, we make the load equal to the Thevenin resistance of the source. This has been done here.

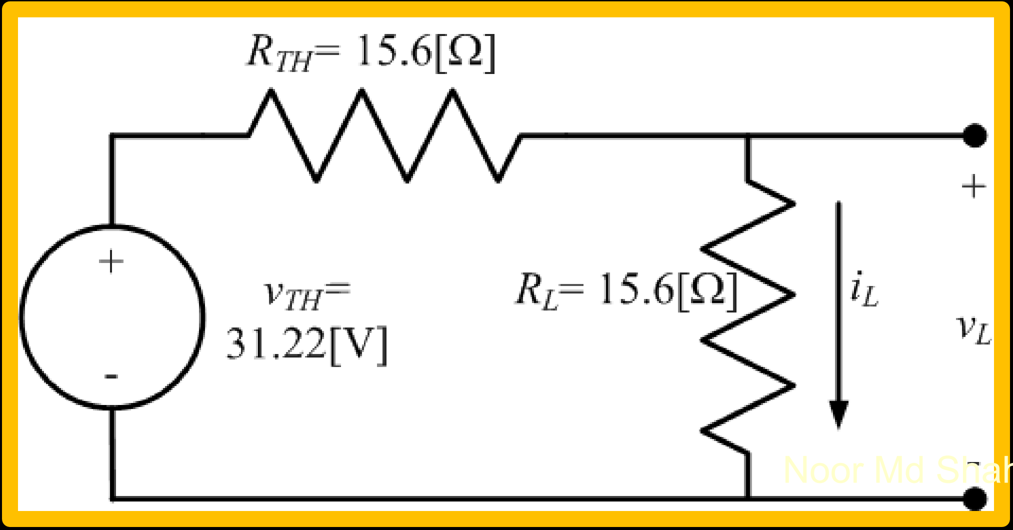


Example Problem – Step 6

Finally, we calculate the power absorbed by the load. Because the resistances are equal, the voltage across the load is half that of the source. We have

$$p_{ABS.BY.R_L} = \frac{{v_L}^2}{R_L} = \frac{\left(v_{TH}/2\right)^2}{R_L} = \frac{\left(31.22[\text{V}]/2\right)^2}{15.6[\Omega]}, \text{ or}$$

$$p_{ABS.BY.R_L} = 15.62[\text{W}].$$



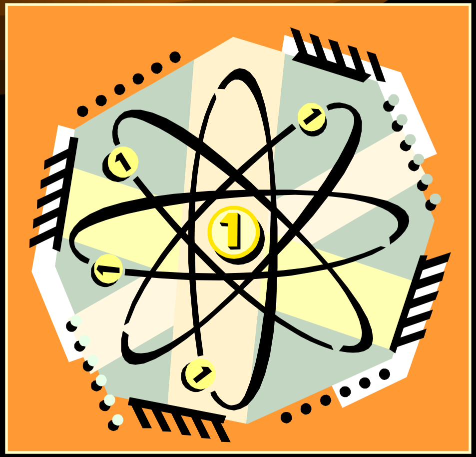
Week -15

Page- (471-488)

Inductors and Capacitors

Circuit Elements

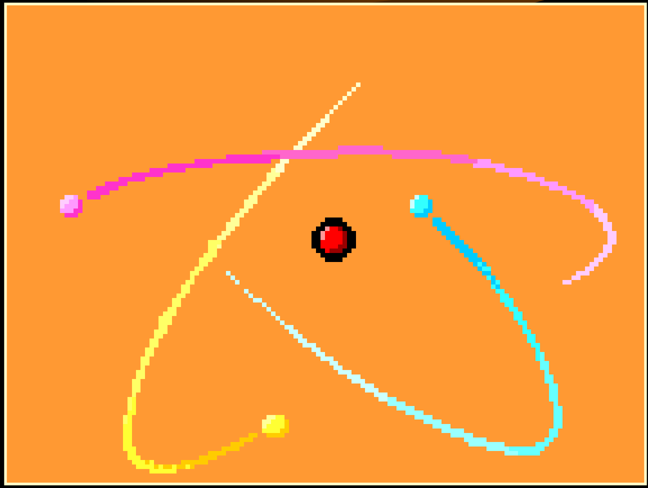
- In circuits, we think about basic **circuit elements** that are the basic “building blocks” of our circuits. This is similar to what we do in Chemistry with chemical elements like oxygen or nitrogen.
- A circuit element cannot be broken down or subdivided into other circuit elements.
- A circuit element can be defined in terms of the behavior of the voltage and current at its terminals.



The 5 Basic Circuit Elements

There are 5 basic circuit elements:

1. Voltage sources
2. Current sources
3. Resistors
4. Inductors
5. Capacitors



We defined the first three elements previously. We will now introduce inductors or capacitors.

Inductors

- An inductor is a two-terminal circuit element that has a voltage across its terminals which is proportional to the derivative of the current through its terminals.
- The coefficient of this proportionality is the defining characteristic of an inductor.
- An inductor is the device that we use to model the effect of magnetic fields on circuit variables. The energy stored in magnetic fields has effects on voltage and current. We use the inductor component to model these effects.

In many cases a coil of wire can be modeled as an inductor.

Inductors – Definition and Units

- An inductor obeys the expression

$$v_L = L_X \frac{di_L}{dt}$$

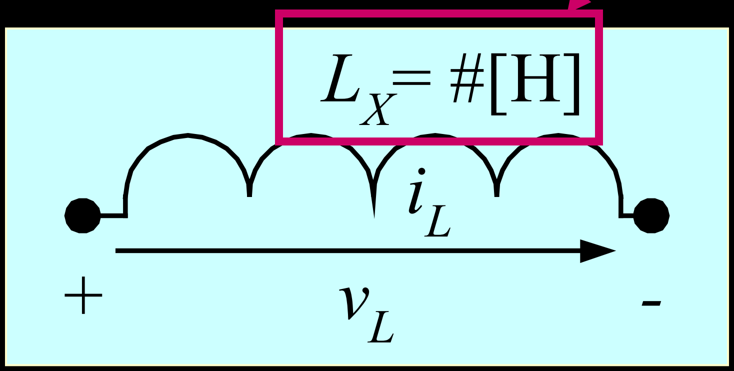
where v_L is the voltage across the inductor, and i_L is the current through the inductor, and L_X is called the inductance.

- In addition, it works both ways. If something obeys this expression, we can think of it, and model it, as an inductor.
- The unit ([Henry] or [H]) is named for Joseph Henry, and is equal to a [Volt-second/Ampere].

There is an inductance whenever we have magnetic fields produced, and there are magnetic fields whenever current flows. However, this inductance is often negligible except when we wind wires in coils to concentrate the effects.

Schematic Symbol for Inductors

The schematic symbol that we use for inductors is shown here.



This is intended to indicate that the schematic symbol can be labeled either with a variable, like L_X, or a value, with some number, and units. An example might be 390[mH]. It could also be labeled with both.

$$v_L = L_X \frac{di_L}{dt}$$

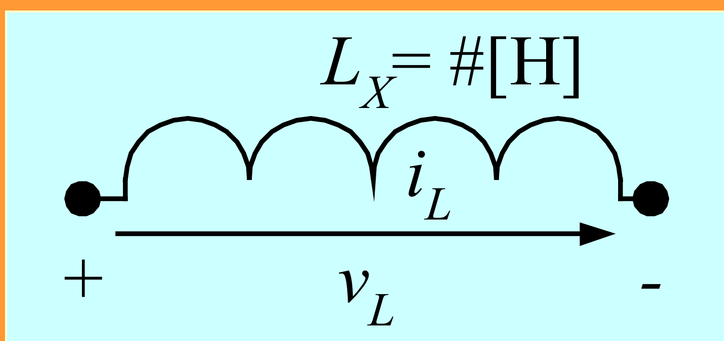
Inductor Polarities

- Previously, we have emphasized the important of reference polarities of current sources and voltages sources. There is no corresponding polarity to an inductor. You can flip it end-for-end, and it will behave the same way.
- However, similar to a resistor, direction matters in one sense; we need to have defined the voltage and current in the **passive sign relationship** to use the defining equation the way we have it here.

$$v_L = L_X \frac{di_L}{dt}$$

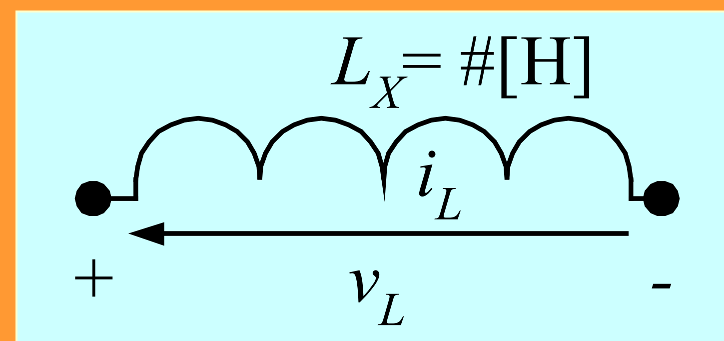
Passive and Active Sign Relationship for Inductors

The sign of the equation that we use for inductors depends on whether we have used the passive sign relationship or the active sign relationship.



$$v_L = L_X \frac{di_L}{dt}$$

Passive Sign Relationship



$$v_L = -L_X \frac{di_L}{dt}$$

Active Sign Relationship

Defining Equation, Integral Form, Derivation

The defining equation for the inductor,

$$v_L = L_X \frac{di_L}{dt}$$

can be rewritten in another way. If we want to express the current in terms of the voltage, we can integrate both sides. We get

$$\int_{t_0}^{t} v_L(t)dt = \int_{t_0}^{t} L_X \frac{di_L}{dt} dt.$$

We pick t_0 and t for limits of the integral, where t is time, and t_0 is an arbitrary time value, often zero. The inductance, L_X, is constant, and can be taken out of the integral. To avoid confusion, we introduce the dummy variable s in the integral. We get

$$\frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds = \int_{t_0}^{t} di_L.$$

We finish the derivation in the next slide.

Defining Equations for Inductors

$$\frac{1}{L_X}\int_{t_0}^{t} v_L(s)\,ds = \int_{t_0}^{t} di_L .$$

We can take this equation and perform the integral on the right hand side. When we do this we get

$$\frac{1}{L_X}\int_{t_0}^{t} v_L(s)\,ds = i_L(t) - i_L(t_0).$$

Thus, we can solve for $i_L(t)$, and we have two defining equations for the inductor,

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)\,ds + i_L(t_0),$$

and

$$v_L = L_X \frac{di_L}{dt}.$$

Remember that both of these are defined for the passive sign relationship for i_L and v_L. If not, then we need a negative sign in these equations.

Defining Equations for Inductors, Active and Passive

For the passive sign relationship for i_L and v_L.

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$

and

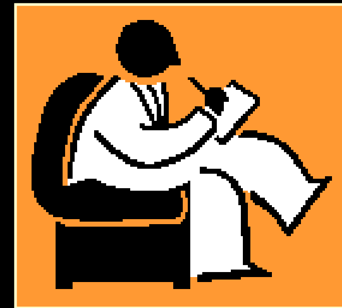
$$v_L = L_X \frac{di_L}{dt}.$$

For the active sign relationship for i_L and v_L.

$$i_L(t) = \frac{-1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$

and

$$v_L = -L_X \frac{di_L}{dt}.$$



Note 1

The implications of these equations are significant. For example, if the current is not changing, then the voltage will be zero. This current could be a constant value, and large, and an inductor will have no voltage across it. This is counter-intuitive for many students. That is because they are thinking of actual coils, which have some finite resistance in their wires. For us, an ideal inductor has no resistance; it simply obeys the laws below.

We might model a coil with both an inductor and a resistor, but for now, all we need to note is what happens with these ideal elements.

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$ and $$v_L = L_X \frac{di_L}{dt}.$$

Step Change

Ask the Step Change question.

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$ and $$v_L = L_X \frac{di_L}{dt}.$$

Note 2

The implications of these equations are significant. Another implication is that we cannot change the current through an inductor instantaneously. If we were to make such a change, the derivative of current with respect to time would be infinity, and the voltage would have to be infinite. Since it is not possible to have an infinite voltage, it must be impossible to change the current through an inductor instantaneously.

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$ and $$v_L = L_X \frac{di_L}{dt}.$$

Energy in Inductors, Derivation

We can take the defining equation for the inductor, and use it to solve for the energy stored in the magnetic field associated with the inductor. First, we note that the power is voltage times current, as it has always been. So, we can write,

$$p_L = \frac{dw}{dt} = v_L i_L = L_X \frac{di_L}{dt} i_L .$$

Now, we can multiply each side by *dt*, and integrate both sides to get

$$\int_0^{w_L} dw = \int_0^{i_L} L_X i_L di_L .$$

Note, that when we integrated, we needed limits. We know that when the current is zero, there is no magnetic field, and therefore there can be no energy in the magnetic field. That allowed us to use 0 for the lower limits. The upper limits came since we will have the energy stored, w_L, for a given value of current, i_L. The derivation continues on the next slide.

Energy in Inductors, Formula

We had the integral for the energy,

$$\int_0^{w_L} dw = \int_0^{i_L} L_X i_L di_L .$$

Now, we perform the integration. Note that L_X is a constant, independent of the current through the inductor, so we can take it out of the integral. We have

$$w_L - 0 = L_X \left(\frac{i_L{}^2}{2} - 0 \right).$$

We simplify this, and get the formula for energy stored in the inductor,

$$w_L = \tfrac{1}{2} L_X i_L{}^2 .$$

Notes

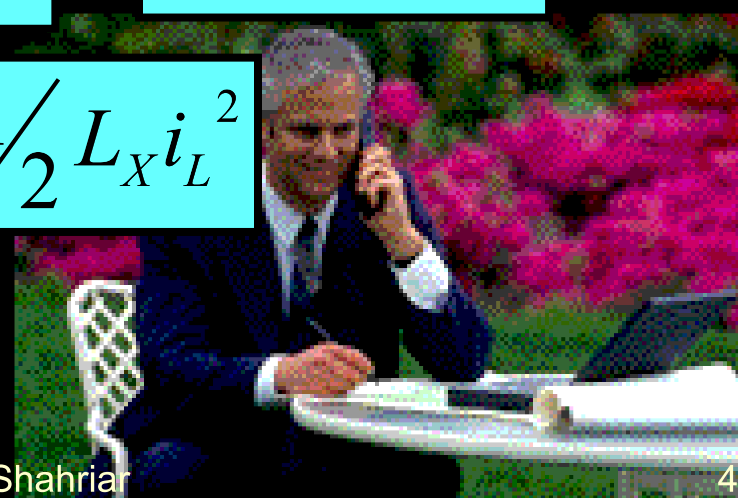
Go back to Overview slide.

1. We took some mathematical liberties in this derivation. For example, we do not really multiply both sides by *dt*, but the results that we obtain are correct here.
2. Note that the energy is a function of the current squared, which will be positive. We will assume that our inductance is also positive, and clearly ½ is positive. So, the energy stored in the magnetic field of an inductor will be positive.
3. These three equations are useful, and should be learned or written down.

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0)$$

$$v_L = L_X \frac{di_L}{dt}$$

$$w_L = \frac{1}{2} L_X i_L^{\,2}$$



Week -16

Page- (489-517)

Capacitors

- A capacitor is a two-terminal circuit element that has a current through its terminals which is proportional to the derivative of the voltage across its terminals.
- The coefficient of this proportionality is the defining characteristic of a capacitor.
- A capacitor is the device that we use to model the effect of electric fields on circuit variables. The energy stored in electric fields has effects on voltage and current. We use the capacitor component to model these effects.

In many cases the idea of two parallel conductive plates is used when we think of a capacitor, since this arrangement facilitates the production of an electric field.

Capacitors – Definition and Units

- A capacitor obeys the expression

$$i_C = C_X \frac{dv_C}{dt}$$

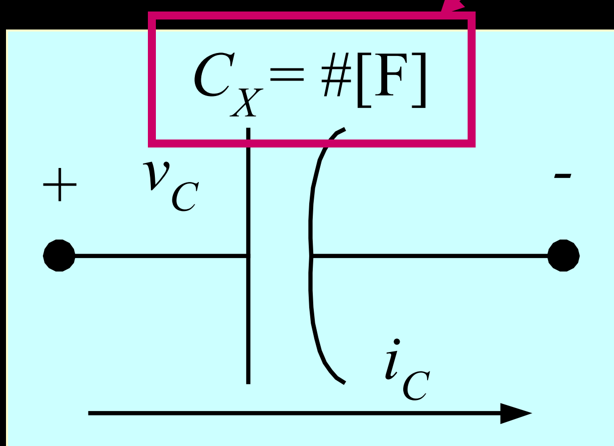
  where v_C is the voltage across the capacitor, and i_C is the current through the capacitor, and C_X is called the capacitance.
- In addition, it works both ways. If something obeys this expression, we can think of it, and model it, as an capacitor.
- The unit ([Farad] or [F]) is named for Michael Faraday, and is equal to a [Ampere-second/Volt]. Since an [Ampere] is a [Coulomb/second], we can also say that a [F] = [C/V].

There is a capacitance whenever we have electric fields produced, and there are electric fields whenever there is a voltage between conductors. However, this capacitance is often negligible.

Schematic Symbol for Capacitors

The schematic symbol that we use for capacitors is shown here.

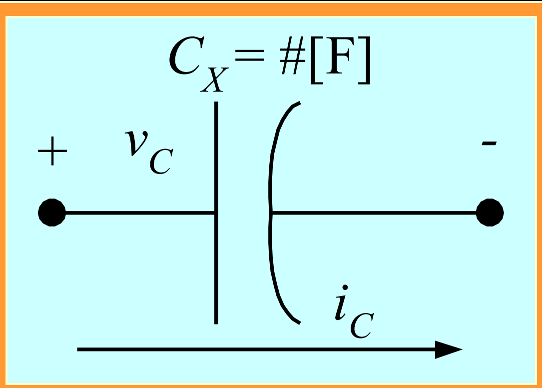
This is intended to indicate that the schematic symbol can be labeled either with a variable, like C_X, or a value, with some number, and units. An example might be 100[mF]. It could also be labeled with both.



$$i_C = C_X \frac{dv_C}{dt}$$

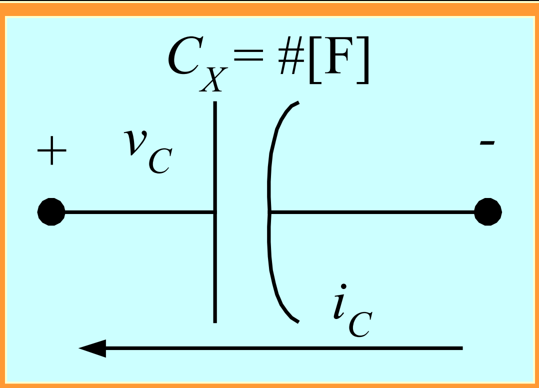
Passive and Active Sign Relationship for Capacitors

The sign of the equation that we use for capacitors depends on whether we have used the passive sign relationship or the active sign relationship.



$$i_C = C_X \frac{dv_C}{dt}$$

Passive Sign Relationship



$$i_C = -C_X \frac{dv_C}{dt}$$

Active Sign Relationship

Defining Equation, Integral Form, Derivation

The defining equation for the capacitor,

$$i_C = C_X \frac{dv_C}{dt}$$

can be rewritten in another way. If we want to express the voltage in terms of the current, we can integrate both sides. We get

$$\int_{t_0}^{t} i_C(t)dt = \int_{t_0}^{t} C_X \frac{dv_C}{dt} dt.$$

We pick t_0 and t for limits of the integral, where t is time, and t_0 is an arbitrary time value, often zero. The capacitance, C_X, is constant, and can be taken out of the integral. To avoid confusion, we introduce the dummy variable s in the integral. We get

$$\frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds = \int_{t_0}^{t} dv_C.$$

We finish the derivation in the next slide.

Defining Equations for Capacitors

$$\frac{1}{C_X}\int_{t_0}^{t} i_C(s)\,ds = \int_{t_0}^{t} dv_C.$$

We can take this equation and perform the integral on the right hand side. When we do this we get

$$\frac{1}{C_X}\int_{t_0}^{t} i_C(s)\,ds = v_C(t) - v_C(t_0).$$

Thus, we can solve for $v_C(t)$, and we have two defining equations for the capacitor,

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)\,ds + v_C(t_0), \quad \text{and} \quad i_C = C_X \frac{dv_C}{dt}.$$

Remember that both of these are defined for the passive sign relationship for i_C and v_C. If not, then we need a negative sign in these equations.

Defining Equations for Capacitors

If we have the passive sign relationship for i_C and v_C then we have

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$

and

$$i_C = C_X \frac{dv_C}{dt}.$$

If we have the active sign relationship for i_C and v_C then we have negative signs in these equations.

$$v_C(t) = \frac{-1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$

and

$$i_C = -C_X \frac{dv_C}{dt}.$$

Note 1

The implications of these equations are significant. For example, if the voltage is not changing, then the current will be zero. This voltage could be a constant value, and large, and a capacitor will have no current through it.

For many students this is easier to accept than the analogous case with the inductor. This is because practical capacitors have a large enough resistance of the dielectric material between the capacitor plates, so that the current flow through it is generally negligible.

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$

and

$$i_C = C_X \frac{dv_C}{dt}.$$

Note 3

Some students are troubled by the introduction of the dummy variable *s* in the integral form of this equation, below. It is not really necessary to introduce a dummy variable. It really doesn't matter what variable is integrated over, because when the limits are inserted, that variable goes away.

The independent variable *t* is in the limits of the integral. This is indicated by the $v_C(t)$ on the left-hand side of the equation.

Remember, the integral here is **not** a function of *s*. It is a function of *t*.

This is a constant.

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$

and

$$i_C = C_X \frac{dv_C}{dt}.$$

Energy in Capacitors, Derivation

We can take the defining equation for the capacitor, and use it to solve for the energy stored in the electric field associated with the capacitor. First, we note that the power is voltage times current, as it has always been. So, we can write,

$$p_C = \frac{dw}{dt} = v_C i_C = v_C C_X \frac{dv_C}{dt}.$$

Now, we can multiply each side by *dt*, and integrate both sides to get

$$\int_0^{w_C} dw = \int_0^{v_C} C_X v_C dv_C.$$

Note, that when we integrated, we needed limits. We know that when the voltage is zero, there is no electric field, and therefore there can be no energy in the electric field. That allowed us to use 0 for the lower limits. The upper limits came since we will have the energy stored, w_C, for a given value of voltage, v_C. The derivation continues on the next slide.

Energy in Capacitors, Formula

We had the integral for the energy,

$$\int_0^{w_C} dw = \int_0^{v_C} C_X v_C dv_C.$$

Now, we perform the integration. Note that C_X is a constant, independent of the voltage across the capacitor, so we can take it out of the integral. We have

$$w_C - 0 = C_X \left(\frac{{v_C}^2}{2} - 0 \right).$$

We simplify this, and get the formula for energy stored in the capacitor,

$$w_C = \frac{1}{2} C_X {v_C}^2.$$

Go back to Overview slide.

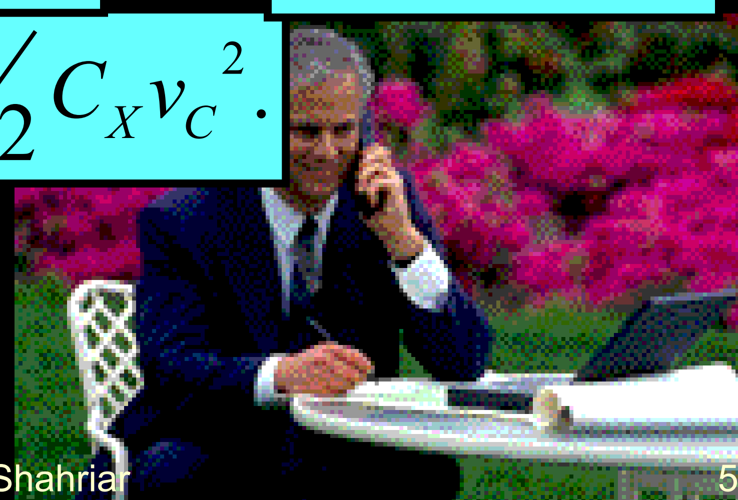
Notes

1. We took some mathematical liberties in this derivation. For example, we do not really multiply both sides by *dt*, but the results that we obtain are correct here.
2. Note that the energy is a function of the voltage squared, which will be positive. We will assume that our capacitance is also positive, and clearly ½ is positive. So, the energy stored in the electric field of an capacitor will be positive.
3. These three equations are useful, and should be learned or written down.

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$

$$i_C = C_X \frac{dv_C}{dt}.$$

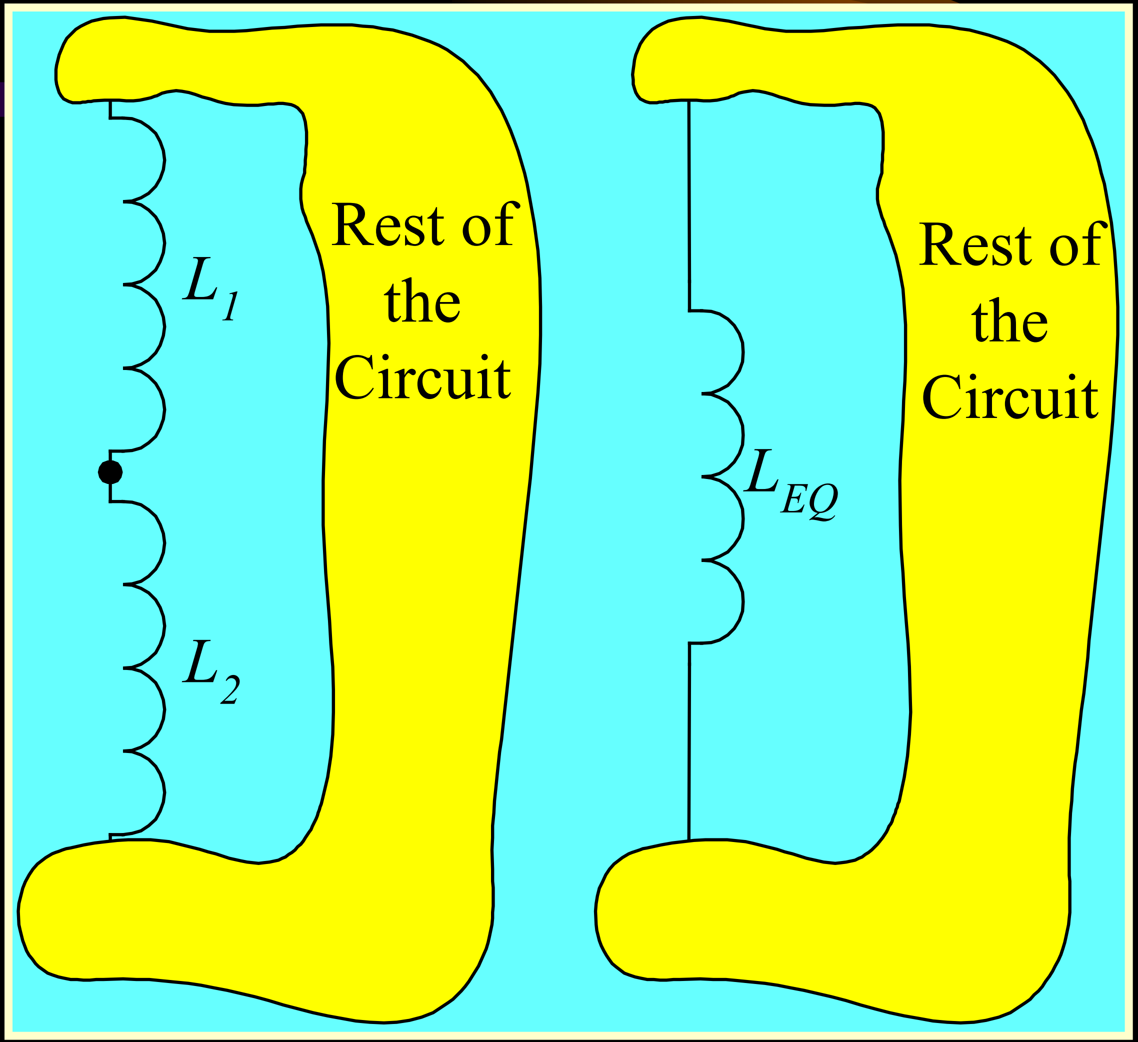
$$w_C = \tfrac{1}{2} C_X {v_C}^2.$$



Series Inductors Equivalent Circuits

Two series inductors, L_1 and L_2, can be replaced with an equivalent circuit with a single inductor L_{EQ}, as long as

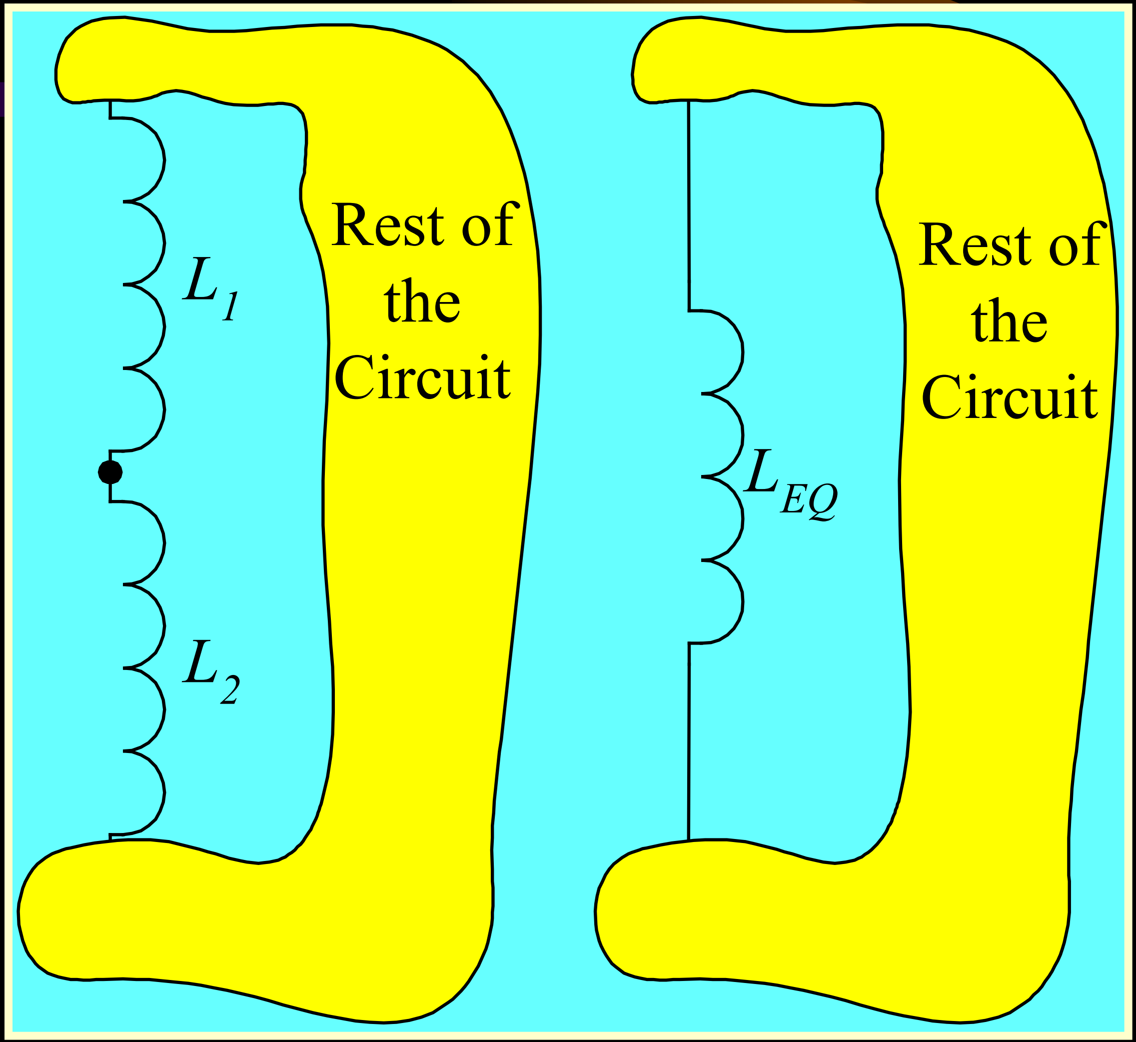
$$L_{EQ} = L_1 + L_2.$$



More than 2 Series Inductors

This rule can be extended to more than two series inductors. In this case, for *N* series inductors, we have

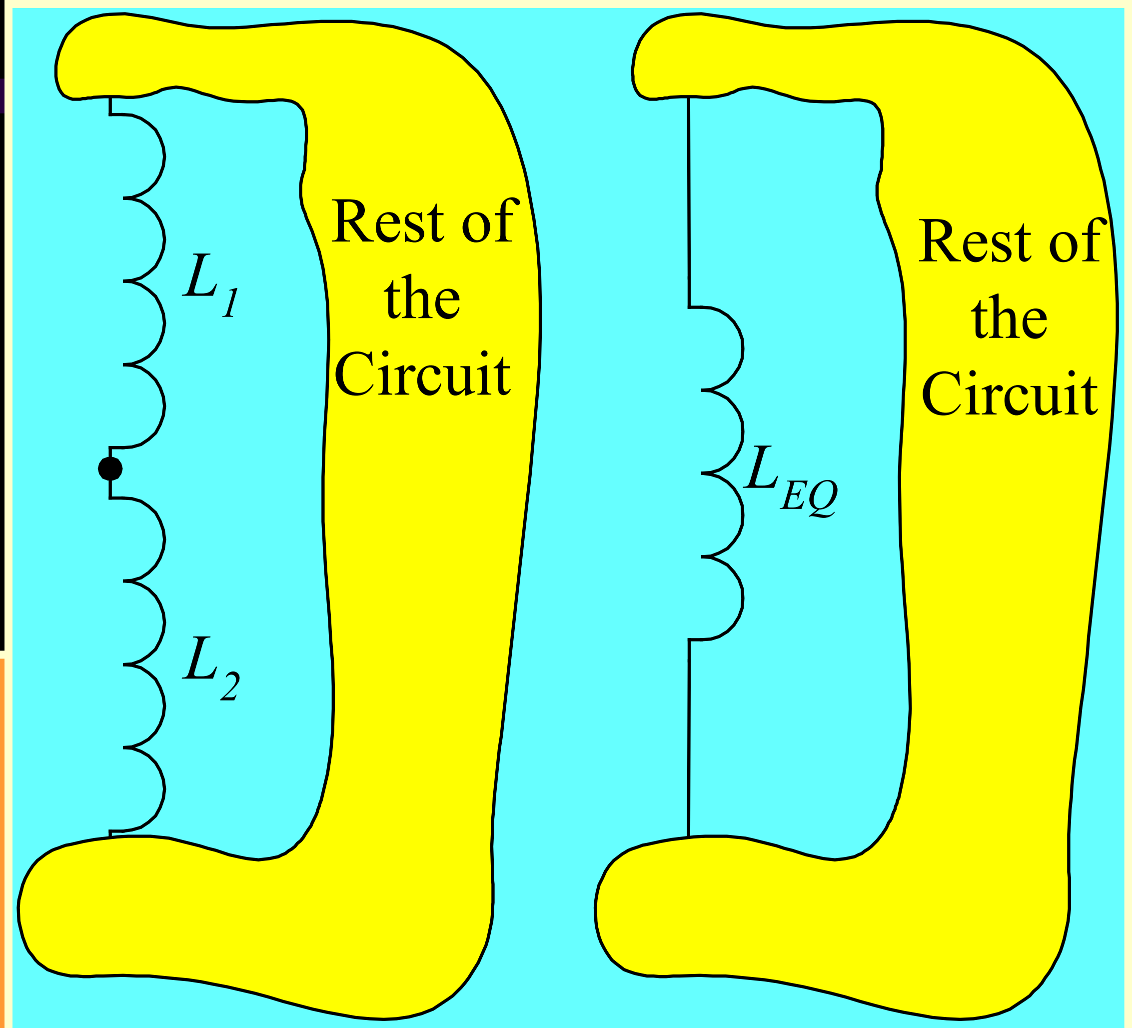
$$L_{EQ} = L_1 + L_2 + \ldots + L_N .$$



Series Inductors Equivalent Circuits: A Reminder

Two series inductors, L_1 and L_2, can be replaced with an equivalent circuit with a single inductor L_{EQ}, as long as

$$L_{EQ} = L_1 + L_2.$$



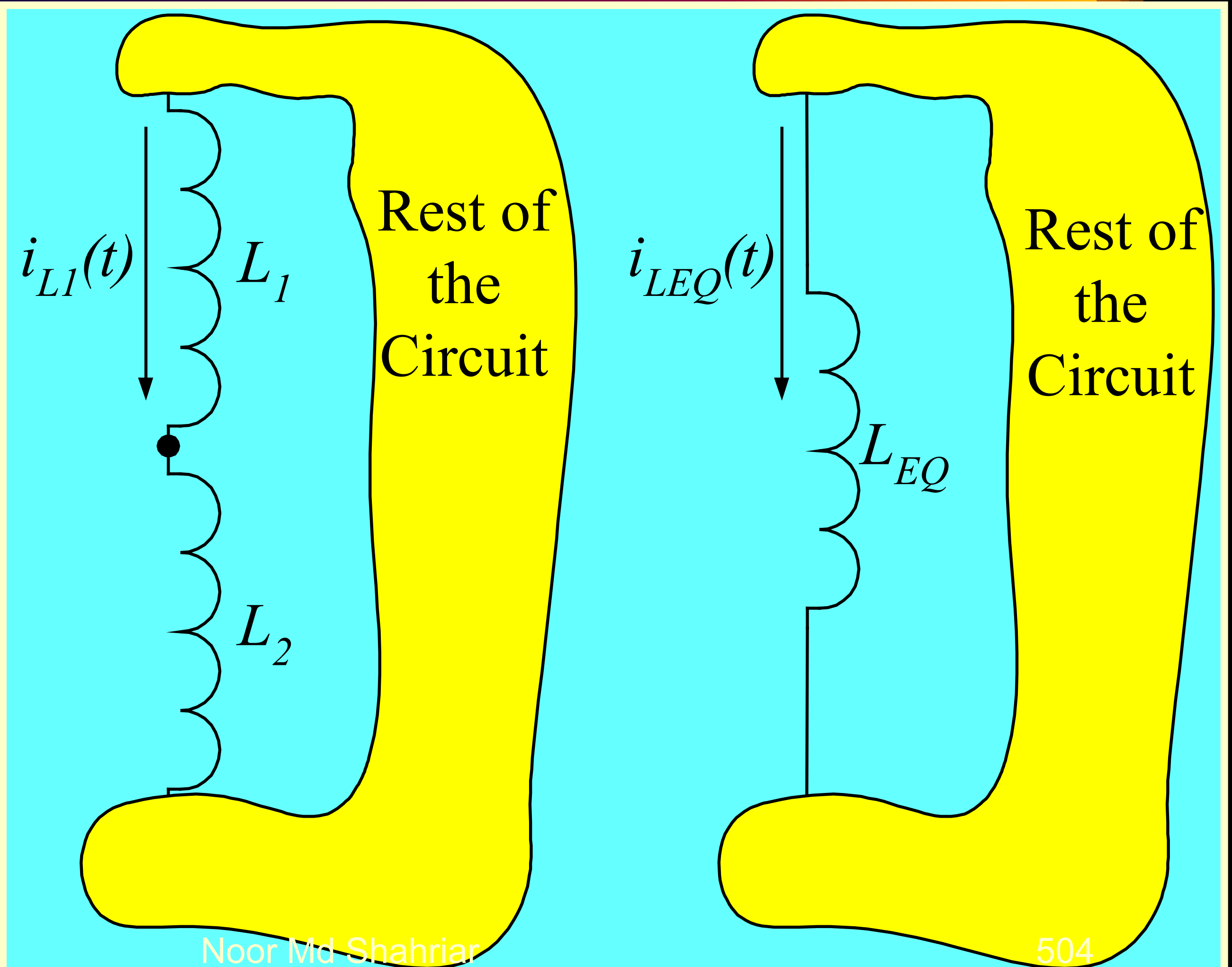
Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)

Series Inductors Equivalent Circuits: Initial Conditions

Two series inductors, L_1 and L_2, can be replaced with an equivalent circuit with a single inductor L_{EQ}, as long as

$$L_{EQ} = L_1 + L_2.$$

To be equivalent with respect to the "rest of the circuit", we must have any initial condition be the same as well. That is, $i_{LEQ}(t_0)$ must equal $i_{L1}(t_0)$.

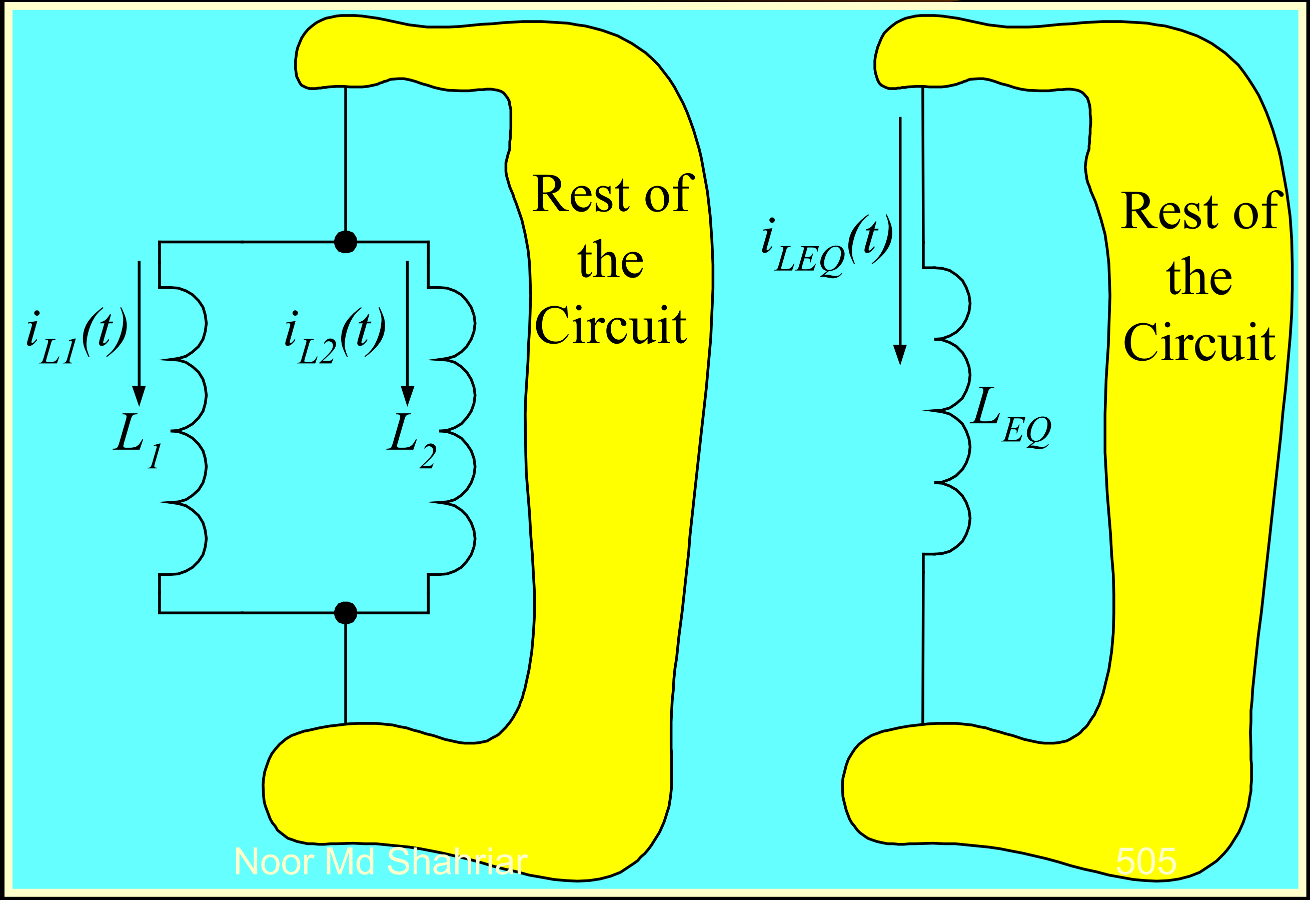


Parallel Inductors Equivalent Circuits

Two parallel inductors, L_1 and L_2, can be replaced with an equivalent circuit with a single inductor L_{EQ}, as long as

$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2}, \text{ or}$$

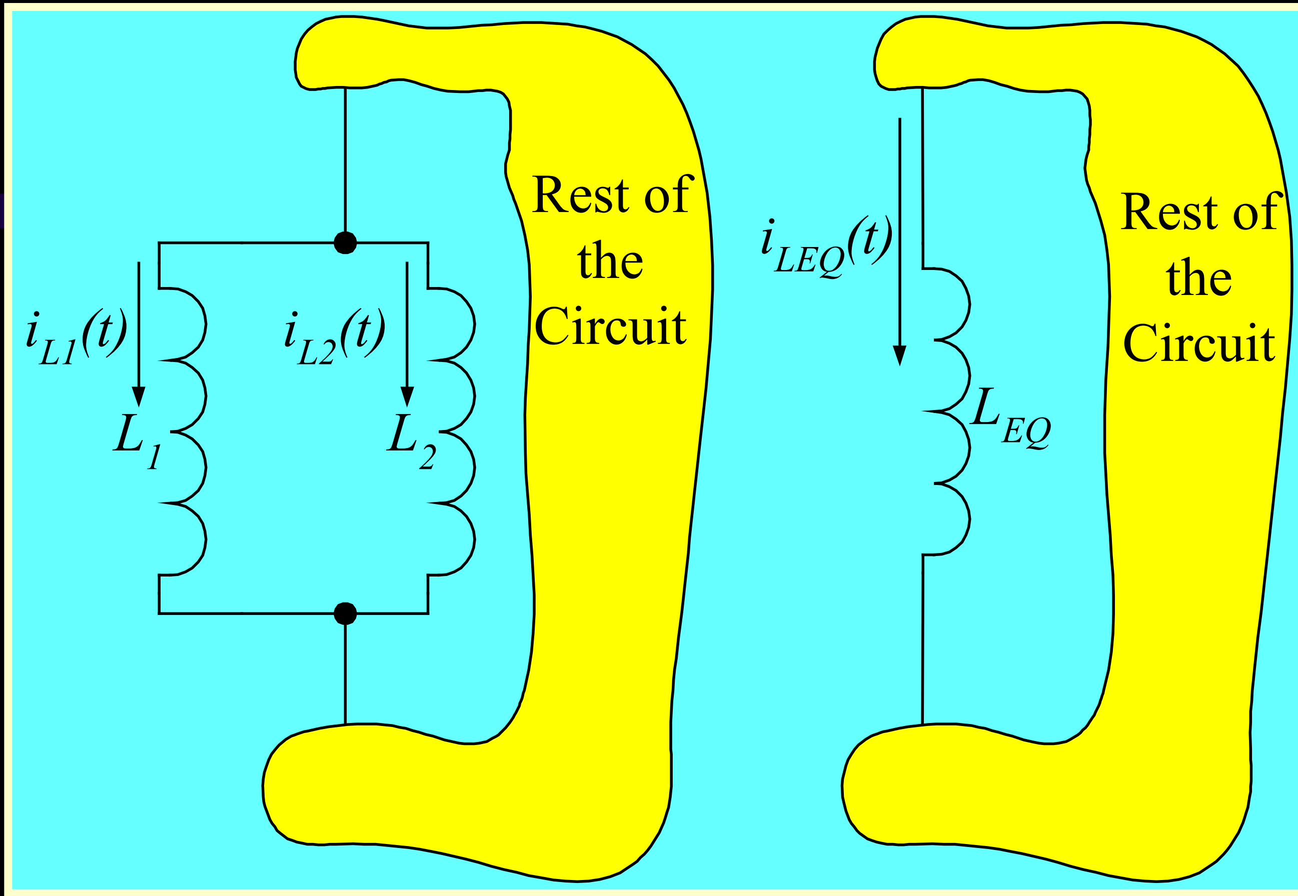
$$L_{EQ} = \frac{L_1 L_2}{L_1 + L_2}.$$



More than 2 Parallel Inductors

This rule can be extended to more than two parallel inductors. In this case, for N parallel inductors, we have

$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}.$$



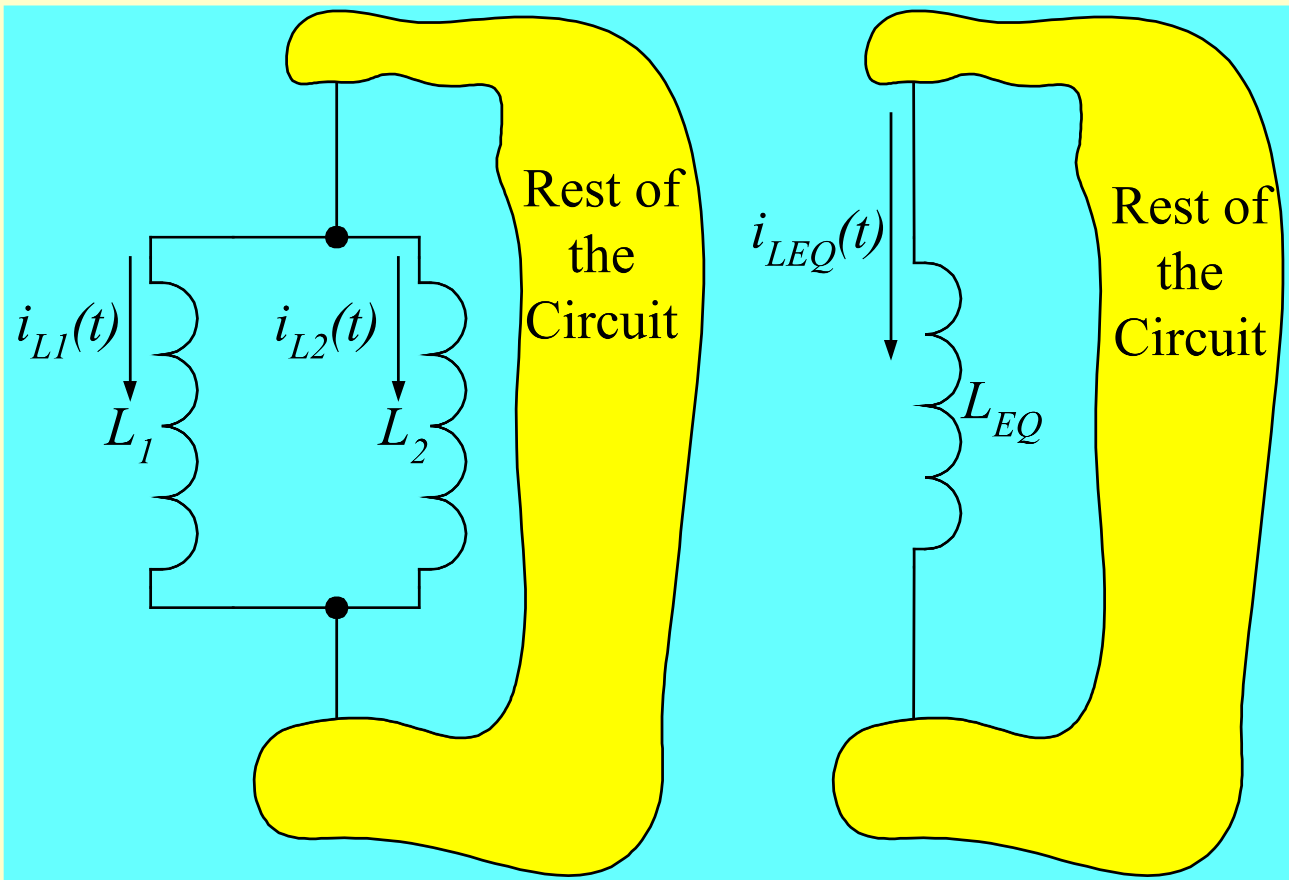
The product over sum rule only works for two inductors.

Parallel Inductors Equivalent Circuits: A Reminder

Two parallel inductors, L_1 and L_2, can be replaced with an equivalent circuit with a single inductor L_{EQ}, as long as

$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2}, \text{ or}$$

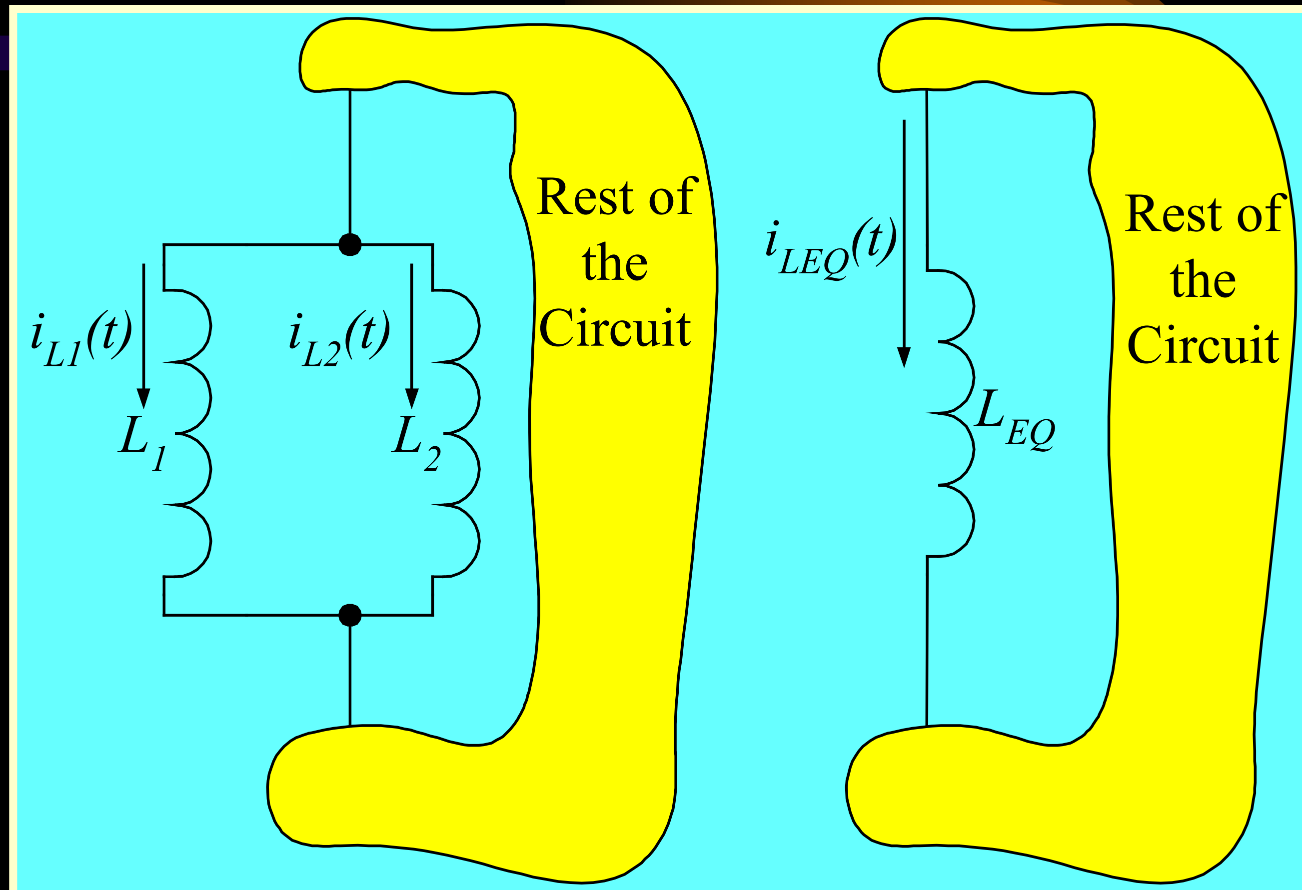
$$L_{EQ} = \frac{L_1 L_2}{L_1 + L_2}.$$



Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)

Parallel Inductors Equivalent Circuits: Initial Conditions

- To be equivalent with respect to the "rest of the circuit", we must have any initial condition be the same as well. That is,

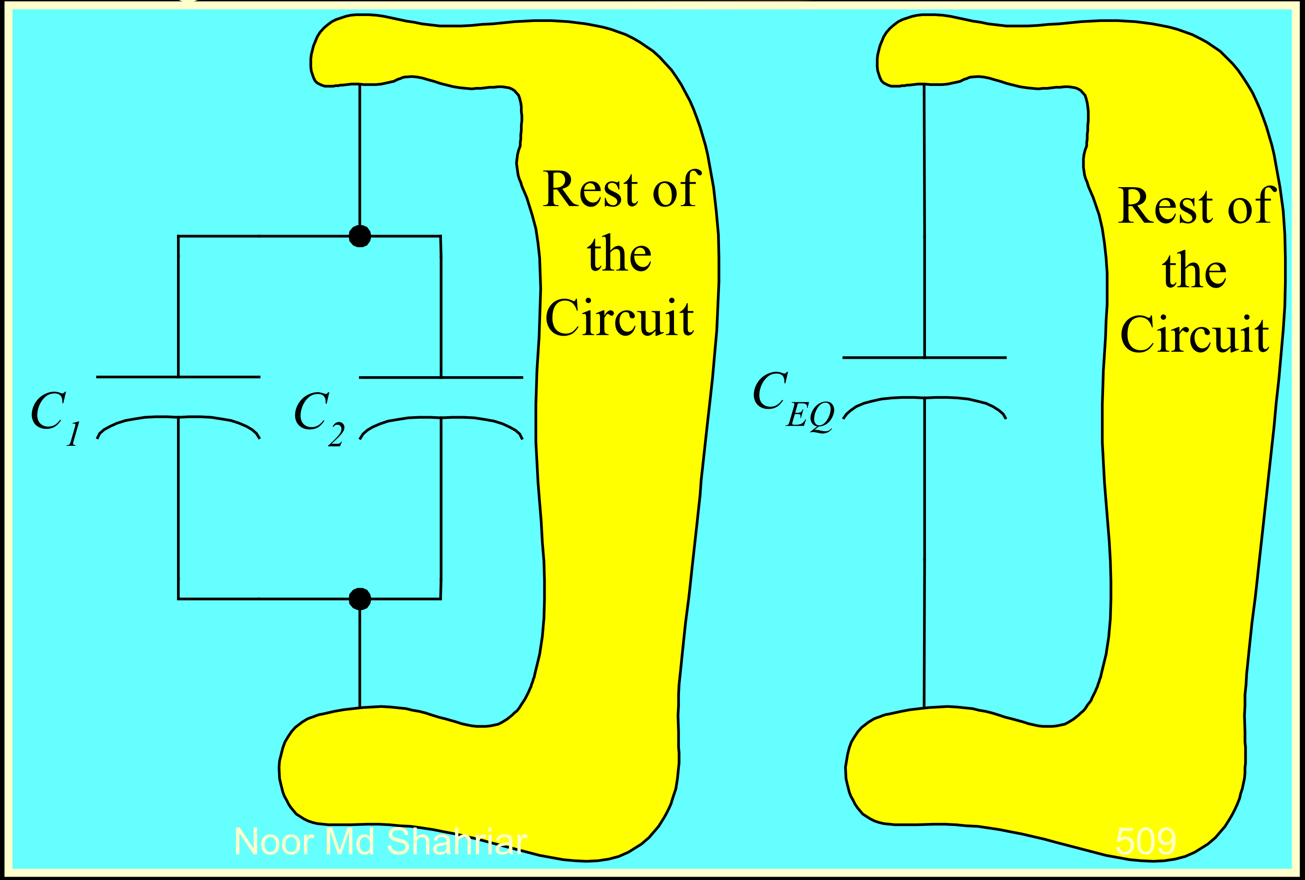


$$i_{LEQ}(t_0) = i_{L1}(t_0) + i_{L2}(t_0).$$

Parallel Capacitors Equivalent Circuits

Two parallel capacitors, C_1 and C_2, can be replaced with an equivalent circuit with a single capacitor C_{EQ}, as long as

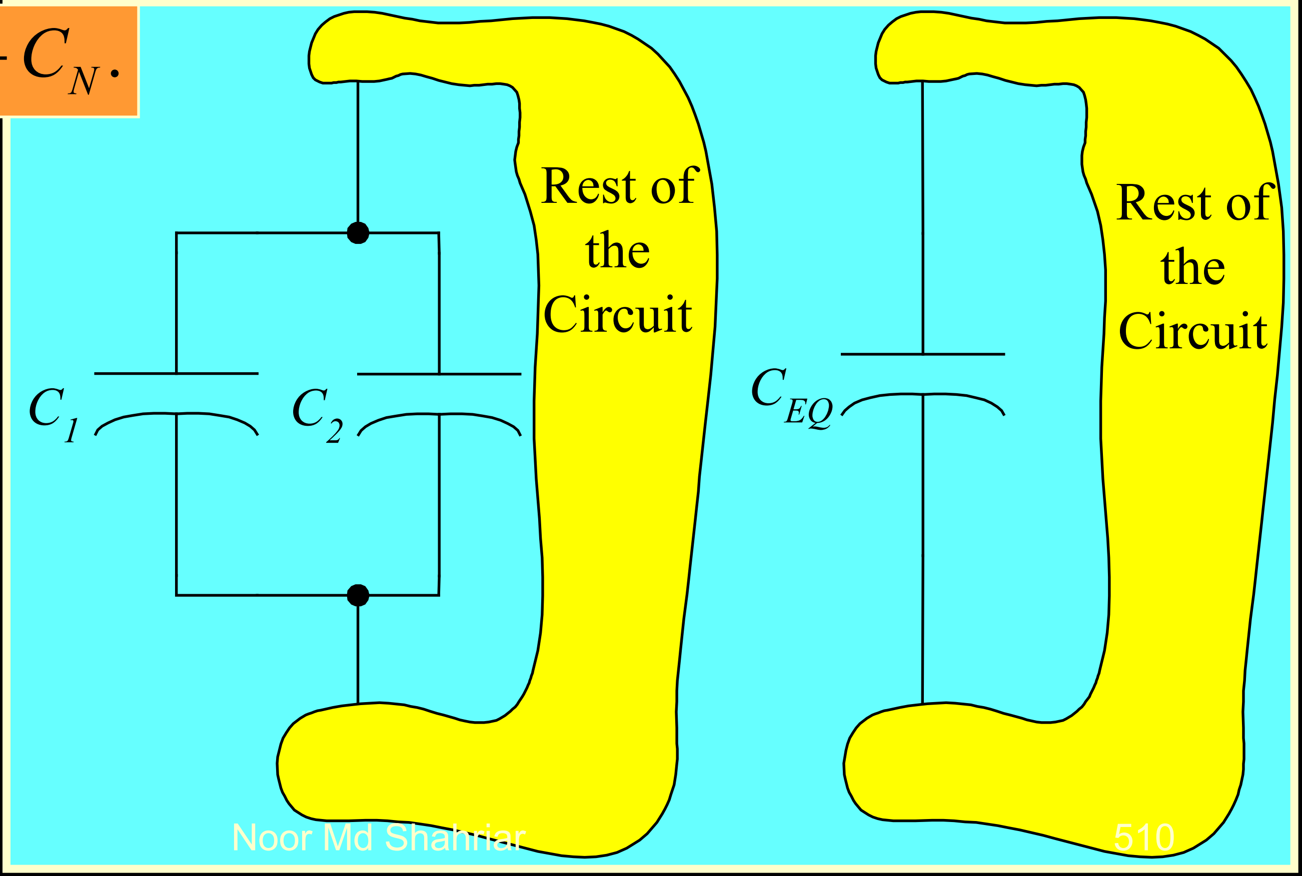
$$C_{EQ} = C_1 + C_2.$$



More than 2 Parallel Capacitors

This rule can be extended to more than two parallel capacitors. In this case, for N parallel capacitors, we have

$$C_{EQ} = C_1 + C_2 + \ldots + C_N .$$

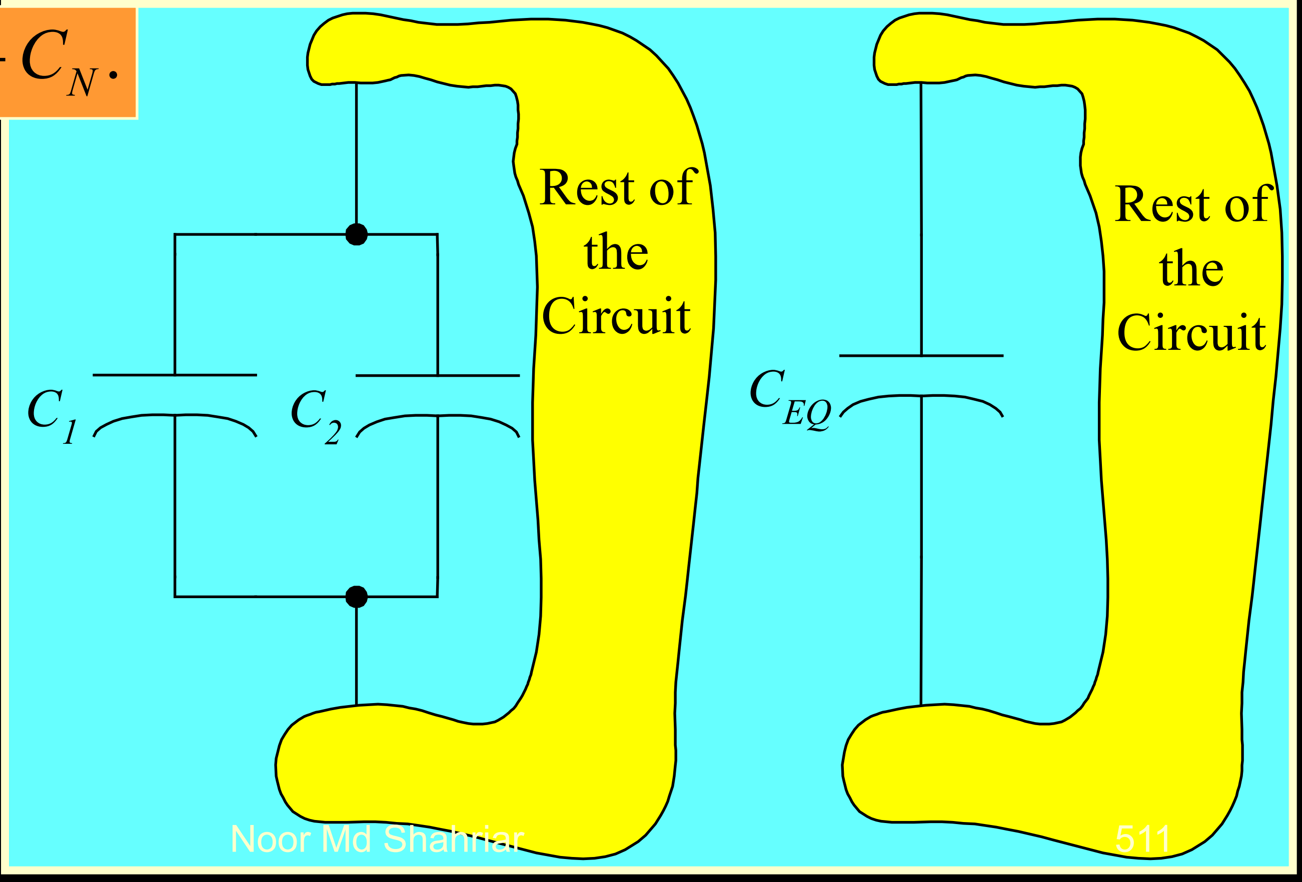


Parallel Capacitors Equivalent Circuits: A Reminder

This rule can be extended to more than two parallel capacitors. In this case, for N parallel capacitors, we have

$$C_{EQ} = C_1 + C_2 + \ldots + C_N .$$

Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)

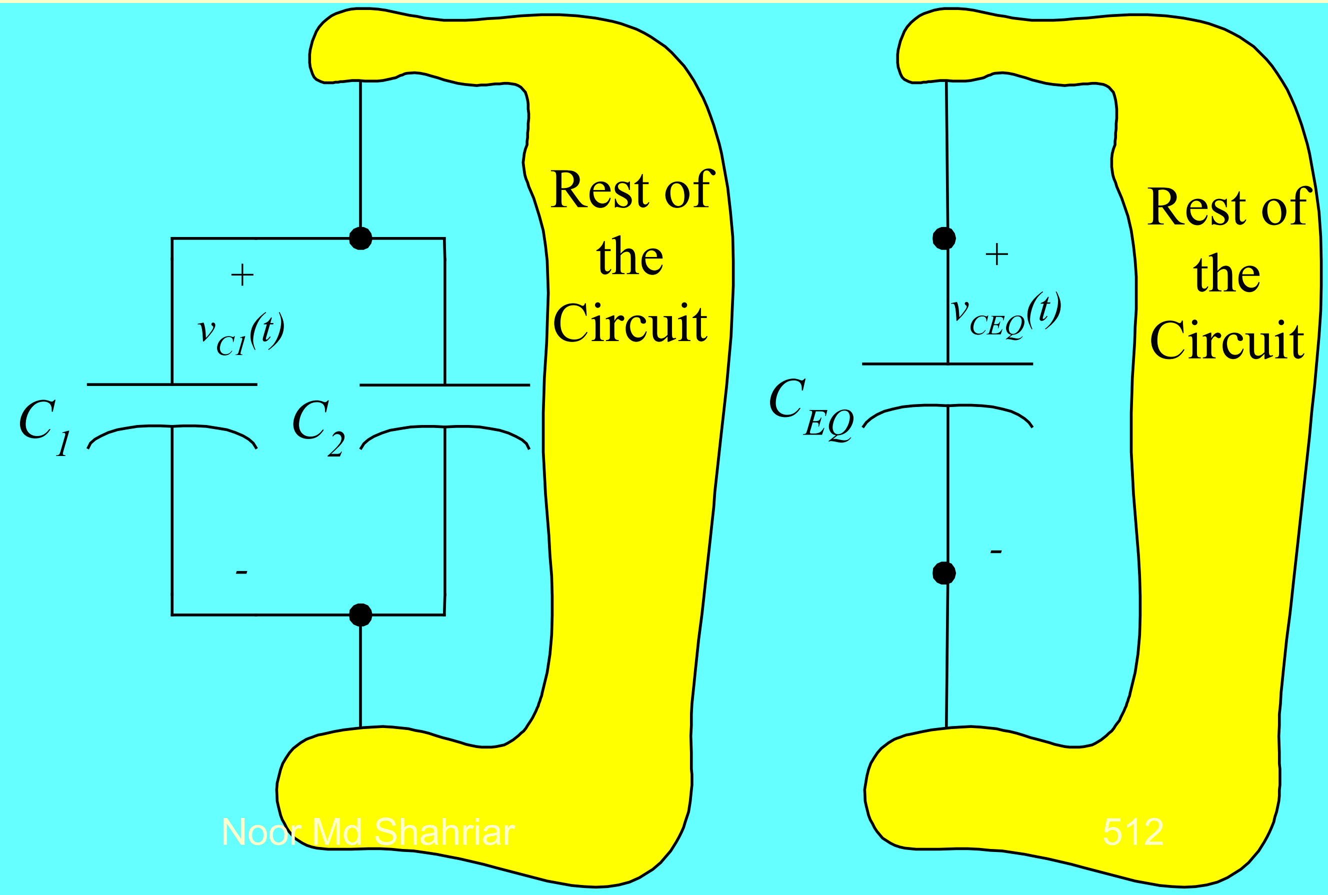


Parallel Capacitors Equivalent Circuits: Initial Conditions

Two parallel capacitors, C_1 and C_2, can be replaced with an equivalent circuit with a single inductor C_{EQ}, as long as

$$C_{EQ} = C_1 + C_2 .$$

To be equivalent with respect to the "rest of the circuit", we must have any initial condition be the same as well. That is, $v_{CEQ}(t_0)$ must equal $v_{C1}(t_0)$.

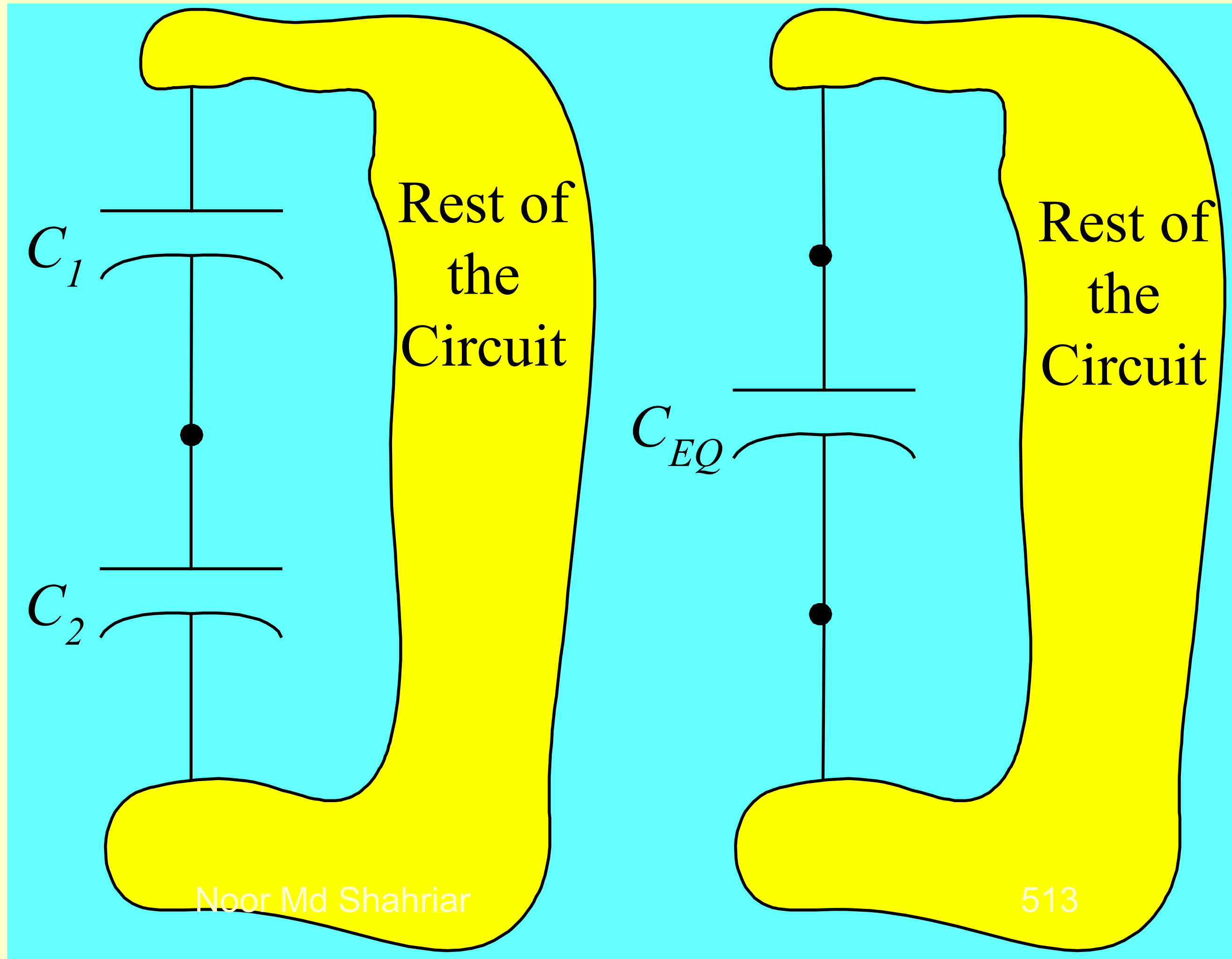


Series Capacitors Equivalent Circuits

Two series capacitors, C_1 and C_2, can be replaced with an equivalent circuit with a single inductor C_{EQ}, as long as

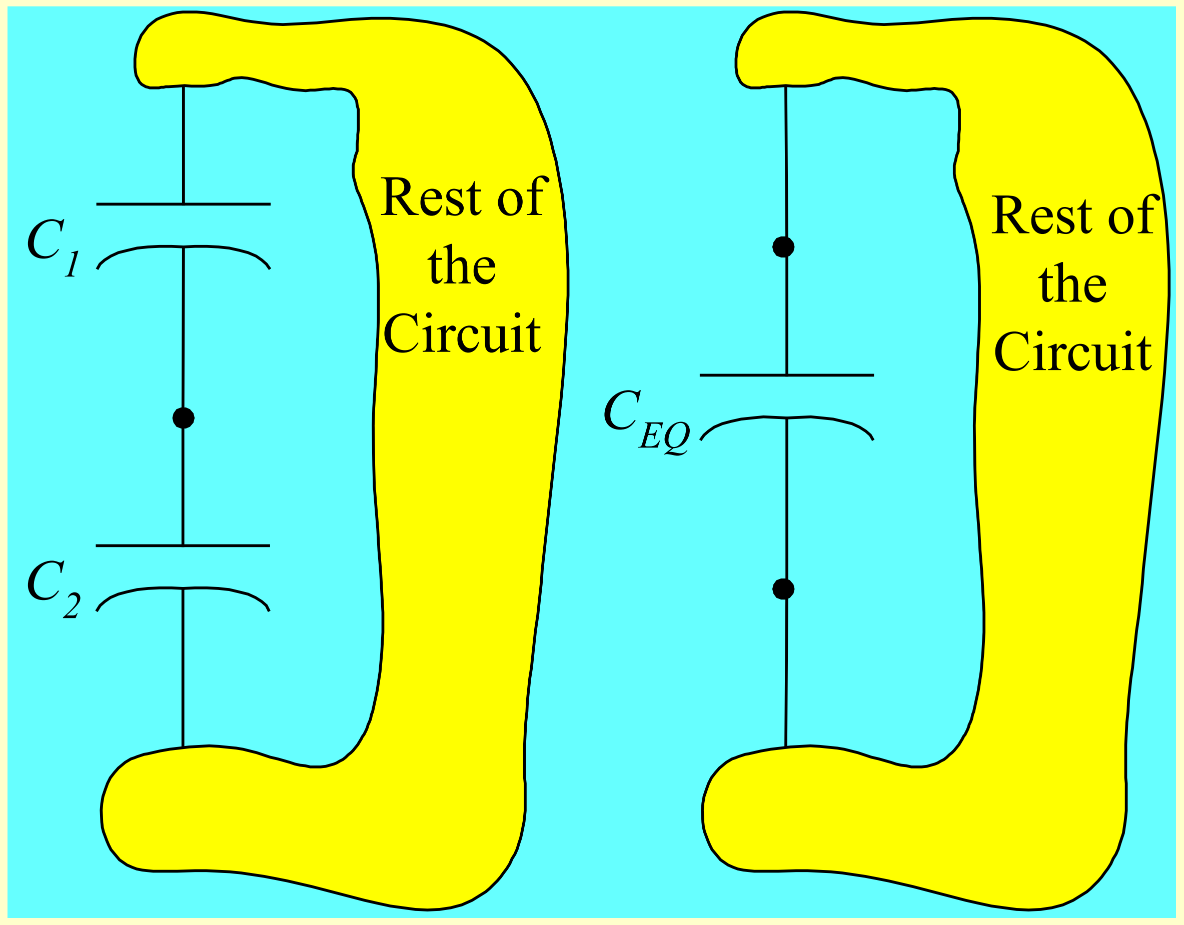
$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2}, \text{ or}$$

$$C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}.$$



More than 2 Series Capacitors

This rule can be extended to more than two series capacitors. In this case, for *N* series capacitors, we have



$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}.$$

The product over sum rule only works for two capacitors.

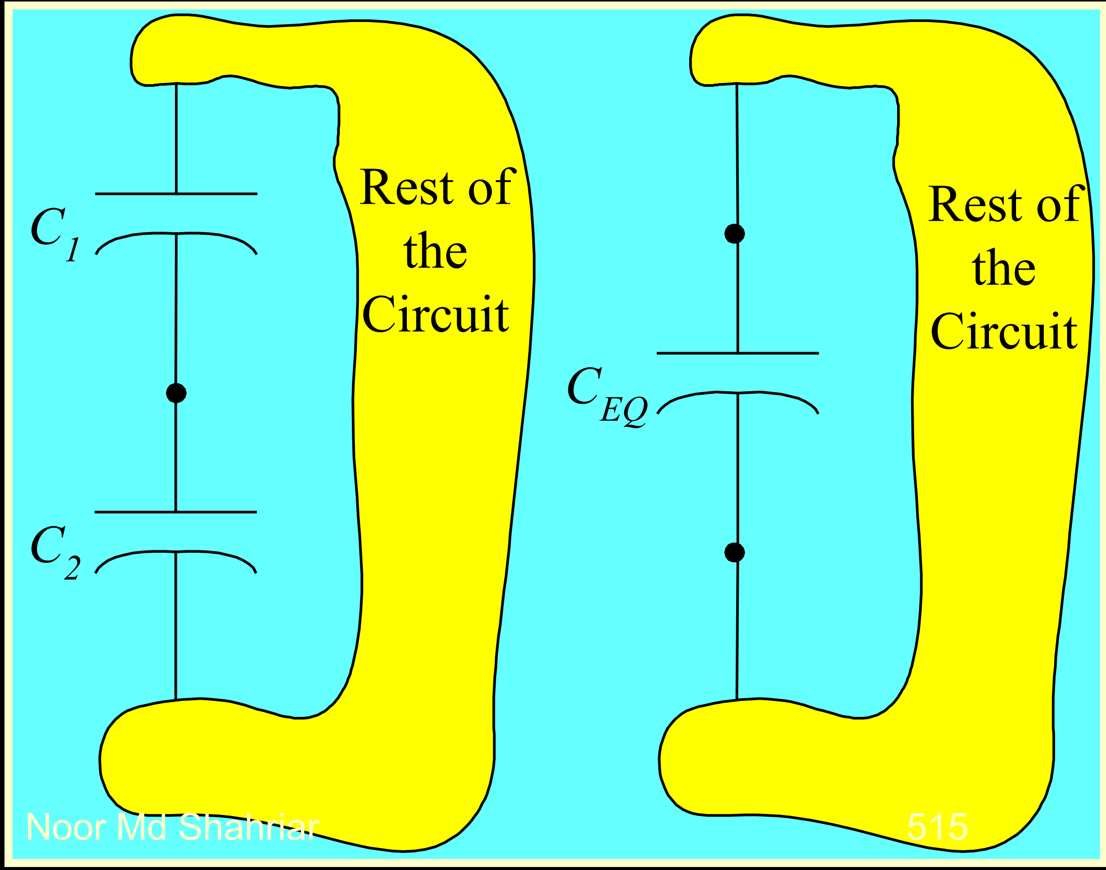
Series Capacitors Equivalent Circuits: A Reminder

Remember that these two equivalent circuits are equivalent only with respect to the circuit connected to them. (In yellow here.)

Two series capacitors, C_1 and C_2, can be replaced with an equivalent circuit with a single capacitor C_{EQ}, as long as

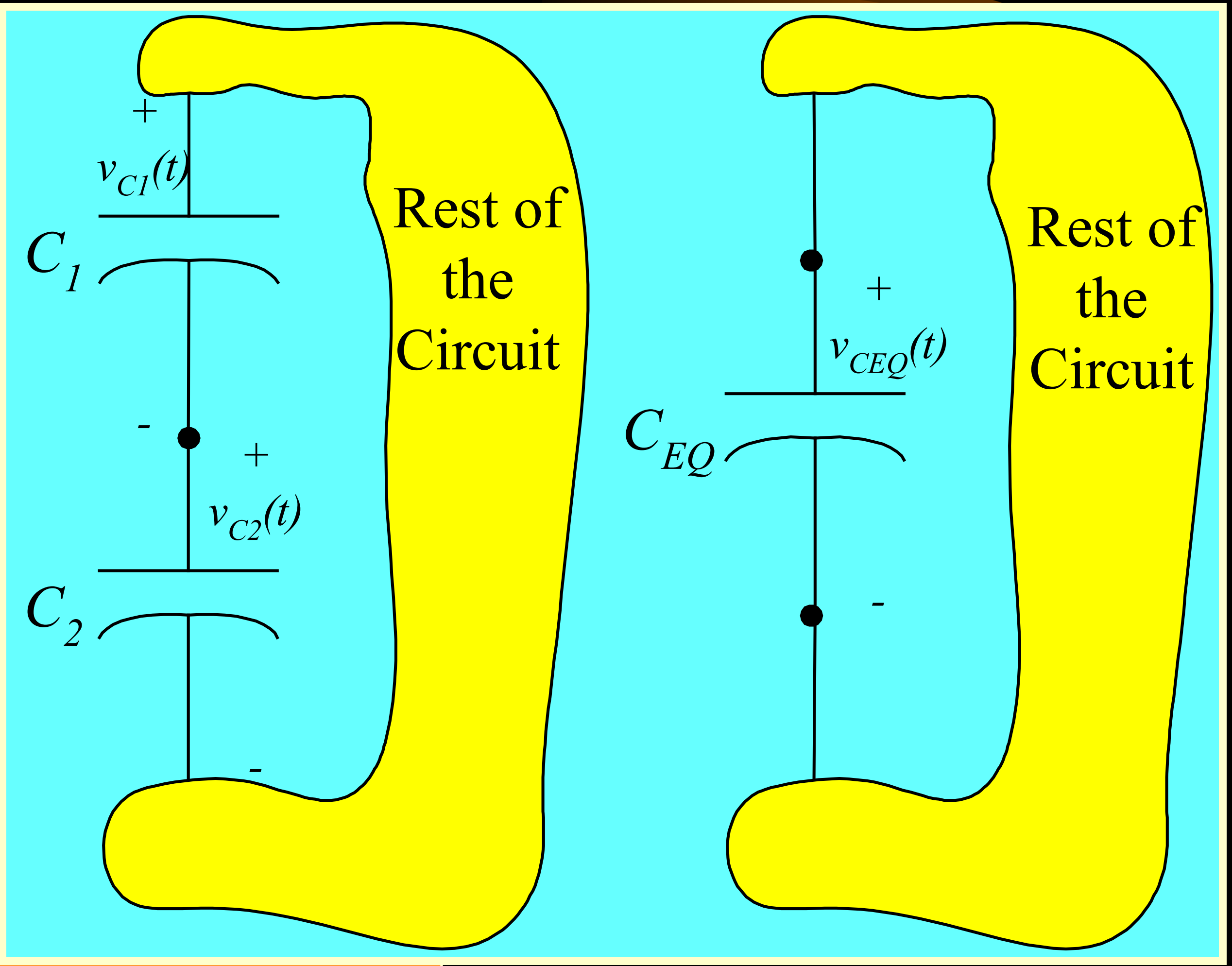
$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2}, \text{ or}$$

$$C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}.$$



Series Capacitors Equivalent Circuits: Initial Conditions

- To be equivalent with respect to the "rest of the circuit", we must have any initial condition be the same as well. That is,



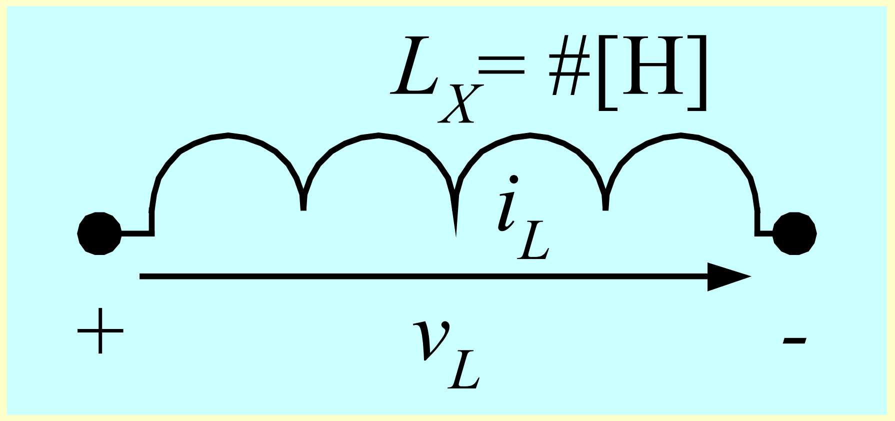
$$v_{CEQ}(t_0) = v_{C1}(t_0) + v_{C2}(t_0).$$

Week -17

Page- (518-527)

Inductor Rules and Equations

- For inductors, we have the following rules and equations which hold:



$$1: v_L(t) = L_X \frac{di_L(t)}{dt}$$

$$2: i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0)$$

$$3: w_L(t) = \left(\tfrac{1}{2}\right) L_X \left(i_L(t)\right)^2$$

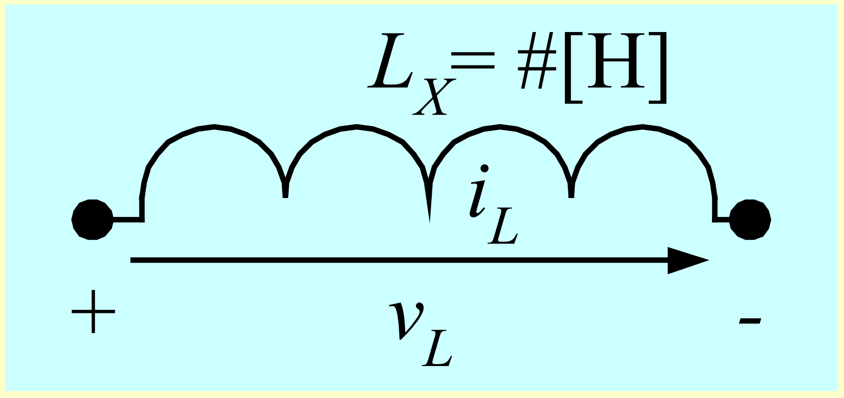
4: No instantaneous change in current through the inductor.

5: When there is no change in the current, there is no voltage.

6: Appears as a short-circuit at dc.

Inductor Rules and Equations – dc Note

- For inductors, we have the following rules and equations which hold:



$$1: v_L(t) = L_X \frac{di_L(t)}{dt}$$

$$2: i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0)$$

$$3: w_L(t) = \left(\tfrac{1}{2}\right) L_X \left(i_L(t)\right)^2$$

4: No instantaneous change in current through the inductor.

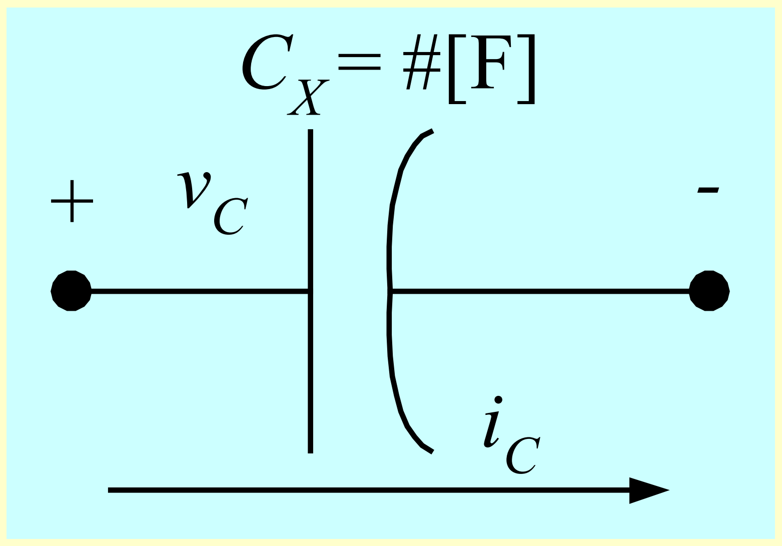
5: When there is no change in the current, there is no voltage.

6: Appears as a short-circuit at dc.

The phrase dc may be new to some students. By “dc”, we mean that nothing is changing. It came from the phrase “direct current”, but is now used in many additional situations, where things are constant. It is used with more than just current.

Capacitor Rules and Equations

- For capacitors, we have the following rules and equations which hold:



$$1: i_C(t) = C_X \frac{dv_C(t)}{dt}$$

$$2: v_C(t) = \frac{1}{C_X} \int_{t_0}^{t} i_C(s)ds + v_C(t_0)$$

$$3: w_C(t) = \left(\frac{1}{2}\right) C_X \left(v_C(t)\right)^2$$

4: No instantaneous change in voltage across the capacitor.

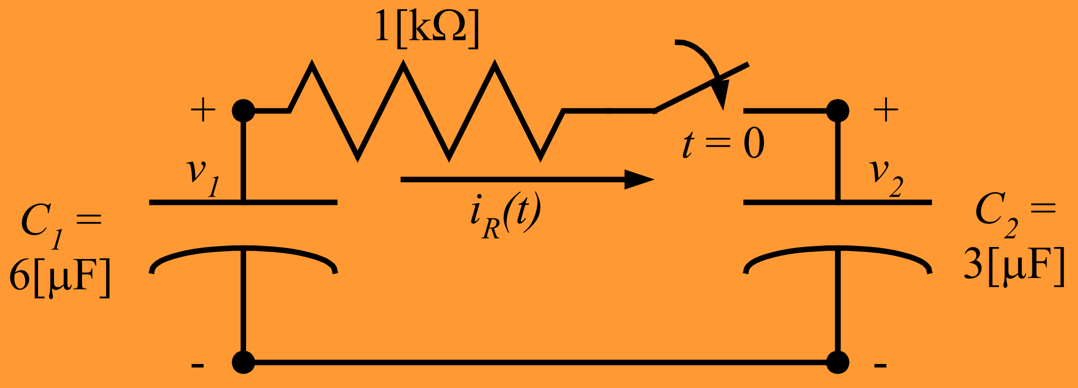
5: When there is no change in the voltage, there is no current.

6: Appears as a open-circuit at dc.

Example Problem #1

1. The circuit shown below has a switch which closed at $t = 0$. The voltages v_1 and v_2 were measured before the switch was closed, and it was found that

$$v_1(t) = 15[\text{V}],\ \text{for } t < 0,\ \text{and } v_2(t) = -7[\text{V}],\ \text{for } t < 0.$$



In addition, for time greater than zero, it was determined that

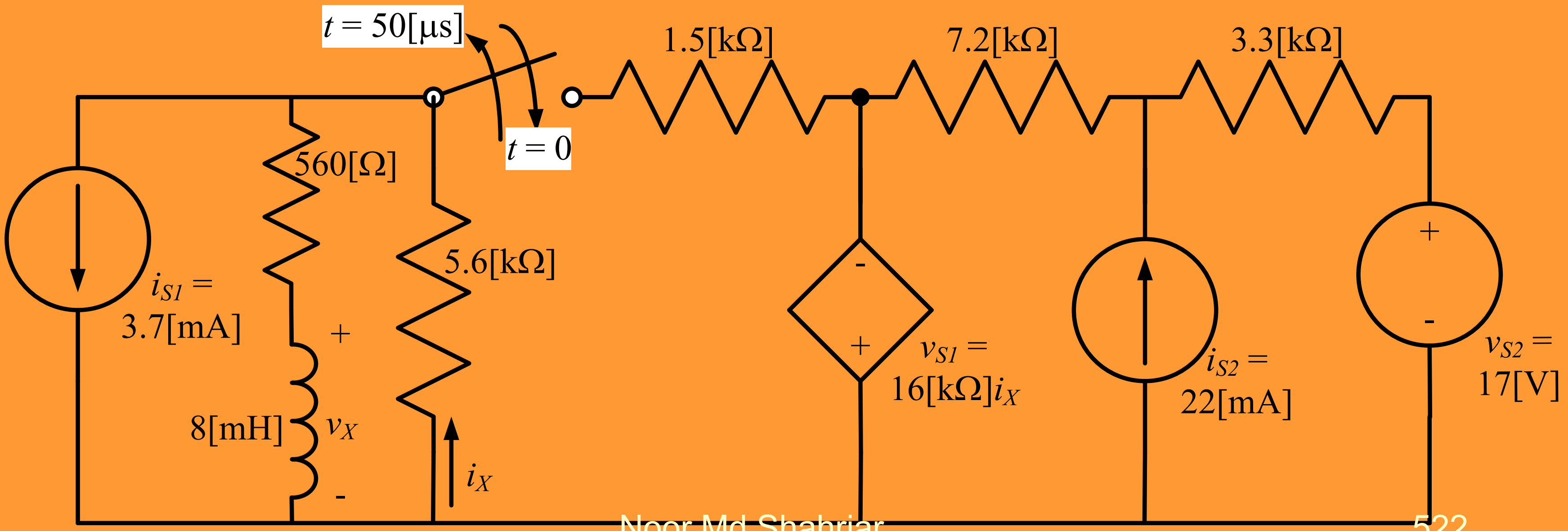
$$i_R(t) = \left(22e^{-500\left[\text{s}^{-1}\right]t}\right)[\text{mA}],\ \text{for } t > 0.$$

Explore the energy stored in the capacitors for $t < 0$, and for $t = \infty$.

Example Problem #2

The switch shown had been open for a long time, then closed at $t = 0$, and opened again at 50[μs].

a) Find $i_X(0^-)$.
b) Find $i_X(0^+)$.
c) Find $v_X(0^-)$.
d) Find $v_X(0^+)$.

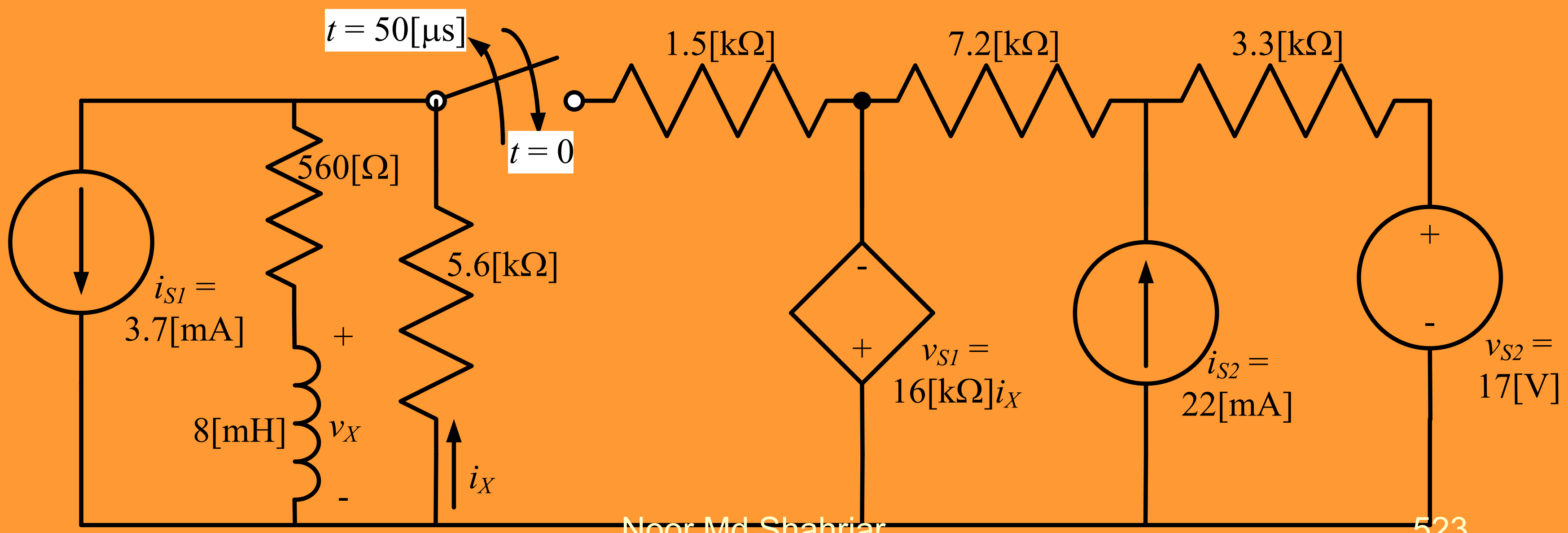


Example Problem #2

The switch shown had been open for a long time, then closed at $t = 0$, and opened again at 50[μs].

a) Find $i_X(0^-)$.
b) Find $i_X(0^+)$.
c) Find $v_X(0^-)$.
d) Find $v_X(0^+)$.

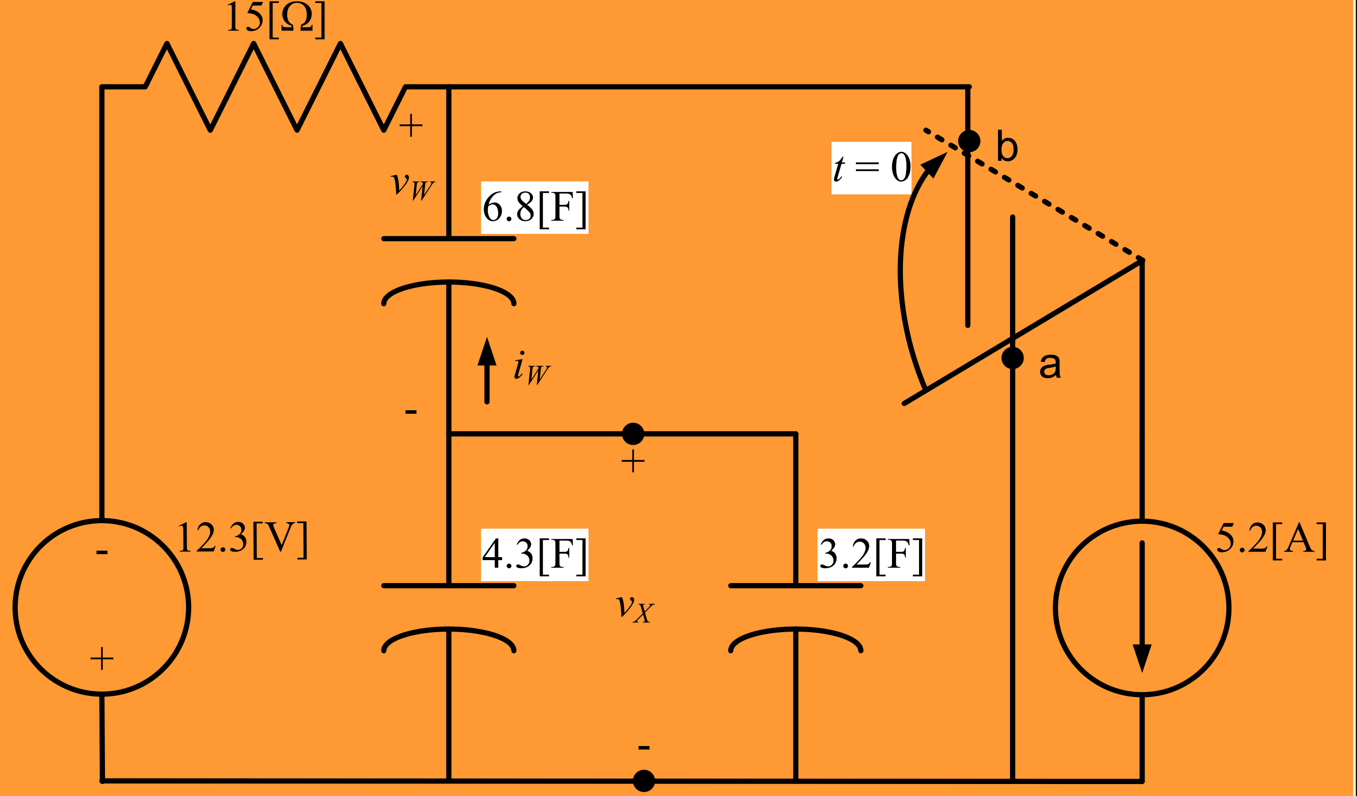
a) $i_X(0^-) = 336.4$[μA]
b) $i_X(0^+) = -56.63$[μA]
c) $v_X(0^-) = 0$
d) $v_X(0^+) = 2.20$[V]



Example Problem #3

For the circuit shown, the switch had been in position a for a long time before moving to position b at $t = 0$. The voltage v_X before $t = 0$ was constant, and equal to -7.34[V].

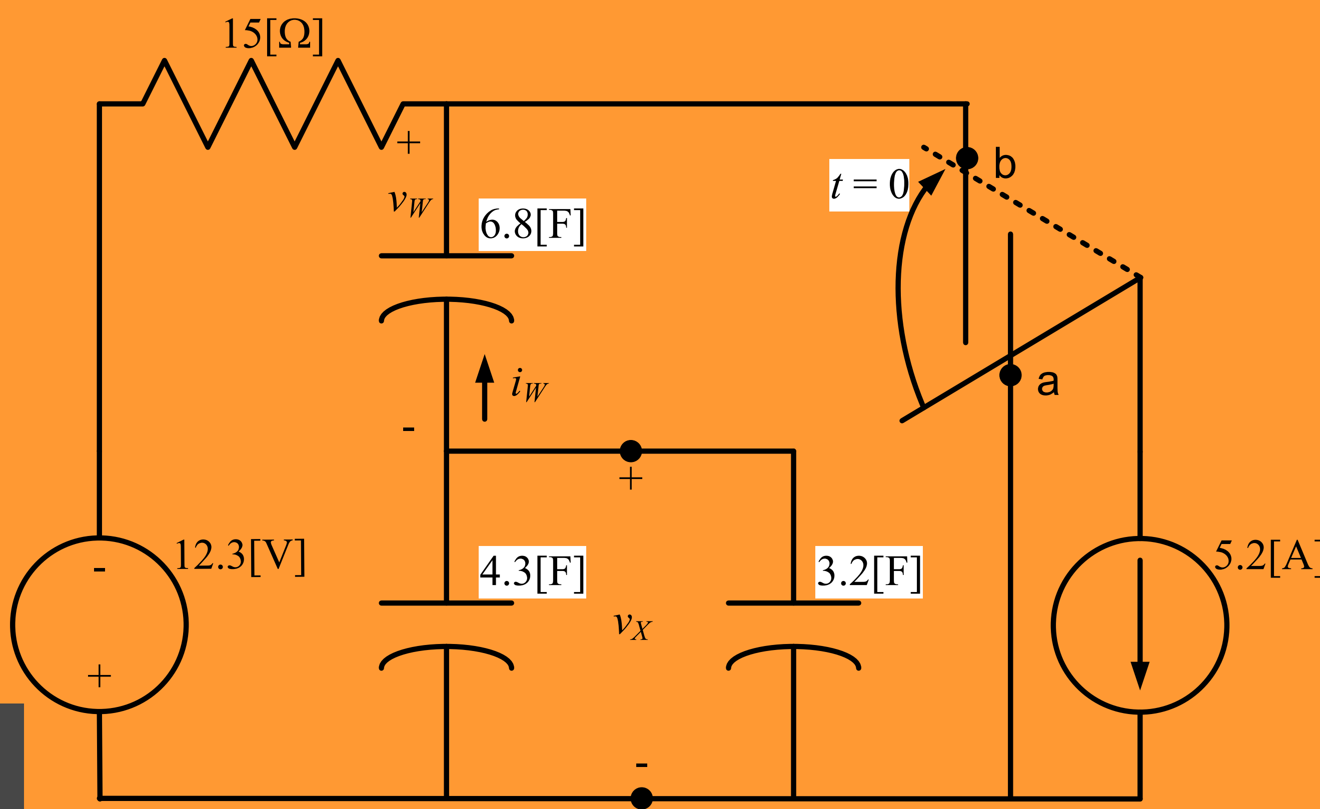
a) Find $v_W(0^-)$.
b) Find $v_W(0^+)$.
c) Find $i_W(0^-)$.
d) Find $i_W(0^+)$.



Example Problem #3

For the circuit shown, the switch had been in position a for a long time before moving to position b at $t = 0$. The voltage v_X before $t = 0$ was constant, and equal to -7.34[V].

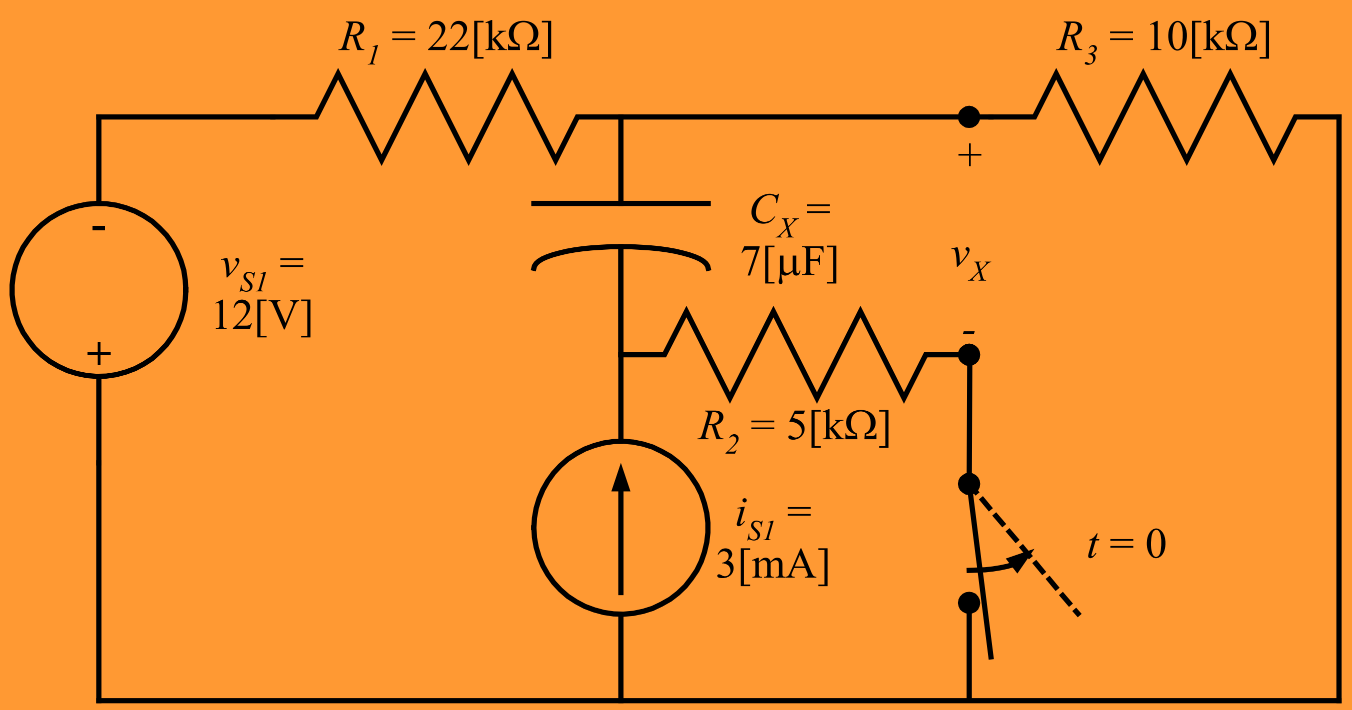
a) Find $v_W(0^-)$.
b) Find $v_W(0^+)$.
c) Find $i_W(0^-)$.
d) Find $i_W(0^+)$.



a) $v_W(0^-) = -4.96$[V]
b) $v_W(0^+) = -4.96$[V]
c) $i_W(0^-) = 0$
d) $i_W(0^+) = 5.2$[A]

Example Problem #4

In the circuit shown, the switch was closed for a long time, before it was opened at $t = 0$. Find v_X(10[ms]).



$$v_X(10[\text{ms}]) = -23.04[\text{V}].$$

DEAR STUDENTS, AS YOU PREPARE FOR YOUR EXAMS, REMEMBER THAT YOUR WORTH IS NOT DEFINED BY A TEST SCORE. YOU ARE TALENTED, CAPABLE, AND DESTINED FOR GREATNESS. BELIEVE IN YOURSELF, GIVE IT YOUR BEST, AND SUCCESS WILL FOLLOW. GOOD LUCK!